DISCOUNTING BENEFITS AND COSTS IN FUTURE TIME PERIODS

EE465/EE463 Project Evaluation

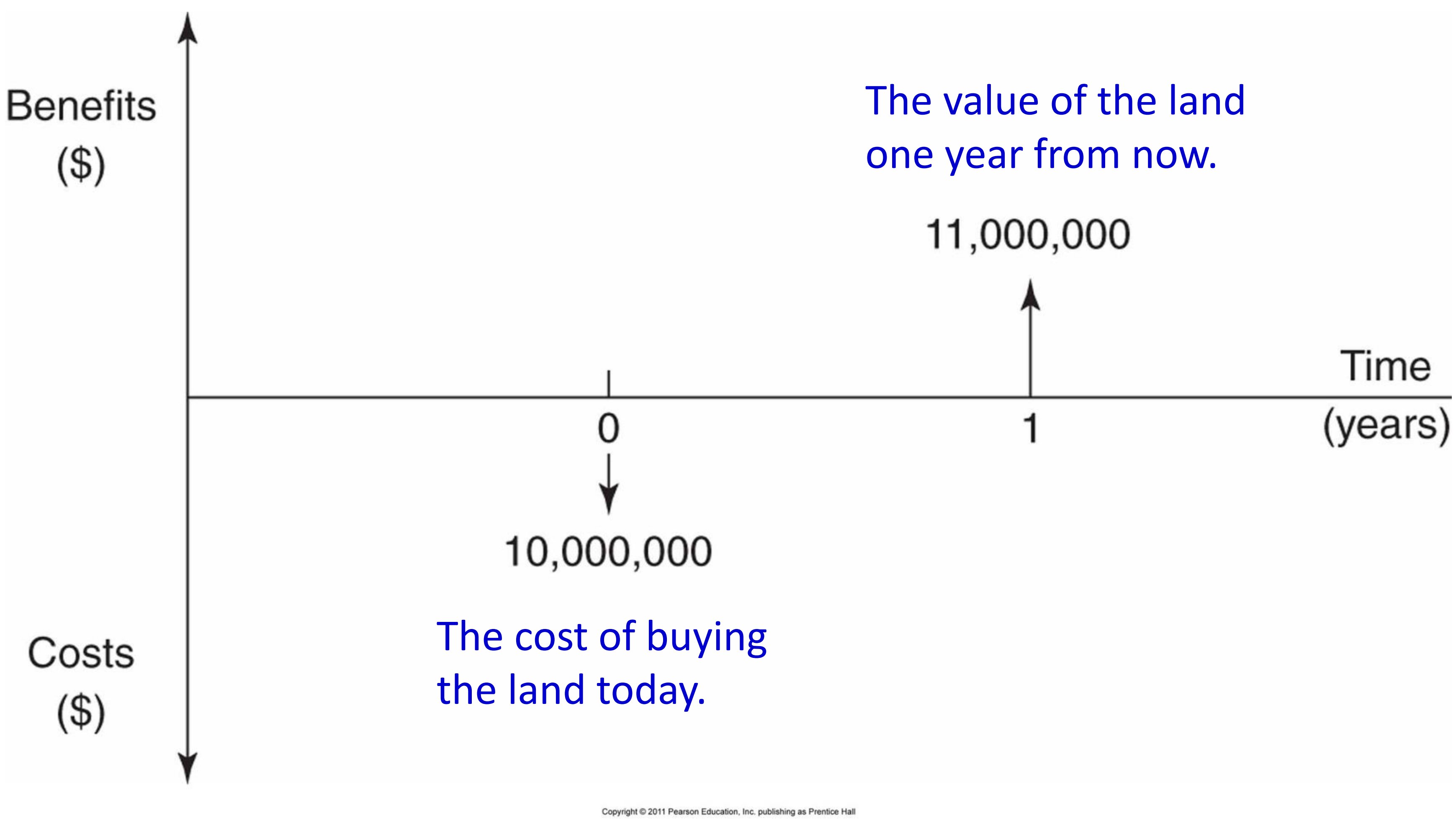
Semester 2/2014

Topics

- The Basics of Discounting
- Compounding and Discounting Over Multiple Years
- Timing of Benefits and Costs
- Long-Lived Projects and Different Time Frames
- Inflation and Real vs. Nominal Dollars
- Relative Price Changes
- Sensitivity Analysis in Discounting

