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Homework

1. Find Cournot equilibrium when there are 3 firms in the market

$$P = a - bQ \quad ; \quad Q = q_1 + q_2 + q_3$$

$$C_1 = C_2 = C_3 = C$$

What is equivbrium price ? : P^*

What are firms' profit ? $\Pi_1 = \Pi_2 = \Pi_3 = ?$

Firm 1

$$\begin{aligned}\Pi_1 &= TR_1 - TC_1 \\ &= pq_1 - cq_1 \\ &= [a - b(q_1 + q_2 + q_3)]q_1 - cq_1 \\ &= [a - bq_1 - bq_2 - bq_3]q_1 - cq_1\end{aligned}$$

$$\frac{\partial \Pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 - C = 0$$

$$2bq_1 = a - c - bq_2 - bq_3$$

$$q_1 = \frac{a - c - bq_2 - bq_3}{2b}$$ => Reaction Function of Firm 1 (R_1)

Firm 2

$$\begin{aligned}\Pi_2 &= TR_2 - TC_2 \\ &= pq_2 - cq_2 \\ &= [a - b(q_1 + q_2 + q_3)]q_2 - cq_2 \\ &= [a - bq_1 - bq_2 - bq_3]q_2 - cq_2\end{aligned}$$

$$\frac{\partial \Pi_1}{\partial q_1} = a - bq_1 - 2bq_2 - bq_3 - C = 0$$

$$2bq_2 = a - c - bq_1 - bq_3$$

$$q_2 = \frac{a - c - bq_1 - bq_3}{2b}$$ => Reaction Function of Firm 2 (R_2)

Firm 3

$$\pi_3 = TR_3 - TC_3$$
$$= pq_3 - cq_3$$
$$= [a - b(q_1 + q_2 + q_3)]q_3 - cq_3$$
$$= [a - bq_1 - bq_2 - bq_3]q_3 - cq_3$$

$$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 - c = 0$$

$$2bq_3 = a - c - bq_1 - bq_2$$

$$q_3 = \frac{a - c - bq_1 - bq_2}{2b}$$ ⇒ Reaction Function of Firm 3 (R_3)

Substitute R_2 in R_1

$$q_1 = \frac{a - c - \cancel{b}\left[\frac{a-c-bq_1-bq_3}{2\cancel{b}}\right] - bq_3}{2b}$$

$$q_1 = \frac{(2a - 2c - a + c + bq_1 + bq_2 - 2bq_3)/2}{2b}$$

$$q_1 = \frac{a - c + bq_1 - bq_3}{4b}$$

$$4bq_1 = a - c + bq_1 - bq_3$$

$$3bq_1 = a - c - bq_3$$

$$q_1 = \frac{a - c - bq_3}{3b} \quad \text{— (1)}$$

Substitute R_2 in R_3

$$q_3 = \frac{a - c - bq_1 - \cancel{b}\left[\frac{a-c-bq_1-bq_3}{2\cancel{b}}\right]}{2b}$$

$$q_3 = \frac{(2a - 2c - 2bq_1 - a + c + bq_1 + bq_3)/2}{2b}$$

$$q_3 = \frac{a - c - bq_1 + bq_3}{4b}$$

$$4bq_3 = a - c - bq_1 + bq_3$$

$$3bq_3 = a - c - bq_1$$

$$q_3 = \frac{a - c - bq_1}{3b} \quad \text{— (2)}$$

Substitute 2 in 1

$$q_1 = \frac{a - c - \cancel{b}\left[\frac{a - c - bq_1}{3\cancel{b}}\right]}{3b}$$

$$3bq_1 = \frac{3a - 3c - a + c + bq_1}{3}$$

$$9bq_1 = 2a - 2c + bq_1$$

$$8bq_1 = 2a - 2c$$

$$q_1^* = \frac{a - c}{4b} \quad \#$$

Substitute q_1^* in 2

$$q_3 = \frac{a - c - bq_1}{3b}$$

$$q_3 = \frac{a - c - \cancel{b}\left[\frac{a - c}{4\cancel{b}}\right]}{3b}$$

$$3bq_3 = \frac{4a - 4c - a + c}{4}$$

$$q_3 = \frac{3a - 3c}{4(3b)}$$

$$q_3^* = \frac{a - c}{4b} \quad \#$$

Substitute q_1^* & q_2^* in R_2

$$q_2 = \frac{a - c - bq_1 - bq_3}{2b}$$

$$2bq_2 = a - c - \cancel{b}\left[\frac{a - c}{4\cancel{b}}\right] - \cancel{b}\left[\frac{a - c}{4\cancel{b}}\right]$$

