Wantitee T.
610464059

BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 from $\hat{\beta}_2 = \dfrac{n \sum_{i=1}^{n} X_i Y_i - \sum_{i=1}^{n} X_i \sum_{i=1}^{n} Y_i}{n \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2}$

$$\hat{\beta}_2 = \frac{8(2012,4) - (621)(25,7)}{8(48,717) - (621)^2}$$

$$= \frac{16099,2 - 15959,7}{389736 - 385641}$$

$$= \frac{139,5}{4095} = 0,034$$

On average if student score increase by 1 their gpa is increasing 0.034 holding all other factors that might effect the GPA constant from $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

m+ m-

$$= \left(\frac{25.7}{8}\right) - (0.034)\left(\frac{621}{8}\right)$$

$$= 3.2125 - 2.63925$$

$$= 0.57325$$

If the score is 0, lowest gpa that will earn equal to 0.57325

1.2 find $\hat{Y}$ & $\hat{u}$ & $\sum_{i=1}^{n} \hat{u}_i = 0$

from $\hat{u}_i = Y_i - \hat{Y}_i$

$$= Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$$

So, $\hat{Y} = Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)$

$$\hat{Y}_1 = (0.57 + 0.034(63)) = 2.712$$
$$\hat{Y}_2 = (0.57 + 0.034(72)) = 3.018$$
$$\hat{Y}_3 = (0.57 + 0.034(78)) = 3.222$$
$$\hat{Y}_4 = (0.57 + 0.034(81)) = 3.324$$
$$\hat{Y}_5 = (0.57 + 0.034(87)) = 3.528$$
$$\hat{Y}_6 = (0.57 + 0.034(75)) = 3.12$$
$$\hat{Y}_7 = (0.57 + 0.034(75)) = 3.12$$
$$\hat{Y}_8 = (0.57 + 0.034(90)) = 3.63$$

then $\hat{u}_i = Y_i - \hat{Y}_i$

$$\hat{u}_1 = 2.8 - 2.712 = 0.088$$
$$\hat{u}_2 = 3.4 - 3.018 = 0.382$$
$$\hat{u}_3 = 3 - 3.222 = -0.222$$
$$\hat{u}_4 = 3.5 - 3.324 = 0.176$$
$$\hat{u}_5 = 3.6 - 3.528 = 0.072$$
$$\hat{u}_6 = 3 - 3.12 = -0.12$$
$$\hat{u}_7 = 2.7 - 3.12 = -0.42$$
$$\hat{u}_8 = 3.7 - 3.63 = 0.07$$

Lastly $\sum_{i=1}^{n} \hat{u}_i = 0$

$$\sum_{i=1}^{8} (0.088 + 0.382 - 0.222 + 0.176 + 0.072 - 0.12 - 0.42 + 0.07)$$

$$\approx 0.026 \approx 0$$

1.3 find $Var(\hat{u}_i)$, $Var(\hat{\beta}_1)$, $Var(\hat{\beta}_2)$

$$Var(\hat{u}_i) = \frac{\sum \hat{u}_i^2}{n-2} = \sigma_u^2$$

$$= \frac{(0.088)^2 + (0.382)^2 + (0.222)^2 + (0.176)^2 + (0.072)^2 + (0.12)^2 + (0.62)^2 + (0.07)^2}{8-2}$$

$$= \frac{0.439}{6} \approx 0.07333$$

$$\mathrm{var}(\hat{\beta}_1) = \frac{\sigma_u^2 \sum X_i^2}{n \sum x_i^2} = \frac{\sigma^2_u \sum X_i^2}{n \sum (X_i - \bar{X})^2}$$

$$= \frac{0.0733(48717)}{8(500.624)}$$

$$\approx \frac{3571}{4005} \approx 0.892$$

$$\mathrm{var}(\hat{\beta}_2) = \frac{\sigma^2 u}{\sum x_i^2}$$

$$= \frac{0.0733}{(500.624)}$$

$$= 0.00014$$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i}$, $\boldsymbol{u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

Note to myself

SRF : $\hat{\beta}_1 + \hat{\beta}_2 X_i = \hat{Y}_i$

$Y_i - \hat{Y}_i = \hat{u}_i$

"Practice makes Perfect."

