

TVM2

- ◊ Perpetuity
- ◊ Growing Annuity
- ◊ Loan

Perpetuities

- A series of level/even/equal sized cash flows that occur at the end of each period for an infinite time period
- Examples of Perpetuities:
 - Consoles issued by British Government
 - Preferred Stock
 - Present Value of a Perpetuity

Growing Annuity

A growing stream of cash flows with a fixed maturity

$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \dots + \frac{C \times (1+g)^{t-1}}{(1+r)^t}$$

$$PV = \frac{C}{r-g} \left[1 - \left(\frac{(1+g)}{(1+r)} \right)^t \right]$$

Growing Annuity: Example

- ❖ A defined-benefit retirement plan offers to pay \$20,000 per year for 40 years and increase the annual payment by three-percent each year.
- ❖ What is the present value at retirement if the discount rate is 10 percent?

$$PV = \frac{\$20,000}{.10 - .03} \left[1 - \left(\frac{1.03}{1.10} \right)^{40} \right] = \$265,121.57$$

Growing Perpetuity

A growing stream of cash flows that lasts forever

$$PV = \frac{C}{(1+r)} + \frac{C \times (1+g)}{(1+r)^2} + \frac{C \times (1+g)^2}{(1+r)^3} + \dots$$

$$PV = \frac{C}{r-g}$$

Example: Growing Perpetuity

The expected dividend next year is \$1.30, and dividends are expected to grow at 5% forever.

If the discount rate is 10%, what is the value of this promised dividend stream?

$$PV = \frac{\$1.30}{.10 - .05} = \$26.00$$

Example I:

- o Suppose you have just won the first prize in a lottery.
- o The lottery offers you two possibilities for receiving your prize.
 - o The first possibility is to receive a payment of \$10,000 at the end of the year, and then, for the next 15 years this payment will be repeated, but it will grow at a rate of 5%.
 - o The second possibility is to receive \$100,000 right now.
 - o The interest rate is 12% during the entire period.
 - o Which of the two possibilities would you take?

Practice Question

- o An investor purchasing a British diplomat is entitled to receive annual payments from the British government forever.
- o What is the price of a diplomat that pays \$120 annually if **the next payment occurs one year from today?**
- o The market interest rate is 5.7 percent.

Practice Question

- o An investor purchasing a British diplomat is entitled to receive annual payments from the British government forever.
- o What is the price of a diplomat that pays \$120 annually if **the next payment occurs one year from today?**
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$$PV = C / r$$

$$PV = \$120 / .057$$

$$PV = \$2,105.26$$

Practice Question

- o Calculating Annuity Present Value An investment offers \$4,300 per year for 15 years, with the first payment occurring one year from now.
- o If the required return is 9 percent, what is the value of the investment?
- o What would the value be if the payments occurred for 40 years?
- o For 75 years?
- o Forever?

Practice Question

- o You are planning to save for retirement over the next 30 years.
- o To do this, you will invest \$700 a month in a stock account and \$300 a month in a bond account.
- o The return of the stock account is expected to be 10 percent, and the bond account will pay 6 percent.
- o When you retire, you will combine your money into an account with an 8 percent return.
- o How much can you withdraw each month from your account assuming a 25-year withdrawal period?

Practice: Annuity

- Suppose you had \$275,000 in your bank account and you invested it at 8.25% per year.
- How much could you withdraw at the beginning of each of the next 20 years?

Practice: Annuity

Your grandmother just died and left you \$100,000 in a trust fund that pays 6.5% interest. You must spend the money on your college education, and you must withdraw the money in 4 equal installments, beginning immediately. How much could you withdraw today and at the beginning of each of the next 3 years and end up with zero in the account?

Practice: Perpetuity

- What annual payment must you receive in order to earn a 6.5% rate of return on a perpetuity that has a cost of \$1,250?

Practice: Growing Annuity

- o Tom Adams has received a job offer from a large investment bank as a clerk to an associate banker.
- o His base salary will be \$45,000. He will receive his first annual salary payment one year from the day he begins to work.
- o In addition, he will get an immediate \$10,000 bonus for joining the company.
- o His salary will grow at 3.5 percent each year.
- o Each year he will receive a bonus equal to 10 percent of his salary.
- o Mr. Adams is expected to work for 25 years.
- o What is the present value of the offer if the discount rate is 12 percent?