$$2bq_2 = \frac{4a - 4c - a + c - a + c}{4}$$

$$q_2 = \frac{2a - 2c}{8b}$$

$$q_2^* = \frac{a - c}{4b} \quad \#$$

$\therefore$ Cournot equilibrium output are $(q_1^*, q_2^*, q_3^*) = \left(\frac{a - c}{4b}, \frac{a - c}{4b}, \frac{a - c}{4b}\right)$

equilibrium price (P^*)

$$P = a - bQ$$

$$P = a - b[q_1 + q_2 + q_3]$$

$$P = a - b\left[\frac{a-c}{4b} + \frac{a-c}{4b} + \frac{a-c}{4b}\right]$$

$$P = a - \cancel{b}\left[3\left(\frac{a-c}{4\cancel{b}}\right)\right]$$

$$P = \frac{4a - 3a + 3c}{4}$$

$$P^* = \frac{a-3c}{4} \quad \#$$

$\therefore$ Cournot equilibrium price (P^*) is $\frac{a-3c}{4}$

Firms' profit ($\Pi_1 = \Pi_2 = \Pi_3$)

$$\Pi_1 = pq_1 - cq_1$$
$$= (p-c)q_1$$
$$= \left[\frac{a+3c}{4} - c\right]\left[\frac{a-c}{4b}\right]$$
$$= \left[\frac{a+3c-4c}{4}\right]\left[\frac{a-c}{4b}\right]$$
$$= \left[\frac{a-c}{4}\right]\left[\frac{a-c}{4b}\right]$$
$$\Pi_1^* = \frac{(a-c)^2}{16b} \quad \#$$

$$\Pi_2 = pq_2 - cq_2$$
$$= (p-c)q_2$$
$$= \left[\frac{a+3c}{4} - c\right]\left[\frac{a-c}{4b}\right]$$
$$= \left[\frac{a+3c-4c}{4}\right]\left[\frac{a-c}{4b}\right]$$
$$= \left[\frac{a-c}{4}\right]\left[\frac{a-c}{4b}\right]$$
$$\Pi_2^* = \frac{(a-c)^2}{16b} \quad \#$$

$$\Pi_3 = pq_3 - cq_3$$
$$= (p-c)q$$
$$= \left[\frac{a+3c}{4} - c\right]\left[\frac{a-c}{4b}\right]$$
$$= \left[\frac{a+3c-4c}{4}\right]\left[\frac{a-c}{4b}\right]$$
$$= \left[\frac{a-c}{4}\right]\left[\frac{a-c}{4b}\right]$$
$$\Pi_3^* = \frac{(a-c)^2}{16b} \quad \#$$

$\therefore$ Cournot equilibrium profit $\Pi_1 = \Pi_2 = \Pi_3 = \frac{(a-c)^2}{16b}$

2. If there are N firm

$$q^*_i = f(N)\ ,\quad P = f(N)\ ,\quad \Pi_i = f(N)$$

$$P = a - bQ \qquad Q = q_1 + q_2 + q_3 + \dots + q_N$$

$$C_1 = C_2 = C_3 = \dots = C_N = C$$

Let's assume $\underbrace{\text{Firm 1}}_{i}$ $\underbrace{\text{Firm 2} \quad \text{Firm 3} \quad \dots \quad \text{Firm N}}_{-i}$

$$Q = q_1 + \underbrace{q_2 + q_3 + \dots + q_N}$$

$$Q = q_1 + q_{-i}$$

Firm i

$$\Pi_i = TR_i - TC_i$$

$$= pq_i - cq_i$$

$$= [a - b(q_i + q_{-i})]\,q_i - cq_i$$

$$\frac{\partial \Pi_i}{\partial q_i} = a - 2bq_i - bq_{-i} - c = 0$$

$$2bq_i = a - c - bq_{-i}$$

$$q_i = \frac{a - c - bq_{-i}}{2b} \Rightarrow \text{Reaction Function of Firm i}$$

from $Q = q_i + q_{-i}$

$q_{-i} = Q - q_i$; Substitute in Reaction function

$$q_i = \frac{a - c - b(Q - q_i)}{2b}$$

$$2bq_i = a - c - bQ + bq_i$$

$$bq_i = a - c - bQ$$

$$q_i = \frac{a - c - bQ}{b}$$

from $C_1 = C_2 = C_3 = \dots = C_N = C$, we know that $q_1 = q_2 = q_3 = \dots = q_n$