THE BASICS OF DISCOUNTING

Projects with Lives of One Year



Future Value Analysis

- The **future value (*FV*)** in one year of an amount *X* invested at interest rate *i* is:

$$FV = X\,(1 + i)$$

- Example:

Suppose the current value of the land is $10 million. If not buying the land, the govt can invest in Treasury Bills and receive an interest rate of 5%.

  - The future value of the T-bills = $10m*(1+0.05) = $10.5 m.
  - The future value of the land = $11 million.
    - Should buy the land now.

Present Value Analysis

- The **present value (*PV*)** of an amount *Y* received in one year is:

$$PV = \frac{Y}{(1+i)}$$

- Example:

Set FV of the land = $11 million, where *i* = 0.05. What's the PV of the land?

- $PV = \frac{11{,}000{,}000}{(1+0.05)} = \$10,476,190$ (better off buying the land by $476,190)

- In general, if the *FV* of a project equals *Y*, then:

$$PV = \frac{FV}{(1+i)}$$

Net Present Values Analysis

- ***Net Present Value (NPV)*** is the sum of the present values of all the benefits and costs of a project (including the initial investment):

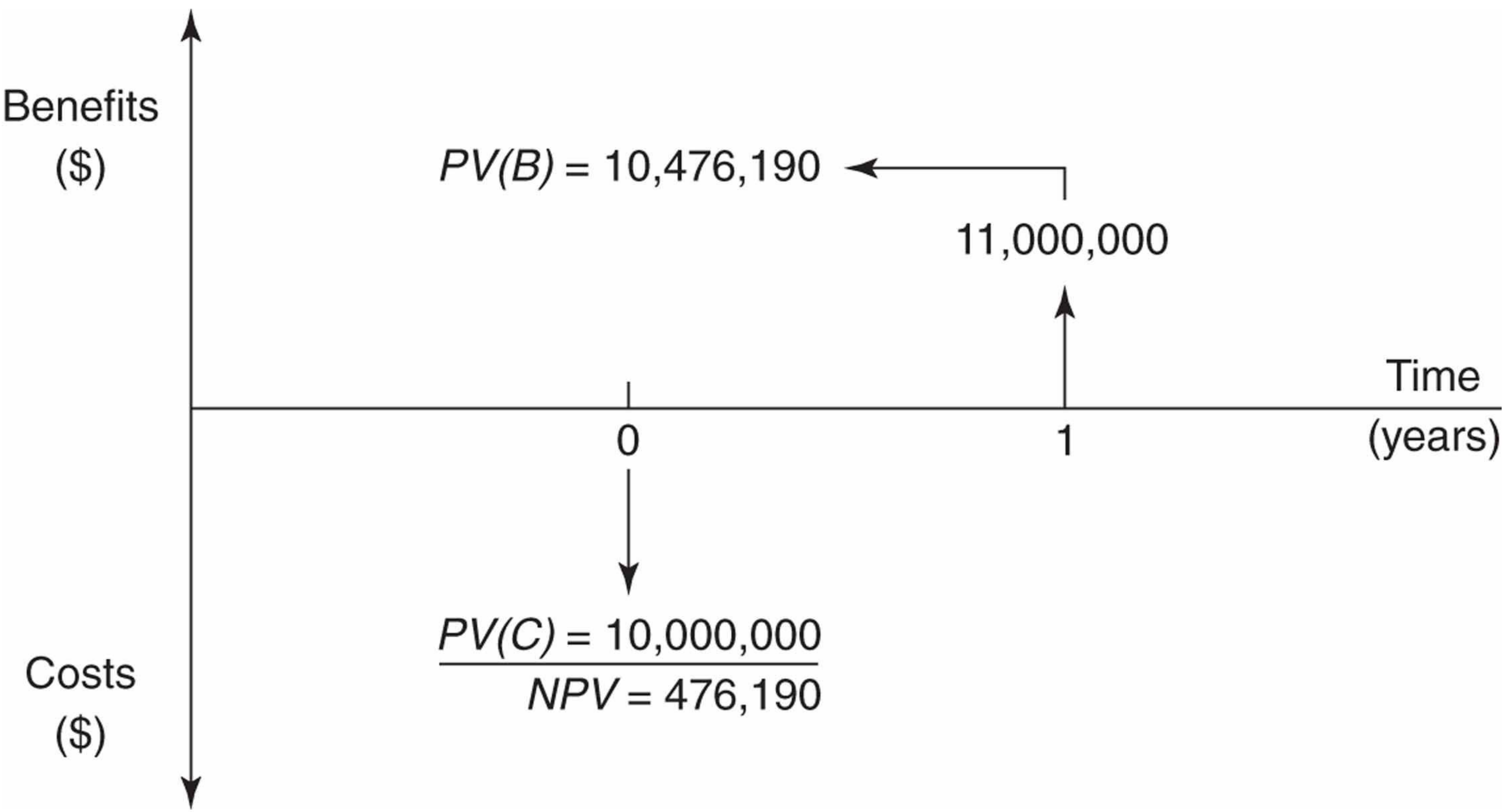
  NPV = PV(B) – PV(C)

