Student Name____ Student ID____________________

EE 325 | Section 02 | Homework Assignment 1

Due date: Tuesday 8th September 2020 before the lecture begins!

1. Find the answers for the following questions

a. $\sum_{i=1}^{5}(a+bx_i)$

b. $\sum_{y=0}^{5} f(x+y)$

c. $\sum_{i=1}^{10} i^2$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

when b is constant number

a. Find the value of b

b. Find the answer for $P(X \leq 2)$

c. Find the answer for $P(-2 \leq X \leq 3)$

d. Find the answer for $\mathrm{P}(\mathrm{X} \geq 1)$

3. **(Optional)** Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

   a. Plot graph for $f(x)$

   b. Find the answer for $P(1 \le X \le 3)$

   c. Find the answer for $P(X \ge 2)$

   d. Find the expected value of X

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.
    a. Construct the joint probability distribution function (PDF) table of X and Y

    b. Find the marginal probability distribution function (PDF) of X

    c. Find the marginal probability distribution function (PDF) of Y

d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

e. Find the expected value of X given Y is equal to 1

f. Find the variance of X given Y is equal to 1

5. Suppose the joint distribution of X and Y is such that $E(X)=5, E(X^2)=30, Var(Y)=81$ $, E(Y|X)=2+3X$

Find the correlation coefficient between X and Y

Are random variables X and Y statistical independence? Why?

6. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent $Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

7. Let $Y \sim N\left(\mu, \sigma^2\right)$ and the estimator for the population mean (μ) have the following two values:

$$\bar{Y} = \sum_{i=1}^{n} \frac{Y_i}{n} \qquad (1)$$

$$\hat{Y} = \frac{n}{\left(n+2\right)^2} \sum_{i=1}^{n} Y_i \qquad (2)$$

Between $\bar{Y}$ and $\hat{Y}$, which one is the better estimator for μ? Why?

8. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

   a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?