Therefore $Q = nq_i$

$$q_i = \frac{a-c-b(nq_i)}{b}$$

$$bq_i = a-c-bnq_i$$

$$bq_i + bnq_i = a-c$$

$$bq_i(n+1) = a-c$$

$$q_i^* = \frac{a-c}{b(n+1)}$$

$\therefore$ Cournot equilibrium output are $(q_1, \ldots, q_N) = \left(\frac{a-c}{b(n+1)}, \ldots, \frac{a-c}{b(n+1)}\right)$ #

$$p = a - bQ$$

$$p = a - b(nq_i)$$

$$p = a - \cancel{b}n\left(\frac{a-c}{\cancel{b}(n+1)}\right)$$

$$p^* = a - n\left(\frac{a-c}{n+1}\right)$$

$\therefore$ Cournot equilibrium price is $a - n\left(\frac{a-c}{n+1}\right)$ #

$$\pi_i = TR_i - TC_i$$

$$= pq_i - cq_i$$

$$= (p-c)q_i$$

$$= \left[a - n\left(\frac{a-c}{n+1}\right) - c\right]\left[\frac{a-c}{b(n+1)}\right]$$

$$\pi_i^* = \left[(a-c) - n\left(\frac{a-c}{n+1}\right)\right]\left[\frac{a-c}{b(n+1)}\right]$$

$\therefore$ Cournot equilibrium profit are $(\pi_1, \ldots, \pi_n) = \left(\left[(a-c) - n\left(\frac{a-c}{n+1}\right)\right]\left[\frac{a-c}{b(n+1)}\right], \ldots, \left[(a-c) - n\left(\frac{a-c}{n+1}\right)\right]\left[\frac{a-c}{b(n+1)}\right]\right)$ #

3. From question 2, what happen if $N \to \infty$

what happen if $N = 1$

from $q_i = \frac{a-c}{b(n+1)}$, $P = a - n\left(\frac{a-c}{n+1}\right)$, $\pi_i = \left[(a-c) - n\left(\frac{a-c}{n+1}\right)\right]\left[\frac{a-c}{b(n+1)}\right]$

If $n \to \infty$ (competitive market) ; ① $q_i = \frac{a-c}{b(n+1)} = \left(\frac{a-c}{b}\right)\left(\frac{1}{n+1}\right)$

if $n \to \infty$, $\frac{1}{n+1} \to 0$ Therefore, $q_i \to \frac{a-c}{b}$ #

② $P = a - n\left(\frac{a-c}{n+1}\right) = a - (a-c)\left(\frac{n}{n+1}\right)$

$$P = \frac{a(n+1) - (a-c)n}{n+1}$$

$$P = \frac{\cancel{an} + a - \cancel{an} + cn}{n+1}$$

$$P - c = \frac{a+cn}{n+1} - c$$

$$P - C = \frac{a + cn - c(n+1)}{n+1}$$

$$P - C = \frac{a + \cancel{cn} - \cancel{cn} - c}{n+1}$$

$$P - C = \frac{a-C}{n+1}$$

if $n \to \infty$, $\frac{a-C}{n+1} \to 0$ Therefore, $P - C = 0$

$P = C$ # (price = marginal cost)

③ $\pi_i = [P-C]q_i$

if $P = C$, $\pi_i = [C-C]q_i$

$\pi_i = 0$ (Normal profit) #

$\therefore$ If $N \to \infty$, $P = MC$, $q_i = \frac{a-c}{b}$ and $\pi_i = 0$ #

If $n=1$; $q_i = \frac{a-c}{b(1+1)} = \frac{a-c}{2b}$ #

(Monopoly)

$$p = a - n\left(\frac{a-c}{n+1}\right)$$

$$= a - \left(\frac{a-c}{1+1}\right)$$

$$= \frac{2a-a-c}{2}$$

$$= \frac{a-c}{2}$$ #

$$\pi_i = \left[(a-c) - n\left(\frac{a-c}{n+1}\right)\right]\left[\frac{a-c}{b(n+1)}\right]$$

$$= \left[(a-c) - (1)\left(\frac{a-c}{1+1}\right)\right]\left[\frac{a-c}{b(1+1)}\right]$$

$$= \left[(a-c) - \left(\frac{a-c}{2}\right)\right]\left[\frac{a-c}{2b}\right]$$

$$= \left[\frac{a-c}{2}\right]\left[\frac{a-c}{2b}\right]$$

$$= \frac{(a-c)^2}{4b}$$ #