1.1(5 points) Interpret each of the slope coefficient estimates $\hat{\beta}_2$ and $\hat{\beta}_4$ in regression equationEq.1; that is, explain in words what the numerical values of the slope coefficient estimates $\hat{\beta}_2$ and $\hat{\beta}_4$ mean.

$\hat{\beta}_2 = -0.9535$

As NOX increases (decreases) by 1% the P will decrease(increases) by 0.9535%, on average.

P is the median house price

NOX is the level of nitrous oxide,

$\hat{\beta}_4 = 0.2545$

As ROOMS increases (decreases) by 1 unit, P will increase (decrease) by $0.2454 \times 100\% = 24.54\%$ on avg.

ROOMS is the avg number of rooms per house

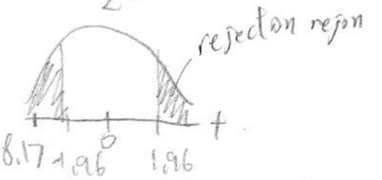
1.2(4 points) Use the estimation results for regression equation Eq.1 to test the individual significance of each of the slope coefficient estimates $\hat{\beta}_2$ for ln(NOX) and $\hat{\beta}_4$ for ROOMS. For each test, state the null and alternative hypotheses, show how you calculate the required test statistic, and draw the conclusion. Which of these two slope coefficient estimates are individually significant at the 5 percent significance level?

$H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\hat{SE}(\beta_2)} = \frac{-0.9535}{0.1167} = -8.17$$

$t_{cri} = t_{\frac{0.05}{2}, 501} \approx t_{0.025,\infty} = 1.96$



since tcal fall into rejection region we reject H_0 that $\beta_2 = 0$ at 0.05 level of significance.

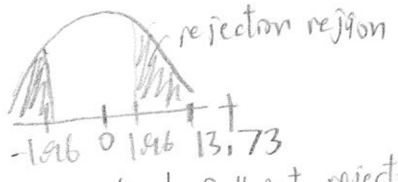
That is β_2 is statistically significant differente from 0 at 0.05 level of significant.

$H_0: \beta_4 = 0$

$H_a: \beta_4 \neq 0$

$$t_{cal} = \frac{\hat{\beta}_4 - \beta_4}{SE(\hat{\beta}_4)} = \frac{0.2545}{0.01853} = 13.73$$

$t_{cri} \approx 1.96$



since tcal fall into rejection region we reject H_0 that $\beta_4 = 0$ at 0.05 level of significance,

That is β_4 is statistically significant difference from 0 at 0.05 level of significant.

That is ln(NOX) is able to explain or That is ROOMS is able to

1.3(5 points) Use the estimation results for regression equation Eq.1 to test the joint significance of all the slope coefficient estimates at the 1 percent significance level (i.e., for significance level $\alpha = 0.01$). State the null and alternative hypotheses, show how you calculate the required test statistic, and state the decision rule you use, and the conclusion you would draw from the test.

$H_0: \beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$

H_a: At least one of β_i is not equal to 0

$$F_{cal} = \frac{ESS/k-1}{RSS/n-k} = \frac{ESS/4}{RSS/501} = \frac{49.3987/4}{35.1835/501} = \frac{12.3497}{0.07}$$

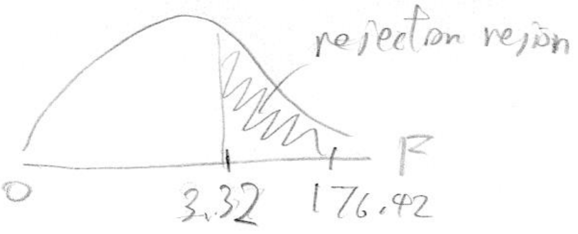
$$= 176.42$$

$TSS = ESS + RSS$

$84.5822 = ESS + 35.1835$

$ESS = 49.3987$

$$F_{cri} = F_{4,501,\,0.01} \approx F_{4,\infty,\,0.01} = 3.32$$



Since Fcal fall into rejection rejon, we reject H_0 that $\beta_2 = \beta_3 = \beta_4 = \beta_5 = 0$ at 0.01 level of significant.