Loan Types and Loan Amortization

- o A loan might be repaid in **equal installments** it might be repaid in a **single lump sum**.
- o In this section, we describe a few forms of repayment that come up quite often, and more complicated forms can usually be built up from these.
- o The three basic types of loans;
 - o Pure discount loans
 - o Interest-only loans
 - o Amortized loans.
- o Working with these loans is a very EASY and straightforward application of the present value principles that we have already developed.

PURE DISCOUNT LOANS

- The *pure discount loan* is the simplest form of loan.
- With such a loan, the borrower receives money today and repays a single lump sum at some time in the future.
- A one-year, 10 percent pure discount loan would require the borrower to repay \$1.10 in one year
- Suppose a borrower was able to repay \$25,000 in five years. If we, acting as the lender, and we wanted a 12% interest rate on the loan, how much would we be willing to lend?

PURE DISCOUNT LOANS

- When the U.S. government borrows money on a short-term basis (a year or less), it does so by selling what are called *Treasury bills*, or *T-bills* for short.
- A T-bill is a promise by the government to repay a fixed amount at some time in the future (3,6,12 months)
- *Treasury bills are pure discount loans.*
- If a T-bill promises to repay \$10,000 in 12 months, and the market interest rate is 7 percent, how much will the bill sell for in the market?

INTEREST-ONLY LOANS

- A second type of loan repayment plan calls for the borrower to pay interest each period and to repay the entire principal at some point in the future.
- Loans with such a repayment plan are called *interest-only loans*.

“if there is just one period, a pure discount loan and an interest-only loan are the same thing”

INTEREST-ONLY LOANS

- For example, with a three-year, 10 percent, interest-only loan of \$1,000
- The borrower would pay
 $\$1,000 \times .10 = \100 in interest at the end of the first and second years.
- At the end of the third year, the borrower would return the \$1,000 along with another \$100 in interest for that year.
- Most corporate bonds have the general form of an interest-only loan.

AMORTIZED LOANS

- o *Amortized loan* which the lender may require the borrower to repay parts of the loan amount over time.
- o The process of providing for a loan to be paid off by making regular principal reductions is called *amortizing* the loan.
- o A simple way of amortizing a loan is to have the borrower pay the interest each period plus some fixed amount.

AMORTIZED LOANS

- o This approach is common with medium-term business loans.
- o For example, suppose a business takes out a \$5,000, five-year loan at 9 percent.
- o The loan agreement calls for the borrower to pay the interest on the loan balance each year and to reduce the loan balance each year by \$1,000.
- o Because the loan amount declines by \$1,000 each year, it is fully paid in five years.

AMORTIZED LOANS

- o In the case we are considering, notice that the total payment will decline each year.
- o The reason is that the loan balance goes down, resulting in a lower interest charge each year, whereas the \$1,000 principal reduction is constant.
- o For example, the interest in the first year will be $\$5,000 \times .09 = \450 .
- o The total payment will be $\$1,000 + 450 = \$1,450$.
- o In the second year, the loan balance is \$4,000,
- o So the interest is $\$4,000 \times .09 = \360 , and the total payment is \$1,360.
- o We can calculate the total payment in each of the remaining years by preparing a simple amortization schedule as follows:

AMORTIZED LOANS

- We can calculate the total payment in each of the remaining years by preparing a simple amortization schedule as follows:

