FN211 Financial Mathematics and Statistics

Mid-term Exercise (Guided Solution)
Semester 1/2020

Problem Set 1
Random Variables and Their Distributions

Guided Solution

Question	Answer	Question	Answer	Question	Answer	Question	Answer
1	C	**13**	B	**25**	I	**37**	B
2	A	**14**	D	**26**	I	**38**	B
3	C	**15**	B	**27**	C	**39**	B
4	C	**16**	A	**28**	IV	**40**	C
5	B	**17**	A	**29**	C	**41**	A
6	C	**18**	C	**30**	A	**42**	C
7	A	**19**	A	**31**	A	**43**	C
8	B	**20**	C	**32**	B	**44**	C
9	C	**21**	C	**33**	A	**45**	A
10	A	**22**	B	**34**	B	**46**	C
11	C	**23**	B	**35**	B	**47**	A
12	C	**24**	A	**36**	D		

1. An investment offers a 10% stated annual return. What is the equivalent return if it is continuously compounded?
   A. 2.20%
   B. 2.30%
   C. 10.52%

2. A coating machine coats film between 120 and 210 microns with a uniform random distribution. Calculate the mean coating thickness.
   A. 165 microns
   B. 330 microns
   C. 210 microns

3. Suppose you have a discrete uniform probability function such that p(X = x) = 20% for X values of 0, 1, 2, 3, and 4. Find F(4).
   A. 0%
   B. 20%
   C. 100%

4. Consider an experiment that consists of removing a card from a deck and identifying its suit. A deck has 52 cards: 13 diamonds, 13 hearts, 13 spades, and 13 clubs. Each card has an equal probability of being selected. What is the probability that a card selected randomly is a heart?
   A. 0.02
   B. 0.07
   C. 0.25

5. An experiment consists of throwing a die and recording the number of dots on the face of the die facing up. What is the probability that the number of dots is greater than 2 but less than 6?
   A. 1/3
   B. 1/2
   C. 1/6

6. A broker selected 20 different stocks that he likes equally. He identified these stocks with alphabet letters from A to T. Assume that an experiment consists of randomly selecting one of these stocks and then buying it. What is the probability that the broker buys stock V?
   A. 0.05
   B. 0.01
   C. None of the above

7. Suppose that the penalty for withdrawing funds early from a certain account follows a uniform distribution on the interval from (5%,12%). Find the variance of the penalty.
   A. 4.08%
   B. 7%
   C. 8.5%

8. Suppose that the penalty for withdrawing funds early from a certain account follows a uniform distribution on the interval from (5%,12%). What is the probability that the penalty is between 6% and 9%?

A. 0.143
B. 0.429
C. 0.882

9. Find the probability of having exactly four girls in seven births. Assume no multiple births and that male and female births are equally likely and independent.
   A. 0.063
   B. 0.571
   C. 0.273

10. Suppose that 20% of all drivers come to a complete stop at an intersection having flashing red lights in all directions when there are no other cars visible. What is the probability that, of 15 randomly selected drivers coming to the intersection under these conditions, exactly 5 will come to a complete stop?
   A. 0.103
   B. 0.188
   C. 0.812

11. A set of four cards consists of two red cards and two black cards. The cards are shuffled thoroughly, and I am dealt two cards. I count the number of red cards X in these two cards. The random variable X has which of the following probability distributions?
   A. The binomial distribution with parameters n = 4 and p = 0.5
   B. The binomial distribution with parameters n = 2 and p = 0.5
   C. None of the above

12. There are five multiple-choice questions on an exam, each with responses a, b, c, or d. Each question is worth 5 points and only one option per question is correct. Suppose a student guesses the answer to each question. His or her guesses from question to question are independent. If the student needs at least 20 points to pass the test, the probability that the student passes is closest to ______.
   A. 0.0146
   B. 0.0010
   C. 0.0156

13. For x, a random variable from a binomial distribution with N = 9 and p = 0.6, p(5) = ______ (to nearest 0.1%).
   A. 27.08%
   B. 25.1%
   C. 1.46%

14. Which of the following is not a binomial experiment?
   A. Flipping a coin 3 times and letting x count the number of heads obtained.
   B. Rolling a die 3 times and letting x count the numbers of 2s obtained.
   C. Randomly selecting a ball (with replacement) three times from a jar containing 1, 2, 2, 5, 6, and 7, and letting x count the number of 2s obtained.
   D. Randomly selecting 3 balls from a jar containing balls numbered 1, 2, 2, 5, 6, 7 and letting x count the number of 2s obtained.

