Last time

$$p = (\alpha - T) - \beta U + g\pi^e \qquad \text{— (4)}$$

$$\frac{d\pi^e}{dt} = j(p - \pi^e) \Rightarrow p(t) = \frac{4}{3}\frac{d\pi^e}{dt} + \pi^e \qquad \text{— (5)}$$

(iv)

$$\frac{dU}{dt} = -k(m - p) \qquad \text{— (6)}$$

Find $p(t)$, $\pi^e(t)$, $U(t)$?

$$\frac{d^2\pi^e}{dt^2} + [\beta k + j(1-g)]\frac{d\pi^e}{dt} + j\beta k\,\pi^e = j\beta k\, m \qquad \text{— (11)}$$

$$\pi^e(t) = e^{-\frac{3}{4}t}\left[A_5 \cos \underbrace{\tfrac{3}{4}t} + A_6 \sin \underbrace{\tfrac{3}{4}t}\right] + m \qquad \text{— (iv)}$$

diff (iv) w.r.t t

$$\frac{d\pi^e}{dt} = -\frac{3}{4}e^{-\frac{3}{4}t}\left[A_5 \cos\tfrac{3}{4}t + A_6 \sin\tfrac{3}{4}t\right] + e^{-\frac{3}{4}t}\left[-\tfrac{3}{4}A_5 \sin\tfrac{3}{4}t + \tfrac{3}{4}A_6\cos\tfrac{3}{4}t\right] \qquad \text{— (v)}$$

sub (iv) and (v) $\Rightarrow$

(v) (iv)

$$p(t) = \frac{4}{3}\frac{d\pi^e}{dt} + \pi^e$$

TRY

$$p(t) = \underbrace{e^{-\frac{3}{4}t}\left(A_6\cos\tfrac{3}{4}t - A_5 \sin\tfrac{3}{4}t\right) + m} \qquad \text{— (vi)}$$

$\hookrightarrow$ this term $\to 0$ as $t \to \infty$

$p(t) \to m$ as m

The inflation rate has a fluctuating time path converging to the equilibrium 'm'

From (i) $U(t) = \frac{1}{3}(\pi^e(t) - p(t)) + \frac{1}{18}$

(iv) (vi)

$$U(t) = \frac{1}{3} e^{-\frac{3}{4}t} \left[(A_5 - A_6)\cos\frac{3}{4}t + (A_5 + A_6)\sin\frac{3}{4}t \right] + \frac{1}{18} \quad \text{—(vii)}$$

The time path of $U(t)$ is damped fluctuation and converges to $\bar{U} = \frac{1}{18}$