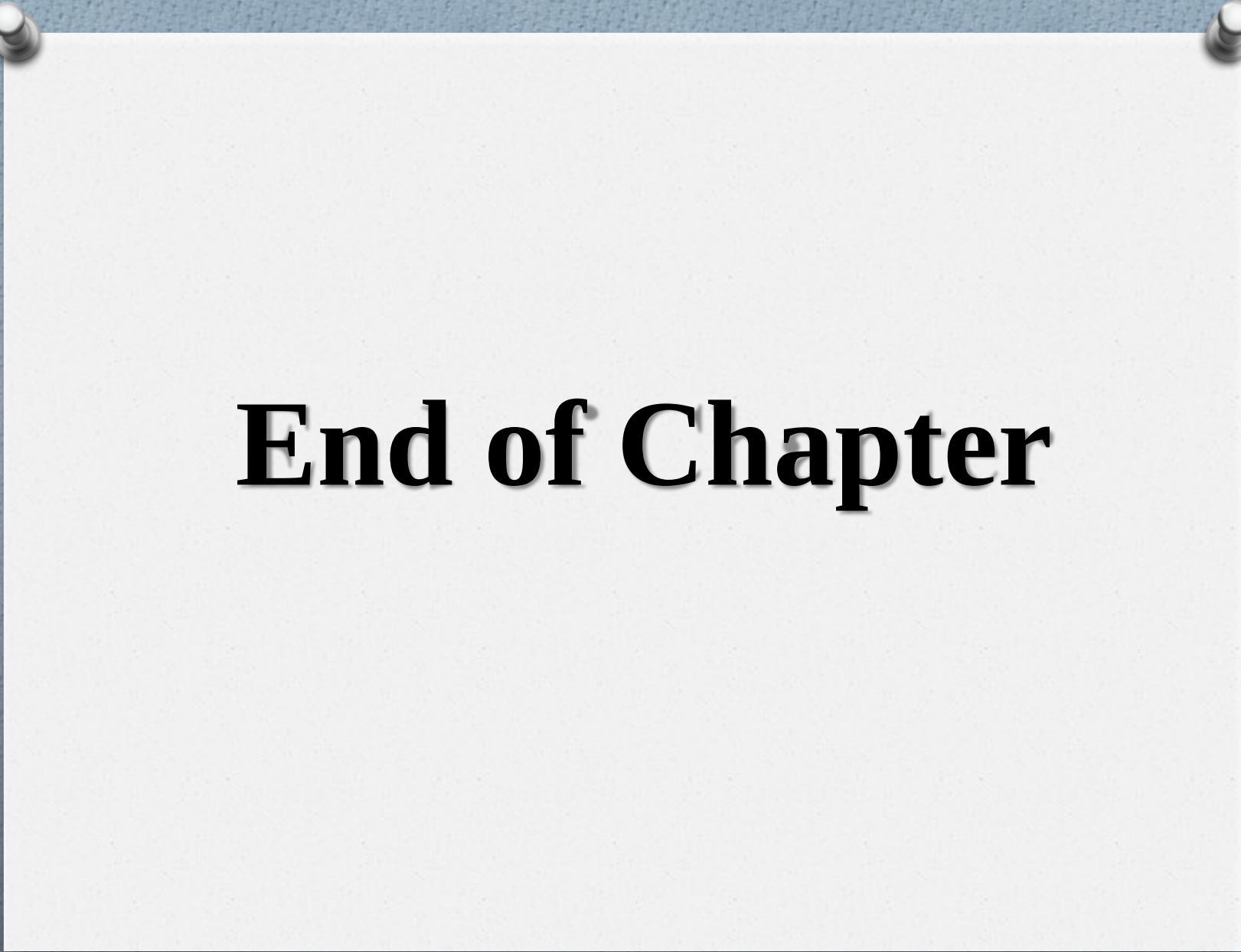
Year	Beginning Balance	Total Payment	Interest Paid	Principal Paid	Ending Balance
1	\$5,000	\$1,450	\$ 450	\$1,000	\$4,000
2	4,000	1,360	360	1,000	3,000
3	3,000	1,270	270	1,000	2,000
4	2,000	1,180	180	1,000	1,000
5	1,000	1,090	90	1,000	0
Totals		\$6,350	\$1,350	\$5,000	

Practice Question6

- o The Happy Hang Glide Company is purchasing a building and has obtained a \$190,000 mortgage loan for 20 years.
- o The loan bears a compound annual interest rate of 17 percent and calls for equal annual installment payments at the end of each of the 20 years.
- o What is the amount of the annual payment?

Practice Question 7

- Calculating Loan Payments You need a 30-year, fixed-rate mortgage to buy a new home for \$250,000.
- Your mortgage bank will lend you the money at a 6.8 percent APR for this 360-month loan.
- However, you can only afford monthly payments of \$1,200, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment.
- How large will this balloon payment have to be for you to keep your monthly payments at \$1,200?