15. The probability of an accident-free day at MRC is 60%. Assuming the accident-free days at MRC are independent, for a random sample of 15 days, the probability of less than 14 accident-free days is ______ (to the nearest 0.1%).

A. 4.7%
B. 99.5%
C. 0.5%

16. Research shows that 72% of consumers have heard of MBI computers. A survey of 500 randomly selected consumers is to be conducted. What is the mean and standard deviation for the number of consumers that have heard of MBI computers?
    A. 360; 10.04
    B. 360; 100.80
    C. 144; 100.80

17. There are five multiple-choice questions on an exam, each having responses a, b, c, or d. Each question is worth 5 points and only one option per question is correct. Suppose a student guesses the answer to each question, and his or her guesses from question to question are independent. The student's mean number of questions correct on the exam should be ______.
    A. 1.25
    B. 2.5
    C. 6.25

18. As part of a promotion for a new type of cracker, free trial samples are offered to shoppers in a local supermarket. The probability that a shopper will buy a packet of crackers after testing the free sample is 0.20. Different shoppers can be regarded as independent trials. Let X be the number among the next twenty shoppers who buy a packet of the crackers after tasting a free sample. The standard deviation of X is ______.
    A. 4.00
    B. 3.20
    C. 1.79

19. Albert is taking a 10-item multiple-choice test (4 choices per item) and randomly guessing on each item. How many items should he expect to get correct?
    A. 2.5
    B. 3
    C. 2

20. For random variable x from a binomial distribution with N = 20 and p = 0.3, the standard deviation is ______ (to nearest 0.01).
    A. 0.30
    B. 4.20
    C. 2.05

21. Given that X is a binomial random variable, with N = 5 and p = 0.3 and that we want to find P(X < 3), which of the following is true?
    A. We can approximate $P(X_B < 3)$ with $P(X_N < 2.5)$.
    B. We can approximate P(X < 3) using a normal with mean = 1.5 and standard deviation = 1.02.
    C. We cannot approximate this binomial distribution with a normal distribution.

22. For X, a binomial random variable with N = 70 and p = 0.3, we can approximate P(X < 19) by using a normal distribution with ______.
    A. mean 70 and standard deviation 0.3
    B. mean 21 and standard deviation 3.83
    C. mean 70 and standard deviation 3.83

23. Suppose that the penalty for withdrawing funds early from a certain account follows a uniform distribution on the interval from (5%,12%). Find the expected value of the penalty.
    A. 7%
    B. 8.5%
    C. 17%

24. Suppose that the penalty for withdrawing funds early from a certain account follows a uniform distribution on the interval from (5%,12%). What is the probability that the penalty is less than 7%?
    A. 0.286
    B. 0.416
    C. 0.714

25. For x, a normal random variable, which of the following is (are) false?
    I. The parameters are N and p.
    II. The parameters are the mean and standard deviation.
    III. The graph is a bell-shaped curve.
    IV. The probability x is equal to a particular value (say, 55) is zero.

26. For x, a normal random variable, which of the following is (are) false?
    I. P(x < mean) = 50%
    II. P(x < 38) = P(x < 38)
    III. The higher the standard deviation the more spread out the distribution.
    IV. P(35 < x < 50) = P(x <50) - P(x < 35)

27. For x, a normal random variable, which of the following is false?
    A. P(Q1 < x < Q3) = 50%
    B. Q1 is 0.67 standard deviations below the mean.
    C. x-score that cuts off the top 10% of the distribution is 1.28 standard deviations below the mean.

28. Which of the following is (are) false regarding the normal curve?
    I. The smaller the standard deviation, the steeper the bell curve.
    II. The mean locates the axis of symmetry.
    III. Total area under the curve is one, with 0.5 area to either side of the mean.
    IV. The mean is greater than the mode.