- Example:

  NPV of the land = $10,476,190 - $10,000,000 = $476,190

- If there is only *one* new potential project (compared to status quo), it should be selected if its NPV > 0.
- If there are *multiple, mutually exclusive alternative* projects,
  - Select the project with the highest NPV (as long as NPV > 0)
  - Maintain status quo if NPV < 0 for all alternative projects.

NPV of Buying the Land



COMPOUNDING AND DISCOUNTING OVER MULTIPLE YEARS

Future Value over Multiple Years

- Interest on reinvested interest is called *compound interest*.
- The future value, *FV*, of an amount *X* invested for *n* years with interest compounded annually at rate *i* is:

$$FV = x(1 + i)^n$$

- The term $(1 + i)^n$ is called the *compound interest factor*.
  - This gives the future value of $1 in n years at annual interest rate *i*.
- Example:

If $10 million is invested for 4 years with interest compounded annually at 7%, then:

$$FV = \$10m*(1+0.07)^4 = \$13.108m$$

Investment of $10 Million with Interest Compounded Annually at 7%

Year	Beginning of Year Balance ($ millions)	Annual Interest ($ millions)	End of Year Balance ($ millions)
1	10.000	0.700	10.700
2	10.700	0.749	11.449
3	11.449	0.801	12.250
4	12.250	0.858	13.108
5	13.108	0.918	14.026