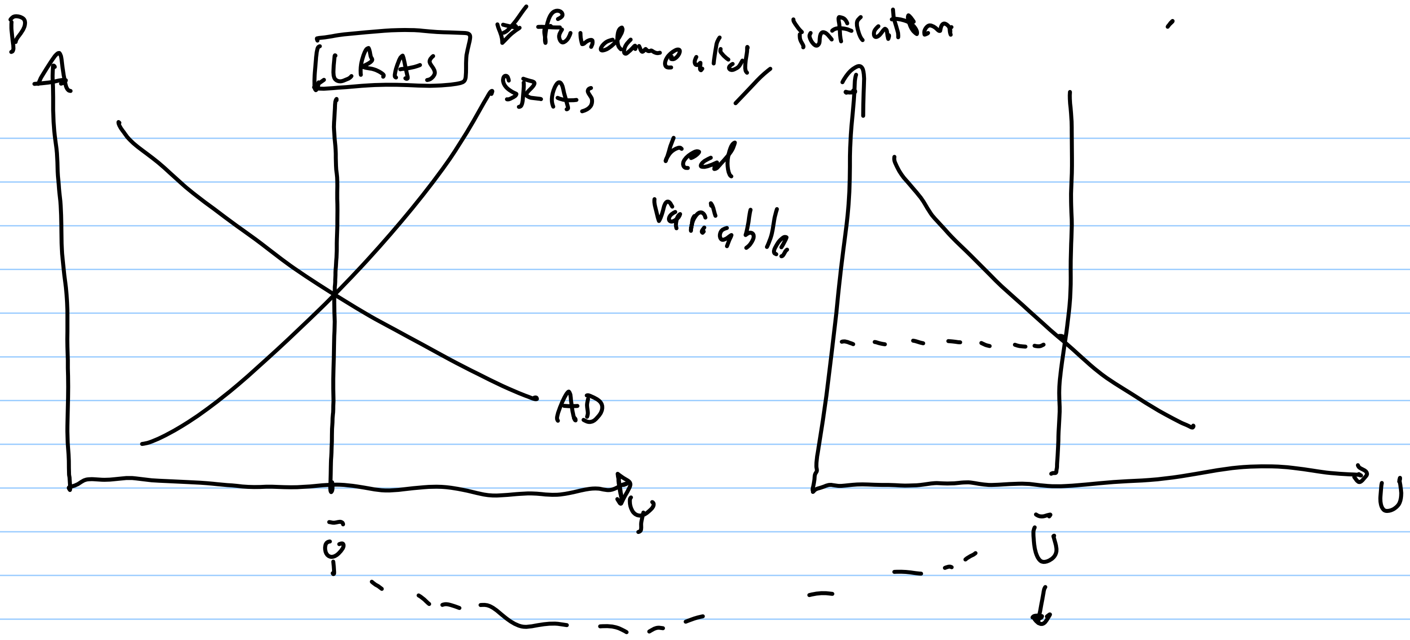
★★★ Intuition

$\Rightarrow$ π^e_p = the intertemporal equilibrium value of π^e = m

$\Rightarrow$ P_p = $\dot{p}$ = m

$\Rightarrow$ Δm affects π^e_p and P_p

$\Rightarrow$ Δm does not affect U_p $(\bar{U} = \frac{1}{18})$



the natural rate of unemployment $\frac{1}{18} \simeq 5.6\%$.

A Market model with price expectation

$$Q_d = \alpha - \beta P$$

$$Q_s = -\gamma + \delta P$$

Now $\quad Q_d = \alpha - \beta P + mP' + nP'' \quad (\alpha, \beta > 0)$

$\qquad\quad Q_s = -\gamma + \delta P \quad (\gamma, \delta > 0)$

$$\left.\begin{aligned} P' &= \frac{dP}{dt} \\ P'' &= \frac{d^2P}{dt^2} \end{aligned}\right\} \text{Price trend information}$$

$m > 0 \Rightarrow$ buyers expect the rising price will continue rise and prefer their purchase now, when the price is still relatively low

$n > 0 \Rightarrow$ the rising price will increase with an increasing rate

$m < 0 \Rightarrow$ the price trend is going down.

$[n < 0]$ buyers would prefer to wait for a lower price

Ex 2

$$\left.\begin{aligned} Q_d &= 42 - 4P - 4P' + P'' \\ Q_s &= -6 + 8P \end{aligned}\right] \Rightarrow$$

$P(0) = 6$ and $P'(0) = 4$

Find the time path $P(t)$

Assume the mkt is clear

$$Q_d = Q_s$$

$$42 - 4P - 4P' + P'' = -6 + 8P$$

$$P'' - 4P' - 12P = -48 \quad \text{— (i)}$$

① Find P_p

try $\left.\begin{aligned} P_p &= k \\ P' &= 0 \end{aligned}\right\}$ sub (i)

$$P'' = 0,$$

$$0 - 4(0) - 12(k) = -48$$

$$k = 4$$

The P_p is 4

The intertemporal equilibrium price is 4.

© To find P_c

We solve the characteristic equation

$$r^2 - 4r - 12 = 0$$

$$r_1, r_2 = \frac{4 \pm \sqrt{16+48}}{2} = 6, -2$$

$$P_c = A_1 e^{6t} + A_2 e^{-2t}$$

The general sol^{n} is

$$P(t) = A_1 e^{6t} + A_2 e^{-2t} + 4$$

→ Find the definite sol^{n} ⇒ $P(t) = e^{6t} + e^{-2t} + 4$

→ P_p is dynamically stable? No b/c of the positive root $r_1 = 6 > 0$

Ex $Q_d = 40 - 2P - 2P' - P''$ $\quad P(0) = 12$

$Q_s = -5 + 3P$ $\quad P'(0) = 1$

TRY

2.2.3 Second-order Linear DE with a variable term

$$y''(t) + a_1 y'(t) + a_2 y(t) = b(t)$$

$b(t)$ is a fn of t

$b(t) \Rightarrow bt^2$, e^{bt} or $b \sin t$

The complementary fn is not affected

Now, to solve for y_p, we use the

"method of undetermined coefficients"

