Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_ไม้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}\left(u_i\right)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

$$b_1 = \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$
$$= \overline{Y} - b_2 \overline{X}$$

(1.a) From the formula : $\hat{\beta}_2 = \frac{n\Sigma xy - \Sigma x \Sigma y}{n\Sigma x^2 - (\Sigma x)^2}$

$$\hat{\beta}_2 = \frac{(30)(7,524) - (366)(631)}{(30)(5,564) - (366)^2}$$

$$\hat{\beta}_2 = \frac{225,720 - 230,946}{166,920 - 133,956}$$

$$\hat{\beta}_2 = -\frac{5,226}{32,964}$$

$$\hat{\beta}_2 = -0\ 1585$$

From the formula : $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$$\hat{\beta}_1 = 21.03 - (-0.1585)(12.20)$$

$$\hat{\beta}_1 = 22.9637$$

Therefore, $Y_i = 22\ 9637 - 0.1585 X_i$

- $E(Y_1 | X_i = 0) = 22\ 9637$, assume other things being equal $(X_i = 0)$. The average or condition mean of $Y_i = 22.9637$ units

- slope $= \hat{\beta}_2 = -0.1585$, if X increase by 1 unit, Y will decrease by 0.1585 units

(1.b) To find r^2 using $r^2 = 1 - \frac{\Sigma \hat{u}_i^2}{\Sigma (Y_i - \bar{Y})^2}$

$$r^2 = 1 - \frac{873.14}{882.97}$$

$$r^2 = 0.0111$$

Hence, r^2 measures how goodness of fit of the fitted regression line comparing to an estimator x can explian by 0.0111, which is the explain part. In this case, r^2 is equal to 0 0111 which is close to 0.

(1.c) $Y_i = 22.9637 - 0.1585X_i$

when $X_i = 5$: $Y_i = 22.9637 - 0.1585(5)$

$Y_i = 22.1712$

Therefore, when X_i is equal to 5 unit, Y_i will equal to 22.1712 units

(1.d) A formula to calculate $var(u_i) = \frac{\sum \hat{u}_i^2}{n-k}$

$= \frac{873.14}{30-2}$

$= 31.1836$

A formula to calculate $var(\hat{\beta}_1) = \frac{\sum x_i^2}{n \sum x_i^2} \sigma^2$

$= \frac{5,564}{30(5,564)} (31.1836)$

$= 1\ 0395$

A formula to calculate $var(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$

$= \frac{31\ 1836}{5,564}$

$= 0.0056$

(1.e) From $Y_1 = 22.9637 - 0.1585(X_i) + u_1$

- Prove β_1

<u>Step 1</u> : State your hypothesis

$H_0 : \beta_1 = 0$ - null hypothesis

$H_a : \beta_1 \neq 0$ - Althenative hypothesis

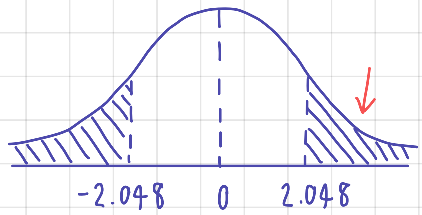
<u>Step 2</u> : Calculate test statistics

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \frac{22.9637 - 0}{1.0196} = 22.5222$$

<u>Step 3</u> : State your decision rule, when $\alpha = 0.05$ use to create CI around $\beta_1 = 0$ based on d.f = 30-2 = 28

The lower bound : $t_{\frac{\alpha}{2}} = -2.048$

The upper bound : $t_{\frac{\alpha}{2}} = 2.048$



<u>Step 4</u> : t_{cal} is 22.5222 which lies beyond 2.048. Hence, we can reject null hypothesis, at the significance level of 95%.

Moreover, β_1 is not zero 95 out of 100 times when we sample

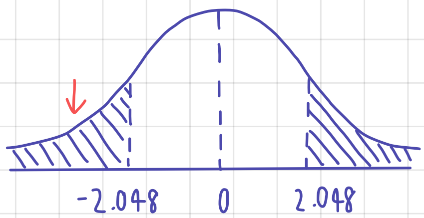
- prove β_2

1. $H_0 : \beta_2 = 0$ - null hypothesis
   $H_a : \beta_2 \neq 0$ - althenative hypothesis

2. $t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \frac{-0.1585 - 0}{0.0748} = -2.1190$

3. State your decision rule, when $\alpha = 0.05$ use to create CI around $\beta_2 = 0$ based on d.f = 30-2 = 28

   - The lower bound : $t_{\frac{\alpha}{2}} = -2.048$
   - The upper bound : $t_{\frac{\alpha}{2}} = 2.048$



4. t_{cal} is -2.1190 which lies beyond CI. Hence, we can reject null hypothesis, at the significance level of 95%.