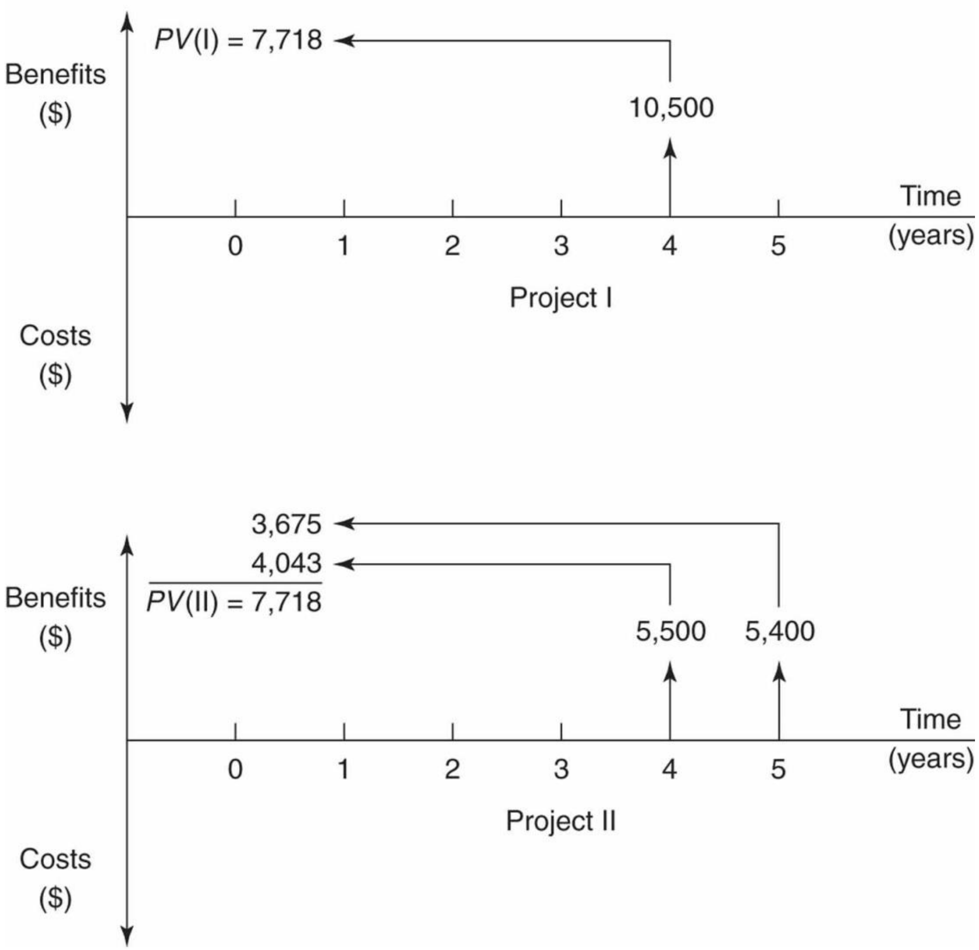
Present Value over Multiple Years

- The present value, *PV,* of an amount Y received in *n* years, with interest compounded annually at rate *i* is:

$$PV = \frac{Y}{(1+i)^n}$$

- The term $1/(1+i)^n$ is called the *present value factor* or *discount factor*.
- Example: Suppose an agency wants to undertake a project that costs $100,000 in three years, and the interest rate is 6%. How much does the agency need to have now to have $100,000 in 3 years?
  - PV = 100000/(1+0.06)^3 = $83,962

Time Lines for Project I and Project II



$$PV(I) = \frac{10{,}500}{1.08^4} = 7{,}718$$

$$PV(II) = \frac{5{,}500}{1.08^4} + \frac{5{,}400}{1.08^5} = 7{,}718$$

Net Present Value of a Project

- **Net present value (NPV)** of a project is the difference between the PV of the benefits and the PV of the costs:

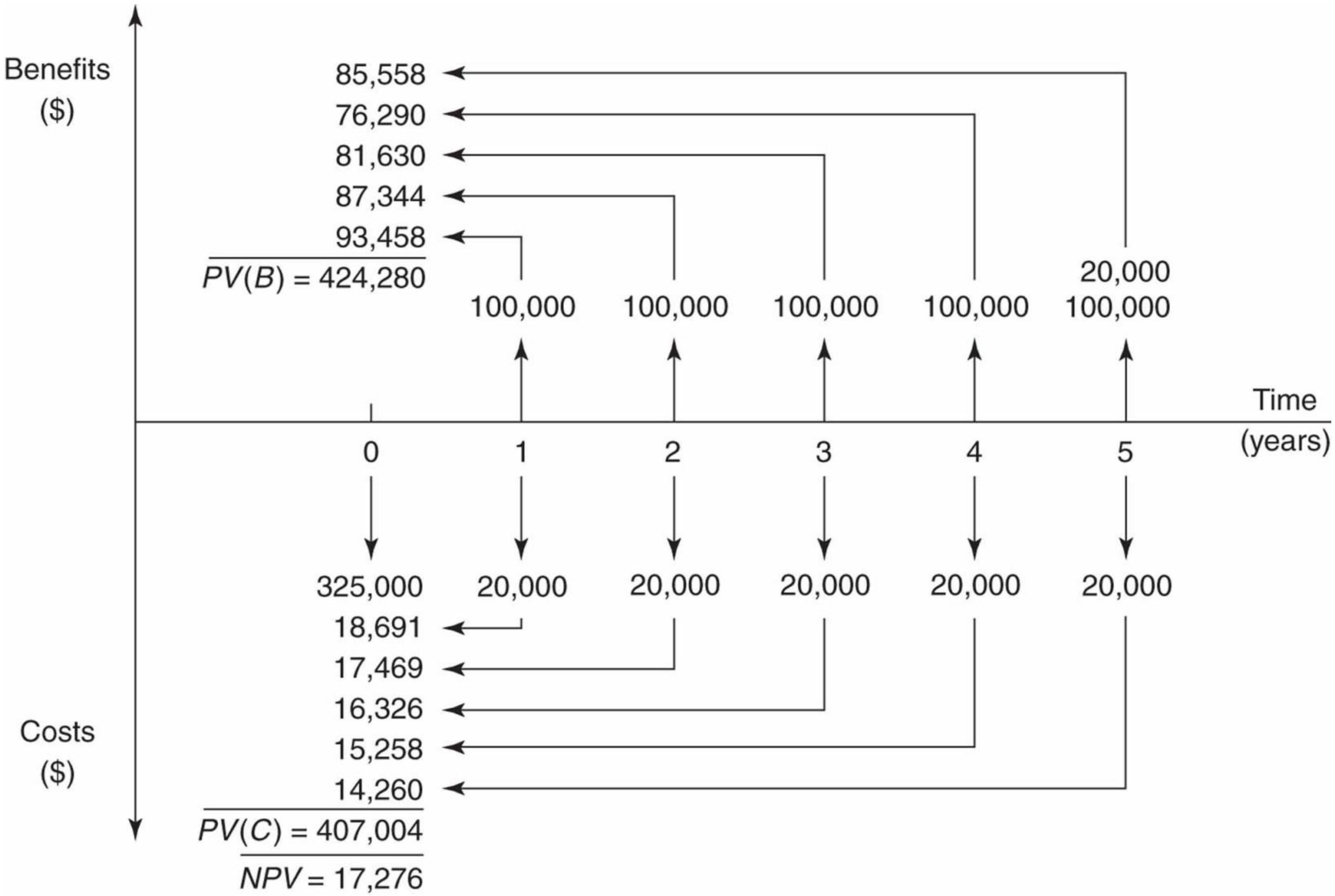
$$NPV = \sum_{t=0}^{n} \frac{B_i}{(1+i)^t} - \sum_{t=0}^{n} \frac{C_i}{(1+i)^t}$$

- Example: Consider an info system which has benefits of $100,000 per year, and the initial set-up cost of $325,000 and the annual maintenance cost of $20,000. The system will be dismantled and sold after five years, and its *terminal value* (or *liquidation value*) is $20,000. (let *i* = 7%)

➔ NPV = $\sum_{t=1}^{5} \frac{100000}{(1+0.07)^t} + \frac{20000}{(1+0.07)^5} - \sum_{t=1}^{5} \frac{20000}{(1+0.07)^t} - 325000$

- Alternatively, NPV of a project is the present value of the net benefits: $NPV = \sum_{t=0}^{n} \frac{NB_t}{(1+i)^t}$ where $NB_t = B_t - C_t$.

