EE320 (2/2012)

INTRODUCTORY MATHEMATICAL ECONOMICS

OPTIMIZATION WITHOUT CONSTRAINTS: ONE-INDEPENDENT VARIABLE CASE

Topics

- Profit maximization
  - Competitive market
  - Monopoly
- Effects of taxes
  - Lump-sum tax
  - Profit tax
  - Excise tax
- Maximization of tax revenue

Introduction

- Main application of finding derivatives is to solve optimization problems.
- Recall: The essence of the optimization problem is to choose the best alternative available.
- Two types of optimization problems:
  - Optimization <u>without</u> constraints
    - Eg. Profit-maximization
  - Optimization <u>with</u> constraints
    - Eg. Consumer's utility maximization subject to budget constraint, cost minimization subject to output constraint

Profit Maximization: Perfect Competition

- Let the profit function be $\pi = TR - TC$ where $TR=R(Q\ ); TC=C(Q)$
- Objective function: $\max_Q \pi = R(Q) - C(Q)$
- Perfectly competitive market: P is constant.

  ➔ Total revenue: $TR = P \times Q$
- First-order necessary condition:

  $$\pi'(Q) = R'(Q) - C'(Q) = 0$$

- Second-order sufficient condition:

Graph: π-max in perfect competition

Example: π-max in perfect competition

- Let P = 30; TC = 100 + 19Q – 5Q^2 + (1/3)Q^3. Find Q* which maximizes profit.

Profit Maximization: Monopoly

- Monopoly: Producer/seller can set the price in the market.
- Let $P = f(Q) = a – bQ$
  - Total revenue:

    $TR = P \times Q$
  - Marginal revenue:

    $MR =$
- Profit maximization:
  - <u>Objective function</u>:

    $\max_Q \pi = TR(Q) – TC(Q)$

Graph: π-max in monopoly market

Example: π-max in monopoly market

- Let $P = 48 - 0.5Q$; $TC = 2 + 60Q - 8Q^2 + Q^3$. Find Q* which maximizes profit.

Case of Duopoly [Extra]

- There are two firms, with identical cost: $TC_i = cQ_i$, $i = 1, 2$, and market demand is $P = a - bQ$, $Q = Q_1 + Q_2$.
- Find Q_1, Q_2 that maximizes each firm's profit.

Effect of Taxes

- Given that $\pi = TR(Q) - TC(Q)$, what will happen to profit-maximizing quantity and price if the government imposes:
  - Lump-sum tax
    - *Fixed* tax amount, e.g. $T = t_0$.
  - Profit tax
    - Tax changes according to profit: $T = t\pi$, where $0 < t < 1$.
  - Specific (or excise) tax
    - Tax varies with quantity: $T = tQ$, where $0 < t < 1$.

Effect of Taxes: Lump-Sum Tax

- Net profit after lump-sum tax: $\pi_N =$
- Objective function:

  $\max_Q \pi_N =$

- Profit-maximizing condition:
- Effect of lump-sum tax on profit-max quantity:

Effect of Taxes: Profit Tax

- Net profit after profit tax: $\pi_N =$
- Objective function:

$$\max_Q \pi_N =$$

- Profit-maximizing condition:
- Effect of profit tax on profit-max quantity:

Effect of Taxes: Excise (or Specific) Tax

- Net profit after tax: $\pi_N =$
- Objective function:

  $\max_Q \pi_N =$

- Profit-maximizing condition:
- Effect of specific tax on profit-max quantity:

Tax Revenue Maximization (1)

- Let $P = a - bQ$, so that $TR = aQ - bQ^2$.
- Let $TC = c_0 + c_1Q + c_2Q^2$.
- Suppose the government imposes a specific tax t baht per unit.

➢ After-tax total cost is:

➢ After-tax profit:

➢ Profit-maximizing condition:

➢ *Total* tax revenue:

Tax Revenue Maximization (2)

- Tax-revenue-maximizing condition:

- **Example**: Let $P = 40 – 0.5Q$, and $TC = 2 - 5Q + 7Q^2$. Find the specific tax rate *t* that maximizes the government tax revenue.