29. Which of the following is not a characteristic of the normal distribution?
    A. unimodal
    B. symmetrical about the median
    C. extends from 0 to infinite

30. Which of the following statements about a normal distribution is the LEAST ACCURATE? A normal distribution ______.
    A. has an excess kurtosis of 3
    B. is completely described by two parameters
    C. can be the linear combination of two or more normal random variables

31. An investor estimated the mean return of a portfolio at 12% and the standard deviation at 16%. What is the 95% confidence interval for the mean return on this portfolio?
    A. [-19.36%, 43.36%]
    B. [-14.32%, 18.32%]
    C. [-5.42%, 18.58%]

32. An analyst wants to calculate a 95% confidence interval for the weighted-average of the projected mean return of a portfolio he manages. His latest calculations show that this portfolio has a weighted-average return of 8% and a variance of 36. What is the 95% confidence interval?
    A. [-62.56%, 78.56%]
    B. [-3.76%, 19.76%]
    C. [-1.87%, 17.87%]

33. For a data collection, if a score has a z-score of 1.5, then ______
    A. the score is 1.5 standard deviations above the mean.
    B. the score is better than 15% of the scores in the data collection.
    C. the score is 1.5 standard deviations below the mean.

34. For a set of data with mean 70 and standard deviation 10, the z-score for 62 is ______.
    A. 0.8
    B. -0.8
    C. -8

35. Which of the following statements is incorrect?
    A. A standard normal random variable has a variance of 1.0 and is centered at zero.
    B. A standard normal random variable has a mode of 1.0.
    C. Any normal random variable can be converted to a standard normal distribution.

36. A computed z-value of -1.75 tells us ______
    A. the value -1.75 has a standard normal distribution.
    B. the mean of the distribution is -1.75.
    C. the standard deviation of the distribution is -1.75.
    D. none of the above

37. The daily sales at a certain cafe follow a normal distribution with a mean of $1,060 and a standard deviation of $310. What is the z-value associated with daily sales of $1,711?
    A. -2.10
    B. 2.10
    C. 36.97

38. Suppose a set of data has a mean of 52 and a standard deviation of 6. According to Chebyshev's Theorem, at least what percentage of the data lies between 40 and 64? If the data is normally distributed, about what percentage of the data lies between 40 and 64, according to the Empirical Rule?
    A. 0%, 68%
    B. 75%, 95%
    C. 94%, 100%

39. As part of a promotion for a new type of cracker, free trial samples are offered to shoppers in a local supermarket. The probability that a shopper will buy a packet of crackers after testing the free sample is 0.20. Different shoppers can be regarded as independent trials. Let X be the number among the next 100 shoppers who buy a packet of the crackers after tasting a free sample. The probability that fewer than 30 buy a packet after tasting a free sample is approximately ______.
    A. 0.20
    B. 0.9938
    C. None of the above

40. The lifetime of a 2-volt non-rechargeable battery in constant use has a normal distribution with a mean of 516 hours and a standard deviation of 20 hours. The proportion of batteries with lifetimes exceeding 520 hours is approximately ______.
    A. 0.20
    B. 0.5793
    C. 0.4207

41. The random variable X denotes the time taken for a computer link to be made between the terminal in an executive's office and the computer at a remote factory site. It is known that X has a normal distribution with a mean of 15 seconds and a standard deviation of 3 seconds. $P(X > 20)$ has value (choose the closest option) ______.
    A. 0.048
    B. 0.952
    C. 1.67

42. For x, a random variable from a normal distribution with mean 40 and standard deviation 5, $P(x > 20)$ = ______ (to the nearest 0.1%).
    A. 0+
    B. 95%
    C. 1-

43. For x, a random variable from a normal distribution with mean 40 and standard deviation 5, $P(28 < x < 38)$ = ______ (to nearest 0.1%).
    A. 35.3%
    B. 34.5%
    C. 33.6%

44. For the MRC test, with scores normally distribution with m = 70 and s = 5, the cutoff for the bottom 10% is ______.
    A. 60
    B. 68.72
    C. 63.6

45. The daily sales at a certain shop follow a normal distribution with a mean of $1,060 and a standard deviation of $310. What is the probability that the daily sales are less than $409?
    A. 0.0179
    B. 0.4821
    C. 0.5179

46. The daily sales at a certain cafe follow a normal distribution with a mean of $1,060 and a standard deviation of $310. What is the probability that the daily sales are between $471 and $1,401?
    A. 0.4713
    B. 0.3643
    C. 0.8356

47. The amount of fill in a certain brand of bottled water is normally distributed with a mean of 16 ounces and a standard deviation of .25 ounce. What is the probability that a bottle has more than 16.25 ounces?
    A. 0.1587
    B. 0.3413
    C. 0.6587

Problem Set 2
Working with Probability and Introduction to Multivariate Distribution

1. Suppose that 5 percent of the stocks meeting your stock-selection criteria are in the telecommunications (telecom) industry. Also, dividend-paying telecom stocks are 1 percent of the total number of stocks meeting your selection criteria. What is the probability that a stock is dividend paying, given that it is a telecom stock that has met your stock selection criteria?