Time Line of the Benefits and Costs of the Library Information System



The NPV of the Library Information System

Year	*Event*	*Annual Benefits*	*Annual Costs*	*Annual Net Benefits*
0	Purchase and install	0	325,000	−325,000
1	Annual benefits and costs	100,000	20,000	80,000
2	Annual benefits and costs	100,000	20,000	80,000
3	Annual benefits and costs	100,000	20,000	80,000
4	Annual benefits and costs	100,000	20,000	80,000
5	Annual benefits and costs	100,000	20,000	80,000
5	Liquidation	20,000	0	20,000
	PV	$424,280	$407,004	$17,276

TIMING OF BENEFITS AND COSTS

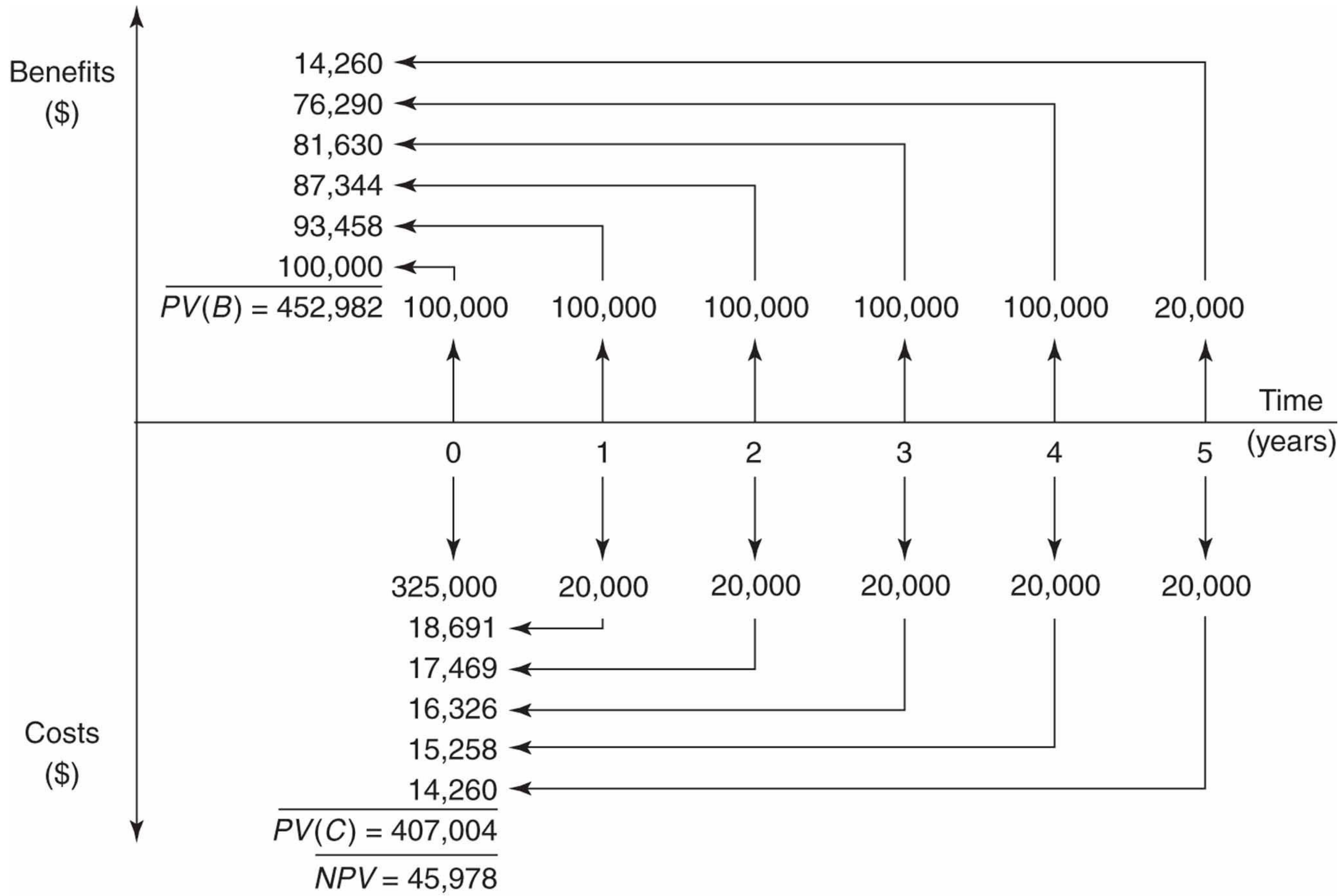
- So far, we have assumed that impacts occur immediately, or at the *end* of the first year, or at the *end* of the second year, and so on.
  - ➢ *NPV* is lower than if it were computed under an alternative assumption.
- However, it is possible that the benefits or costs can occur at the *beginning* or in the *middle* of the year.