That is at least one of slope coefficient $(\beta_2, \beta_3, \beta_4, \beta_5)$ is statistically significant difference from 0 at 0.01 level of significant

1.4(6 points) Use the estimation results for regression equation Eq.1 to test the proposition that $\beta_2 = \beta_3$, i.e., the marginal effect of ln(NOX) on ln(P) equals the marginal effect of ~~ln(DIST) on ln(P)~~.
Explain in words what this proposition means. Perform the test at the 1 percent significance level.
State the null hypothesis H_0 and the alternative hypothesis H_1. Write the restricted regression equation implied by the null hypothesis H_0.
OLS estimation of this **restricted regression equation** yields a **Residual Sum-of-Squares value of RSS = 41.9532**. Use this information, together with the results from OLS estimation of equation Eq.1, to calculate the required test statistic, and state the decision rule you use, and the conclusion you would draw from the test.

1 $H_0 : \beta_2 = \beta_3$
$H_a : \beta_2 \neq \beta_3$

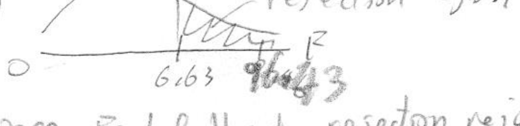
1 Restricted equation is $\ln(P)_i = \beta_1 + \beta_2 \ln(NOX)_i + \beta_2 \ln(Dist)_i + \beta_4 ROOMS_i + \beta_5 START_i + u_i$

$\ln(P)_r = \beta_1 + \beta_2 (\ln(Nox)_i + \ln(Dist)_i) + \beta_4 ROOMS_i + \beta_5 START_i + u_i$

1 This proposition is that the marginal effect of ln(NOX) and ln(DIST) on ln(P) is equal to each other, That is $\beta_2 = \beta_3$

$$F_{cal} = \frac{(RSS_R - RSS_{UR})/m}{(RSS_{UR})/n-k} = \frac{(41.9532 - 35.1835)/1}{35.1835/501} = \frac{6.7697}{0.0702} = 96.43$$

1 $F_{cri} = F^{0.01}_{1,501} \approx F^{0.01}_{1,\infty} = 6.63$

1 

1 since Fcal fall into rejection region we reject H_0 that $\beta_2 = \beta_3$ at 0.01 level of significant. That is the marginal effect of ln(NOX) & ln(DIST) is statistically significant difference from each other.

2 (20 points) Consider an equation to explain salaries of CEOs in terms of annual firm sales, return on equity (ROE in percentage form), and return on the firm's stock (ROS, in percentage form):

$$log(salary) = \beta_0 + \beta_1 log(sales) + \beta_2 ROE + \beta_3 ROS + u$$

2.1 (5 points) In terms of the model parameters, state the null hypothesis that, after controlling for sales and ROE, ROS has no effect on CEO salary. State the alternative hypothesis that better stock market performance increase a CEO's salary.

$H_0 : \beta_3 = 0$
$H_a : \beta_3 > 0$

2.2 (5 points) Using the sample data, the following equation was obtained by OLS:

$$\widehat{\log(\text{salary})} = \underset{(0.3200)}{4.32} + \underset{(0.0350)}{0.280} \log(\text{sale}) + \underset{(0.0041)}{0.0174} \text{ ROE} + \underset{(0.00054)}{0.00024} \text{ ROS} \quad \text{(Eq.2)}$$

$$R^2 = 0.283 \quad n = 209$$

By what percentage is salary predicted to increase if ROS increase by 50 points?
Does ROS have a practically large effect on salary?

If ROS increases by 50 basis point or 0.5 unit, salary will increase by $0.5 \times 0.00024 \times 100 = 0.012\%$ on average.