   Solution:
   P(stock is dividend paying | telecom stock that meets criteria)
   = P(stock is dividend paying and telecom stock that meets criteria) / P(telecom stock that meets criteria)
   = 0.01/0.05
   = 0.20.

2. You apply both valuation criteria and financial strength criteria in choosing stocks. The probability that a randomly selected stock (from your investment universe) meets your valuation criteria is 0.25. Given that a stock meets your valuation criteria, the probability that the stock meets your financial strength criteria is 0.40. What is the probability that a stock meets both your valuation and financial strength criteria?

   Solution:
   The multiplication rule for probabilities $P(AB) = P(A \mid B)P(B)$, defining
   A as the event that *a stock meets the financial strength criteria* and
   B as the event that *a stock meets the valuation criteria*.
   Then $P(AB) = P(A \mid B)P(B) = 0.40 \times 0.25 = 0.10$.
   The probability that a stock meets both the financial and valuation criteria is 0.10.

3. Suppose the prospects for recovering principal for a defaulted bond issue depend on which of two economic scenarios prevails. Scenario 1 has probability 0.75 and will result in recovery of \$0.90 per \$1 principal value with probability 0.45, or in recovery of \$0.80 per \$1 principal value with probability 0.55. Scenario 2 has probability 0.25 and will result in recovery of \$0.50 per \$1 prin- cipal value with probability 0.85, or in recovery of \$0.40 per \$1 principal value with probability 0.15.
   A. Compute the probability of each of the four possible recovery amounts: \$0.90, \$0.80, \$0.50, and \$0.40.
   B. Compute the expected recovery, given the first scenario.
   C. Compute the expected recovery, given the second scenario.
   D. Compute the expected recovery.
   E. Graph the information in a tree diagram.

   Solution:
   A
   Outcomes associated with Scenario 1: With a 0.45 probability of a \$0.90 recovery per \$1 principal value, given Scenario 1, and with the probability of Scenario 1 equal to 0.75, the probability of recovering \$0.90 is 0.45 (0.75) = 0.3375. By a similar calculation, the probability of recovering \$0.80 is 0.55(0.75) = 0.4125.
   Outcomes associated with Scenario 2: With a 0.85 probability of a \$0.50 recovery per \$1 principal value, given Scenario 2, and with the probability of Scenario 2 equal to 0.25, the

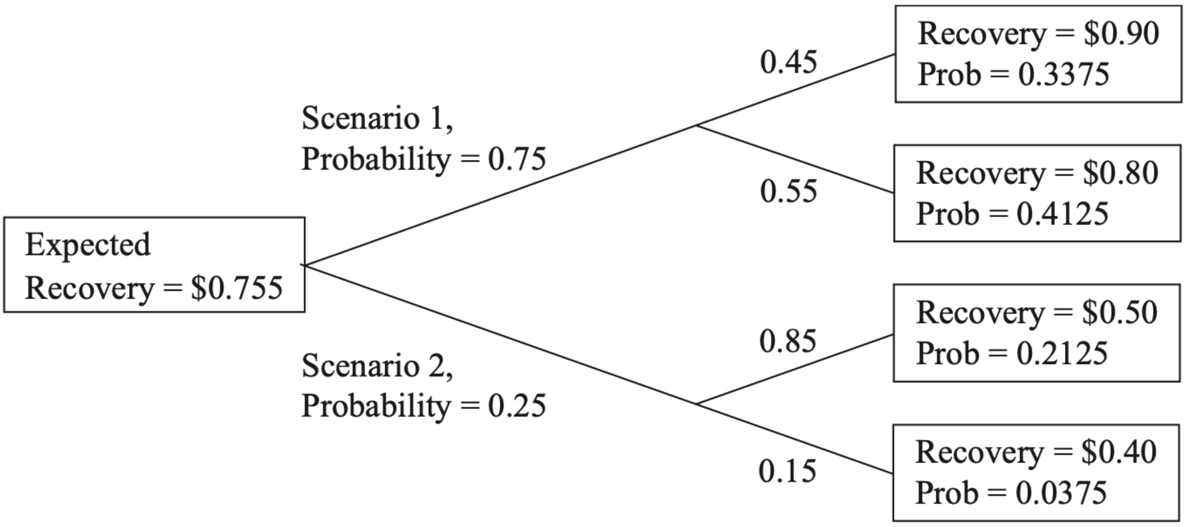
probability of recovering \$0.50 is 0.85(0.25) = 0.2125. By a similar calculation, the probability of recovering \$0.40 is 0.15(0.25) = 0.0375.