$b(t)$	suggested guess for y_p
1) constant b	1) constant k
2) bt^n ; $n \Rightarrow$ an integer number	2) $B_n t^n + B_{n-1} t^{n-1} + \dots + B_1 t + B_0$
$n = 2$	$B_2 t^2 + B_1 t + B_0$
3) be^{rt}	3) Be^{rt}
4) $b\cos(rt)$ or $b\sin(rt)$	4) $B_1 \cos(rt) + B_2 \sin(rt)$

5) $b t^n \cos(rt)$

or

$b t^n \sin(rt)$

5) $(A_n t^n + A_{n-1} t^{n-1} + \dots + A_1 t + A_0)\cos(rt)$

$+ (B_n t^n + B_{n-1} t^{n-1} + \dots + B_1 t + B_0)\sin(rt)$

Ex $y''(t) + 5y'(t) + 3y(t) = 6t^2 - t - 1 \quad -(i)$

⊙ Find y_p

$$\left.\begin{aligned} y_p(t) &= B_2 t^2 + B_1 t + B_0 \\ y'(t) &= 2B_2 t + B_1 \\ y''(t) &= 2B_2 \end{aligned}\right\} \text{sub } (i)$$

$$2B_2 + 5(2B_2 t + B_1) + 3(B_2 t^2 + B_1 t + B_0) = 6t^2 - t - 1$$

$$2B_2 + 10B_2 t + 5B_1 + 3B_2 t^2 + 3B_1 t + 3B_0 = 6t^2 - t - 1$$

$$3B_2 t^2 + (10B_2 + 3B_1)t + (2B_2 + 5B_1 + 3B_0) = 6t^2 - t - 1$$

$$\left.\begin{aligned} 3B_2 &= 6 \\ 10B_2 + 3B_1 &= -1 \\ 2B_2 + 5B_1 + 3B_0 &= -1 \end{aligned}\right\} \begin{aligned} &\Rightarrow B_2 = 2 \\ &\Rightarrow B_1 = -7 \\ &\Rightarrow B_0 = 10 \end{aligned}$$

$$y_p = 2t^2 - 7t + 10$$

Find y_c

Solve the characteristic eqn

$$r^2 + 5r + 3 = 0$$

TRY

Ex $$y''(t) + 3y'(t) - 4y(t) = 2e^{-4t} \quad -(i)$$

Find y_p

$$\left.\begin{aligned} y_p &= Be^{-4t} \\ y'(t) &= -4Be^{-4t} \\ y''(t) &= 16Be^{-4t} \end{aligned}\right\} \text{sub into (i)}$$

$$16Be^{-4t} + 3(-4Be^{-4t}) - 4(Be^{-4t}) = 2e^{-4t}$$

$$16Be^{-4t} - 12Be^{-4t} - 4Be^{-4t} = 2e^{-4t}$$

$$0 = 2e^{-4t}$$

new guess $$y_p = Bte^{-4t}$$

$$y'(t) = -4Bte^{-4t} + Be^{-4t}$$

$$y''(t) = 16Bte^{-4t} - 4Be^{-4t} - 4Be^{-4t}$$

sub into (i)

$$16Bte^{-4t} - 8Be^{-4t}$$

$$(16Bte^{-4t} - 8Be^{-4t}) + 3(-4Bte^{-4t} + Be^{-4t}) - 4(Bte^{-4t}) = 2e^{-4t}$$

$$\underbrace{(16Bte^{-4t} - 12Bte^{-4t} - 4Bte^{-4t})}_{=0} \underbrace{- 8Be^{-4t} + 3Be^{-4t}}_{-5Be^{-4t}} = 2e^{-4t}$$

$$= 2e^{-4t}$$

$$B = -\frac{2}{5}$$

$$\therefore \quad y_p = -\frac{2}{5}te^{-4t}$$ ✓✓

$y_c = ?$ TRY