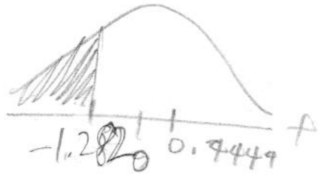
2.3 (5 points) Test the null hypothesis that ROS has no effect on salary against the alternative that ROS has a positive effect. Carry out the test at the 10 %

$H_0\ \beta_3 = 0$

$H_a\ \beta_3 > 0$

$$t_{cal} = \frac{\hat{\beta}_3 - \beta_3}{SE(\hat{\beta}_3)} = \frac{0.00024}{0.00054} = 0.4444$$

$$t_{cri} = t_{0.1,205} \approx t_{0.1,\infty} = -1.282$$



since tcal doesn't fall into rejection rejon, we cannot reject H_0 that $B_3 = 0$ at 0.1 level of significant.
That is ROS isn't statistically significant difference from 0 at 0.1 level of significane.

* On the other hand, we also have no statistical evidence to claim that ROS has positive effect on salary at 0.1 level of si

2.4 (5 points) Would you include ROS in a final model explaining CEO compensation in terms of firm performance? Explain.

Any correct answer according to the hypothesis test in 2.3 will recieve 4 point.

To get the additional 1 point, you need to mention about something eles such as multicolineaity / hetero. or other errors

3 (15 points) Consider the estimated model explaining college GPA (colGPA), with the hight school GPA (hsGPA), the ACT Test (ACT), and the average number of lectures missed ~~per week (skipped)~~

The estimated model is

$$\widehat{\text{colGPA}} = \underset{(0.33)}{1.39} + \underset{(0.094)}{0.412}\ \text{hsGPA} + \underset{(0.011)}{0.015}\ \text{ACT} - \underset{(0.026)}{0.083}\ \text{skipped} \qquad \text{~~(Eq.3)~~}$$

$$R^2 = 0.234 \quad n = 141$$

3.1 (5 points) Using the standard normal approximation, find the 95% confidence interval for β_{hsGPA}.

The 95% confidence interval of β_{hsGPA} is

$\hat{\beta}_{hsGPA} \pm t_{\frac{\alpha}{2}} SE(\hat{\beta}_{hsGPA})$

$0.412 \pm t_{\frac{0.05}{2}} (0.094)$

0.412 ± 0.184

$(0.228,\ 0.596)$

In repeated sampling, 95% of the time, the constructed interval with this method will contain the true value of β_{hs}

3.2 (5 points) Can you reject the hypothesis $H_0 : \beta_{hsGPA} = 0.4$ against the two-sided alternative at the 5% level?

H_0 $\beta_{hsGPA} = 0.4$

H_a $\beta_{hsGPA} \neq 0.4$

$$t_{cal} = \frac{\hat{\beta}_{hsGPA} - \beta_{hsGPA}}{SE(\hat{\beta}_{hsGPA})} = \frac{0.412 - 0.4}{0.094} = 0.1276$$

$$t_{cri} = t_{\frac{0.05}{2}, 137} \approx t_{0.025, \infty} = 1.96$$

since tcal doesn't fall into rejection rejion, we cannot reject Ho that $\beta_{hsGPA} = 0.4$ at 0.05 level of significant. That is, β_{hsGPA} is not statistically significant difference f. at 0.05 lev

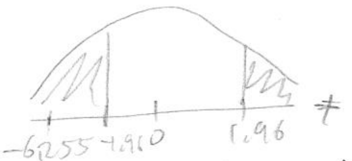
3.3 (5 points) Can you reject the hypothesis $H_0 : \beta_{hsGPA} = 1$ against the two-sided alternative at the 5% level?

H_0 ; $\beta_{hsGPA} = 1$

H_a ; $\beta_{hsGPA} \neq 1$

$$t_{cal} = \frac{\hat{\beta}_{hsGPA} - \beta_{hsGPA}}{SE(\hat{\beta}_{hsGPA})} = \frac{0.412 - 1}{0.094} = -6.255$$

$t_{cri} = 1.96$



since tcal fall into rejection rejion, we reject Ho that $\beta_{hsGPA} = 1$ at 0.05 level of significant. That is, β_{hsGPA} is statistically significant difference from 1 at 0.05 Level of sign