

EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is **studentID\_PT**
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14<sup>th</sup>, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by  $P = 10 - Q^2$  and the market supply is given by  $Q = a + P$ , where  $P$  is the unit price,  $Q$  is the quantity of output, and  $a$  is the coefficient in the supply equation.

1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of  $a$  equal to  $-14$ .

1.2) Solve for the market equilibrium quantity ( $Q^*$ ) and price ( $P^*$ ) when  $a = -14$ . Show your work.

1.3) If " $a$ " increases to  $-12$ , what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

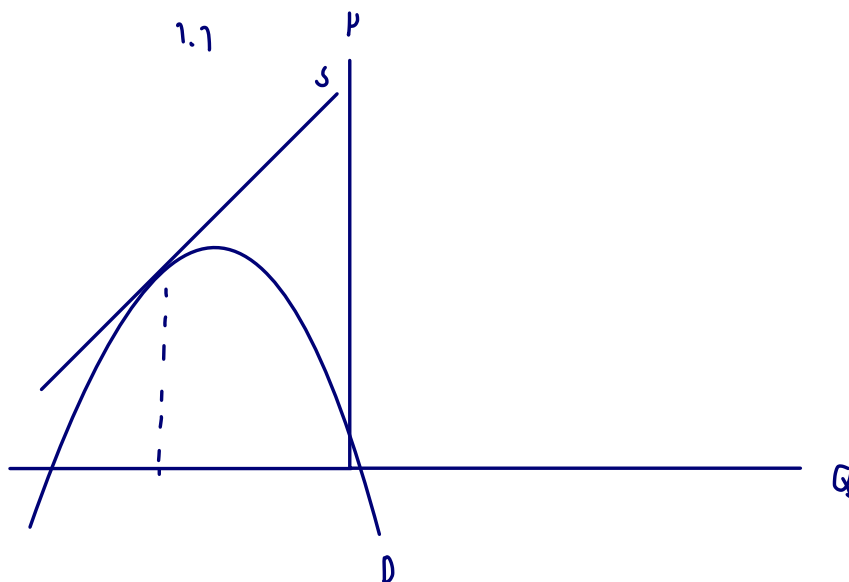
$$P = 10 - Q^2, \quad Q = -14 + P \quad P = Q + 14$$

$$Q + 14 = 10 - Q^2$$

$$Q^2 + Q + 4 = 0$$

$$Q = -\frac{1}{2} + \frac{\sqrt{15}}{2}i, \quad -\frac{1}{2} - \frac{\sqrt{15}}{2}i$$

1.2



$$1.3 \quad P = 10 - Q^2$$

$$Q = -12 + P$$

$$P = Q + 12$$

$$Q + 12 = 10 - Q^2$$

$$Q^2 + Q + 2 = 0$$

$$(Q + 2)(Q - 1) = 0$$

$$Q = -2, 1$$

$$\text{if } Q = -2; P = 10$$

$$\text{if } Q = 1; P = 13$$

2. Suppose that the revenue function is given by  $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right)$ ,  $Q \geq 0$ . Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$\begin{aligned} R'(Q) &= \left( \ln(Q^2 + 1) + \frac{3Q}{Q+1} \right) \\ &= \frac{d}{dQ} (\ln(Q^2 + 1)) + \frac{d}{dQ} \left( \frac{3Q}{Q+1} \right) \\ &= \frac{1}{Q^2 + 1} \times 2Q + \frac{3(Q+1) - 3Q}{(Q+1)^2} \\ &= \frac{2Q}{Q^2 + 1} + \frac{3}{(Q+1)^2} \end{aligned}$$

3. Suppose that the profit function is given by  $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$  where  $Q$  is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\pi'(Q) = -Q^2 - 2Q + 8$$

$$\pi''(Q) = -2Q - 2$$

4. Suppose that  $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ , calculate the following object. Show your work.

4.1  $A + B$  no ans

$$4.2 \ A * B = \begin{bmatrix} 42 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

$$4.3 \ \det(A) = 88 - 90 \\ = -2$$

4.4  $\det(B)$  no ans

4.5  $\det(C)$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{vmatrix} = (45 + 84 + 96) - (105 + 48 + 72) \\ = 225 - 225 \\ = 0$$

5. Suppose that  $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$ . Use the partial derivative technique, calculate  $\frac{\partial U}{\partial x}$  and  $\frac{\partial U}{\partial y}$ .

$$\frac{d\pi(Q)}{dQ} = -Q^2 - 2Q + 8 = 0$$

$$= -(Q^2 + 2Q - 8) = 0$$

$$(Q+4)(Q-2) = 0$$

$$Q = 2, -4$$

$$Q = 2, \pi = 25/3$$

$$\frac{d\pi'(Q)}{dQ} = 2Q + 2 = 0$$

$$Q = -1 < 0$$