2.1

$\bar{X} = 20$, $\bar{Y} = 9.1$

X_i	Y_i	x_i	y_i	x^2	$x_i y_i$
10	0	−10	−9.1	100	91
12	2	−8	−7.1	64	56.8
14	5	−6	−4.1	36	24.6
16	6	−4	−3.1	16	12.4
18	7	−2	−2.1	4	4.2
22	10	2	0.9	4	1.8
24	10	4	0.9	16	3.6
26	15	6	5.9	36	35.4
28	16	8	6.9	64	55.2
30	20	10	10.9	100	109

$\sum x^2 = 440$ $\sum x_i y_i = 394$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{394}{440} = 0.8954$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8954)(20)$$
$$= -8.808$$

2.2 $\hat{y}_i$ & $\hat{u}_i$ & $\sum \hat{u}_i = 0$

from $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

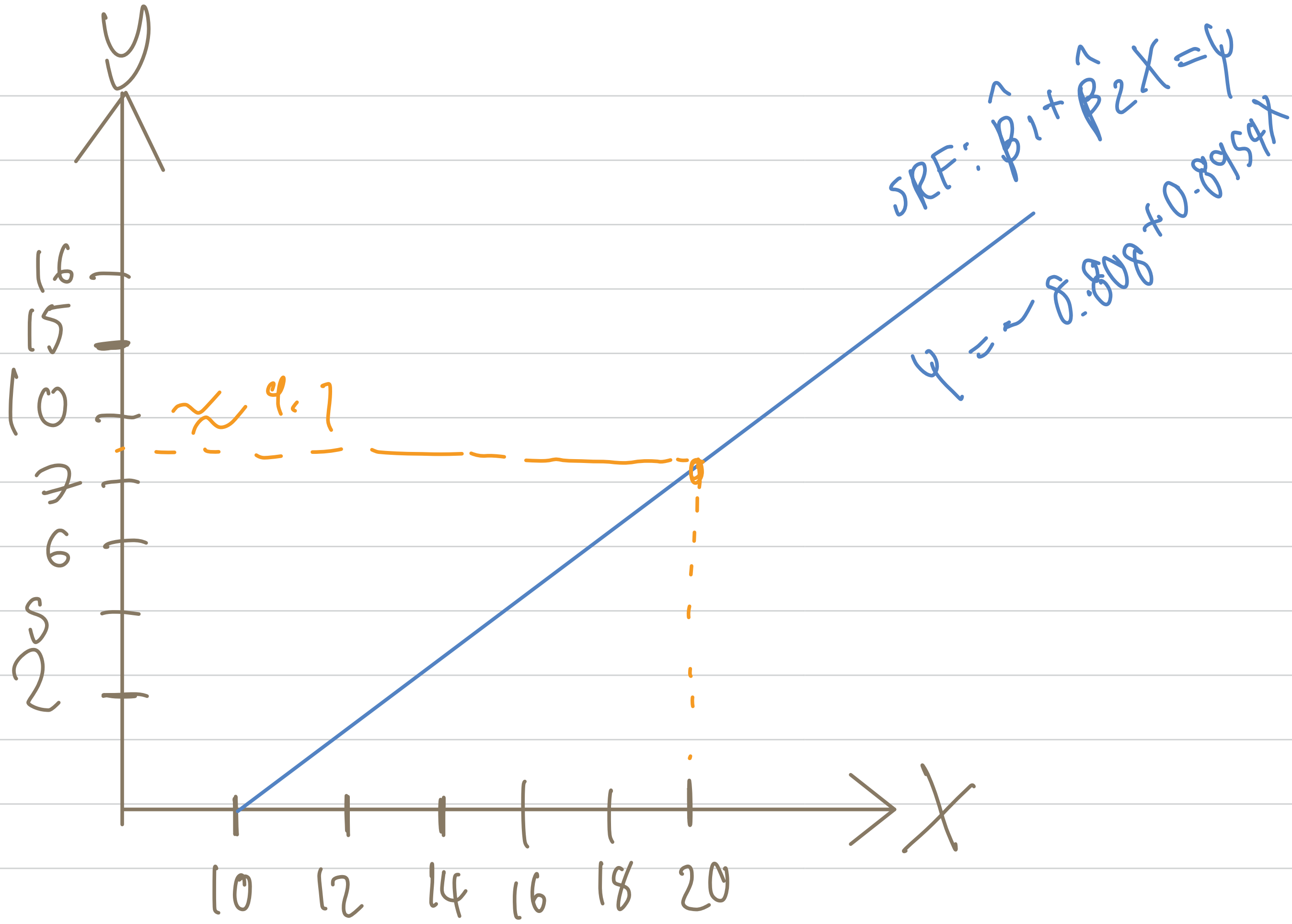
$$\hat{Y}_1 = (-8.808 + 0.8954(10)) = 0.145$$
$$\hat{Y}_2 = (-8.808 + 0.8954(12)) = 1.936$$
$$\hat{Y}_3 = (-8.808 + 0.8954(14)) = 3.727$$
$$\hat{Y}_4 = (-8.808 + 0.8954(16)) = 5.518$$
$$\hat{Y}_5 = (-8.808 + 0.8954(18)) = 7.309$$
$$\hat{Y}_6 = (-8.808 + 0.8954(22)) = 10.891$$
$$\hat{Y}_7 = (-8.808 + 0.8954(24)) = 12.682$$
$$\hat{Y}_8 = (-8.808 + 0.8954(26)) = 14.473$$
$$\hat{Y}_9 = (-8.808 + 0.8954(28)) = 16.264$$
$$\hat{Y}_{10} = (-8.808 + 0.8954(30)) = 18.055$$