   Moreover, β_2 is not zero 95 out of 100 times when we sample

(1.f) - Prove β_1

Step 1 : State your hypothesis

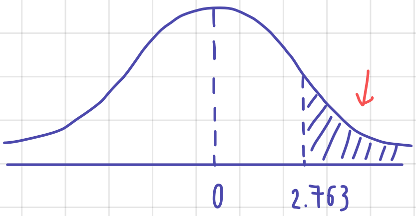
$H_0 : \beta_1 \leqslant 0$ null hypothesis

$H_a : \beta_1 > 0$ althenative hypothesis

Step 2 : $t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \frac{22.9637 - 0}{1.0196} = 22.5222$

Step 3 : when $\alpha = 0.01$ use to create one tail acceptance region when $\beta_1 \leqslant 0$ based on d.f. = 30-2 = 28

- The upper bound : $t_{\frac{\alpha}{2}} = 2.763$



Step4 t_{cal} is 22.5222 which lie beyond 2.763, we can reject the null hypothesis, at the significant level of 99%. we cannot sure that β_1 is less than 0 99 out of 100 times when we sample.

- prove β_2

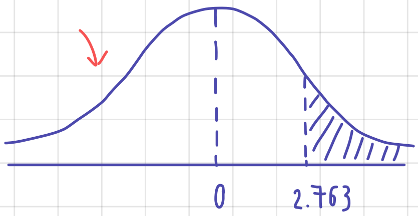
Step 1 $H_0 : \beta_2 \leqslant 0$ null hypothesis

$H_a : \beta_2 > 0$ althenative hypothesis

Step 2 : $t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \frac{-0.1585 - 0}{0.0748} = -2.1190$

Step 3 : when $\alpha = 0.01$ use to create one tail acceptance region when $\beta_2 \leqslant 0$ based on d.f. = 30-2 = 28

- The upper bound : $t_{\frac{\alpha}{2}} = 2.763$



Step4 t_{cal} is -2.1190 which lie whichin any boundary CI, we cannot reject the null hypothesis, at the significant level of 99%. we are sure that β_2 is less than 0 99 out of 100 times when we sample.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

(2.a) Yes, it follows economic sense since $\hat{\beta}_2 = -502.4$ The sign of $\hat{\beta}_2$ is negative meaning that when car is older for 1 year, the market price of car will decrease by 502.4 USD

(2.b) From $\hat{Y}_i = 7836 - 502.4X_i$, $X = 5$

$\hat{Y}_i = 7836 - 502.4(5)$

$\hat{Y}_i = 5,324$

when $\hat{Y}_0$ is an estimator of $E(Y|X_0)$

1. From $var(\hat{Y}_0) = \sigma^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\Sigma(X_i - \bar{X})^2} \right]$

$= (212,877) \left[\frac{1}{11} + \frac{(5 - 7.45)^2}{78.73} \right]$

$= 35,582.5345$

2. Find $\sigma_{\hat{Y}_0} = \sqrt{35,582.5345} = 188.6333$

3 Find 95% CI for $E(Y|X_0 = 5)$

- upper : $\hat{Y}_0 + (t_{\frac{\alpha}{2}} \sigma_{\hat{Y}_0}) = 5324 + (2.262 \cdot 188.6333) = 5750.6885$

- lower : $\hat{Y}_0 - (t_{\frac{\alpha}{2}} \sigma_{\hat{Y}_0}) = 5324 - (2.262 \cdot 188.6333) = 4897.3115$

- Hence, if we create a CI over the mean value, 95 out of 100 times that the CI will cover true value $E(Y|X_0)$.
- when car is 5 yearolds, the market price is between 4897.3115 to 5750.6885 USD

(2.c) $\hat{Y}_i = 7836 - 502.4X_i$

New: $\hat{Y}_i = 7836 - 502.4(10)X_i$

so, $\hat{Y}_i = 7836 - 5024X_i$

when x increase 1 year, the market price of car decreases by 5024 USD

Standard error of the old one is 411.8 and standard error when changed x is 411.8(10) = 4118

Hence, new SRF when multiply x with 10 : $\hat{Y}_i = 7836 - 5024X_i$
se (52) (4118)

(2.d) $\hat{y}_i = 7836 - 502.4x_i$ when $x = 10$, $\hat{y} = 7836 - 502.4(10)$

$\hat{y} = 2812$

$$\frac{d\hat{y}_i}{dx_i} = -502.4$$

$$\left(\frac{dy}{dx}\right) \cdot \frac{x}{y} = -502.4$$

$$\frac{dy}{dx} \cdot \frac{x}{y} = (-502.4)\left(\frac{10}{2812}\right)$$

$$= -1.7866$$

Ans when car is 10 yearolds the elasticity of market price is -1.7866