B $E(recovery \mid Scenario\ 1) = 0.45(\$0.90) + 0.55(\$0.80) = \$0.845$

C $E(recovery \mid Scenario\ 2) = 0.85(\$0.50) + 0.15(\$0.40) = \$0.485$

D $E(recovery) = 0.75(\$0.845) + 0.25(\$0.485) = \$0.755$

E



4. You have developed a set of criteria for evaluating distressed credits. Companies that do not receive a passing score are classed as likely to go bankrupt within 12 months. You gathered the following information when validating the criteria:
   - Forty percent of the companies to which the test is administered will go bankrupt within 12 months: P(nonsurvivor) = 0.40.
   - Fifty-five percent of the companies to which the test is administered pass it: P(pass test) = 0.55.
   - The probability that a company will pass the test given that it will subse- quently survive 12 months, is 0.85: P(pass test | survivor) = 0.85.

   A. What is P(pass test | nonsurvivor)?
   B. Using Bayes' formula, calculate the probability that a company is a survivor, given that it passes the test; that is, calculate P(survivor | pass test).
   C. What is the probability that a company is a nonsurvivor, given that it fails the test?
   D. Is the test effective?

Solution:

A

We can set up the equation using the total probability rule:

$$P(pass\ test) = P(pass\ test \mid survivor)P(survivor) + P(pass\ test \mid nonsurvivor)P(nonsurvivor)$$

We know that $P(survivor) = 1 - P(nonsurvivor) = 1 - 0.40 = 0.60$. Therefore, $P(pass\ test) = 0.55 = 0.85(0.60) + P(pass\ test \mid nonsurvivor)(0.40)$. Thus $P(pass\ test \mid nonsurvivor) = [0.55 - 0.85(0.60)]/0.40 = 0.10$.

B

$$P(survivor \mid pass\ test) = \left[P(pass\ test \mid survivor)/P(pass\ test)\right]P(survivor) = (0.85/0.55)0.60 = 0.927273$$

The information that a company passes the test causes you to update your probability that it is a survivor from 0.60 to approximately 0.927.

C

According to Bayes' formula, $P(nonsurvivor \mid fail\ test) = [P(fail\ test \mid nonsurvivor)/P(fail\ test)]P(nonsurvivor) = [P(fail\ test \mid nonsurvivor)/0.45]0.40$.

We can set up the following equation to obtain $P(fail\ test \mid nonsurvivor)$:

$$P(fail\ test) = P(fail\ test \mid nonsurvivor)P(nonsurvivor) + P(fail\ test \mid survivor)P(survivor)$$

$$0.45 = P(fail\ test \mid nonsurvivor)0.40 + 0.15(0.60)$$

where $P(fail\ test \mid survivor) = 1 - P(pass\ test \mid survivor) = 1 - 0.85 = 0.15$. So $P(fail\ test \mid nonsurvivor) = [0.45 - 0.15(0.60)]/0.40 = 0.90$. Using this result with the formula above, we find $P(nonsurvivor \mid fail\ test) = (0.90/0.45)0.40 = 0.80$. Seeing that a company fails the test causes us to update the probability that it is a nonsurvivor from 0.40 to 0.80.

D

A company passing the test greatly increases our confidence that it is a survivor. A company failing the test doubles the probability that it is a nonsurvivor. Therefore, the test appears to be useful.

5. After estimating the probability that an investment manager will exceed his benchmark return in each of the next two quarters, an analyst wants to forecast the probability that the investment manager will exceed his benchmark return over the two-quarter period in total. Assuming that each quarter's performance is independent of the other, which probability rule should the analyst select?
   A. Addition rule
   B. Multiplication rule
   C. Total probability rule

   Solution:
   B is correct. Because the events are independent, the multiplication rule is most appropriate for forecasting their joint probability. The multiplication rule for independent events states that the joint probability of both A and B occurring is P(AB) = P(A)P(B).