$$\hat{U}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_1 = 0 - 0.145 = -0.145$$
$$\hat{u}_2 = 2 - 1.936 = 0.064$$
$$\hat{u}_3 = 5 - 3.727 = 1.273$$
$$\hat{u}_4 = 6 - 5.518 = 0.482$$
$$\hat{u}_5 = 7 - 7.309 = -0.309$$
$$\hat{u}_6 = 10 - 10.891 = -0.891$$
$$\hat{u}_7 = 10 - 12.682 = -2.682$$
$$\hat{u}_8 = 15 - 14.473 = -0.527$$
$$\hat{u}_9 = 16 - 16.264 = -0.244$$
$$\hat{u}_{10} = 20 - 18.055 = 1.945$$

$$\sum_{i=1} \hat{u}_i = 0 \rightarrow -0.145 + 0.064 + 1.273 + 0.482 - 0.309 - 0.891 - 2.682 - 0.527 - 0.244 + 1.945 = 0.018 \approx 0$$

2.3



2.4 if $X_i = 18$ Y estimate

$Y = (-8.808) + 0.8954(X)$

$= (-8.808) + (0.8954(18))$

$= 7.31$

2.5 find $var(\hat{u}_1)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$var(\hat{u}_i) = \frac{\Sigma \hat{U}_i^2}{n-2} = \hat{\sigma}_u^2$$

$$= \frac{14.08075}{10-2} = 1.760$$

$$Var(\hat{\beta}_2) = \frac{\hat{\sigma}_u^2}{\Sigma \cancel{X}_i^2}$$

$$= \frac{1.760}{440} = 0.004$$

$$Var(\hat{\beta}_1) = \frac{\hat{\sigma}_u^2 \Sigma X_i^2}{n \Sigma x_i^2}$$

$$= \frac{1.760\,(\cancel{4440})}{\cancel{10}\,(4\cancel{40})}$$

$$= 1.760$$

3. Consider the below regression function: consider the two-variable model

$$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\hat{\beta}_1 = \frac{\sum x^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - (\sum X_i)^2}$$

$$= \bar{y} = \hat{\beta}_2 \bar{X}$$

Proof that is unbiased estimator

$$E(\hat{\beta}_1) = E(\bar{y} - \hat{\beta}_2 \bar{X})$$

$$= E(\bar{y}) - \hat{\beta}_2 E(\bar{X})$$

$$= \beta_1 + \beta_2 \bar{X} - \bar{X}\beta_2$$

$$\therefore E(\hat{\beta}_1) = (\beta_1)$$