End of Chapter

- o A local finance company quotes a 15 percent interest rate on one-year loans.
- o So, if you borrow \$26,000, the interest for the year will be \$3,900.
- o Because you must repay a total of \$29,900 in one year, the finance company requires you to pay $\$29,900/12$, or \$2,491.67, per month over the next 12 months.
- o Is this a 15 percent loan? What rate would legally have to be quoted? What is the effective annual rate?

- o An insurance company is offering a new policy to its customers.
- o Typically the policy is bought by a parent or grandparent for a child at the child's birth.
- o The details of the policy are as follows: The purchaser (say, the parent) makes the following six payments to the insurance company:

First birthday:	\$ 800
Second birthday:	\$ 800
Third birthday:	\$ 900
Fourth birthday:	\$ 900
Fifth birthday:	\$1,000
Sixth birthday:	\$1,000

- o After the child's sixth birthday, no more payments are made. When the child reaches age 65, **he or she receives \$350,000**. If the relevant **interest rate is 11 percent** for the first six years and 7 percent for all subsequent years, is the policy worth buying?

- o A financial planning service offers a college savings program.
- o The plan calls for you to make six annual payments of \$8,000 each, with the first payment occurring today, your child's 12th birthday.
- o Beginning on your child's 18th birthday, the plan will provide \$20,000 per year for four years.
- o What return is this investment offering?

- o Your Christmas ski vacation was great, but it unfortunately ran a bit over budget.
- o All is not lost: You just received an offer in the mail to transfer your \$9,000 balance from your current credit card, which charges an annual rate of 18.6 percent, to a new credit card charging a rate of 8.2 percent.
- o How much faster could you pay the loan off by making your planned monthly payments of \$200 with the new card?
- o What if there was a 2 percent fee charged on any balances transferred?

1. What is the present value of an annuity of \$5,000 per year, with the first cash flow received three years from today and the last one received 25 years from today? Use a discount rate of 8 percent.
2. What is the value today of a 15-year annuity that pays \$750 a year? The annuity's first payment occurs six years from

- A **15-year annuity** pays \$1,500 per month, and payments are made at the end of each month.
- If the interest rate is 13 percent compounded monthly for the **first seven years**, and 9 percent compounded monthly thereafter, what is the present value of the annuity?

- o You are saving for the college education of your two children.
- o They are two years apart in age; one will begin college 15 years from today and the other will begin 17 years from today.
- o You estimate your children's college expenses to be \$35,000 per year per child, payable at the beginning of each school year.
- o The annual interest rate is 8.5 percent.
- o How much money must you deposit in an account **each year** to fund your children's education?
- o Your deposits begin one year from today.
- o You will make your last deposit when your oldest child enters college. Assume four years of college.

- o You have recently won the super jackpot in the Washington State Lottery.
- o You have the following two options:
 - o 1. You will receive 31 annual payments of \$175,000, with the first payment being delivered today.
 - o The income will be taxed at a rate of 28 percent.
 - o Taxes will be withheld when the checks are issued.
 - o 2. You will receive \$530,000 now, and you will not have to pay taxes on this amount.
 - o In addition, beginning one year from today, you will receive \$125,000 each year for 30 years.
 - o The cash flows from this annuity will be taxed at 28 percent.
- o Using a discount rate of 10 percent, which option should you select?

- o You need a 30-year, fixed-rate mortgage to buy a new home for \$250,000.
- o Your mortgage bank will lend you the money at a 6.8 percent APR for this 360-month loan.
- o However, you can only afford monthly payments of \$1,200, so you offer to pay off any remaining loan balance at the end of the loan in the form of a single balloon payment.
- o How large will this balloon payment have to be for you to keep your monthly payments at \$1,200?

- On September 1, 2007, Susan Chao bought a motorcycle for \$25,000.
- She paid \$1,000 down and financed the balance with a five-year loan at a stated annual interest rate of 8.4 percent, compounded monthly.
- She started the monthly payments exactly one month after the purchase (i.e., October 1, 2007).
- Two years later, at the end of October 2009, Susan got a new job and decided to pay off the loan.
- If the bank charges her a 1 percent **prepayment penalty** based on the loan balance, how much must she pay the bank on November 1, 2009?