6. An analyst developed two scenarios with respect to the recovery of $100,000 principal from defaulted loans:

Scenario	Probability of Scenario (%)	Amount Recovered ($)	Probability of Amount (%)
1	40	50,000	60
		30,000	40
2	60	80,000	90
		60,000	10

The amount of the expected recovery is closest to:
   A. $36,400.
   B. $63,600.
   C. $81,600.

   Solution:
   B is correct. If Scenario 1 occurs, the expected recovery is
      60% ($50,000) + 40% ($30,000) = $42,000, and
   if Scenario 2 occurs, the expected recovery is
      90% ($80,000) + 10%($60,000) = $78,000.
   Weighting by the probability of each scenario, the expected recovery is
      40%($42,000) + 60%($78,000) = $63,600.
   Alternatively, first calculating the probability of each amount occurring, the expected recovery is
      (40%)(60%)($50,000) + (40%)(40%)($30,000) + (60%)(90%) ($80,000)
         + (60%)(10%)($60,000)
      = $63,600.

7. US and Spanish bonds have return standard deviations of 0.64 and 0.56, respectively. If the correlation between the two bonds is 0.24, the covariance of returns is closest to:
   A. 0.086.
   B. 0.670.
   C. 0.781.

Solution:

A is correct. The covariance is the product of the standard deviations and correlation using the formula:

Cov(US bond returns, Spanish bond returns)
= σ(US bonds) × σ (Spanish bonds) × ρ(US bond returns, Spanish bond returns)
= 0.64 × 0.56 × 0.24
= 0.086.

8. The covariance of returns is positive when the returns on two assets tend to:
   A. have the same expected values.
   B. be above their expected value at different times.
   C. be on the same side of their expected value at the same time.

Solution:

C is correct. The covariance of returns is positive when the returns on both assets tend to be on the same side (above or below) their expected values at the same time, indicating an average positive relationship between returns.

9. Which of the following correlation coefficients indicates the weakest linear relationship between two variables?
   A. –0.67
   B. –0.24
   C. 0.33

Solution:

B is correct. Correlations near +1 exhibit strong positive linearity, whereas correlations near –1 exhibit strong negative linearity. A correlation of 0 indi- cates an absence of any linear relationship between the variables. The closer the correlation is to 0, the weaker the linear relationship.

10. An analyst develops the following covariance matrix of returns:

	Hedge Fund	Market Index
Hedge fund	256	110
Market index	110	81

The correlation of returns between the hedge fund and the market index is closest to:
   A. 0.005.
   B. 0.073.
   C. 0.764.

Solution:

C is correct. The correlation between two random variables R_i and R_j is defined as $\rho(R_i,R_j) = \text{Cov}(R_i,R_j)/\sigma(R_i)\sigma(R_j)$. Using the subscript i to represent hedge funds and the subscript j to represent the market index, the standard deviations are $\sigma(R_i) = 256^{1/2} = 16$ and $\sigma(R_j) = 81^{1/2} = 9$. Thus, $\rho(R_i,R_j) = \text{Cov}(R_i,R_j)/\sigma(R_i)\sigma(R_j) = 110/(16 \times 9) = 0.764$.

11. The probability distribution for a company's sales is:

Probability	Sales ($ millions)
0.05	70
0.70	40
0.25	25

The standard deviation of sales is closest to:

A. $9.81 million.
B. $12.20 million.
C. $32.40 million.

Solution:

A is correct. The analyst must first calculate expected sales as 0.05 × $70 + 0.70 × $40 + 0.25 × $25 = $3.50 million + $28.00 million + $6.25 million = $37.75 million.

After calculating expected sales, we can calculate the variance of sales:

$= \sigma^2$ (Sales)
$= P(\$70)[\$70 - E(\text{Sales})]^2 + P(\$40)[\$40 - E(\text{Sales})]^2 + P(\$25)[\$25 - E(\text{Sales})]^2$
$= 0.05(\$70 - 37.75)^2 + 0.70(\$40 - 37.75)^2 + 0.25(\$25 - 37.75)^2$
= $52.00 million + $3.54 million + $40.64 million = $96.18 million.

The standard deviation of sales is thus $\sigma = (\$96.18)^{1/2}$ = $9.81 million.