➢ Example: Reconsider the library information system. Assume instead that the annual benefits of $100,000 occur at the beginning of the year (instead of the end of the year).

  - The PV of benefits = $\sum_{t=0}^{4} \frac{100000}{(1+0.07)^t} + \frac{20000}{(1+0.07)^5}$ ➔ NPV = $45,978

- When costs or benefits occur over the course of the year, a more appropriate approach is to treat as if they occur in the *middle* of year.
  - t = 0.5, 1.5, 2.5, 3.5, …

Time Line of the Benefits and Costs of the Library Information System Assuming User Benefits Occur at the Beginning of Each Year



COMPARING PROJECTS WITH DIFFERENT TIME FRAMES

Projects with Different Time Frames

- Projects with different time frames are not directly comparable.
- Consider 2 alternative projects:
    1. Large hydroelectric dam (HED) –75 years; NPV = $30m
    2. Cogeneration plant (CGP) – 15 years; NPV = $24m.
- Is HED preferred to CGP?
    - No!
- Two methods for evaluating projects with different time frames (which always lead to the same conclusion):
    1. Roll-over method
    2. Equivalent annual net benefit (EANB)

Roll-Over Method

- If project A spans *n* times the number of years as project B, then assume that project B is repeated *n* times and compare the *NPV* of *n* repeated project B's to the NPV of (one) project A.
- <u>Example</u>:

  HED lasts 75 years and CGP lasts 15 years.

  - Compare the NPV of 75-year HED to the *NPV* of 5 back-to-back CGP's:

$$\text{NPV(5CGP)} = 24m + \frac{24m}{(1+0.08)^{15}} + \frac{24m}{(1+0.08)^{30}} + \frac{24m}{(1+0.08)^{45}} + \frac{24m}{(1+0.08)^{60}}$$

= $34.94 m

➔ NPV(5CGP) > NPV(HED)

Equivalent Annual Net Benefits (EANB) Method

- The EANB is the amount received each year for the life of the project that has the same *NPV* as the project itself.
- EANB is computed by dividing the NPV of the project by the *annuity factor* a_i^n:

$$EANB = \frac{NPV}{a_i^n}$$

where $a_i^n = \frac{1-(1+i)^{-n}}{i}$ is the PV of an annuity of $1 for the life of the project. (see appendix A for details.)

- Example:
  - EANB(HED) = $30/12.461 = $2.407m
  - EANB(CGP) = $24/8.559 = $2.804m

INFLATION AND REAL VERSUS NOMINAL DOLLARS

Real vs. Nominal Dollars

- Conventional private sector financial analysis measures monetary amounts in *nominal dollars* (a.k.a. *current dollars*).
- But, due to inflation, one cannot buy as many goods and services with a dollar today as one could one, two or more years previously
  - i.e., “a dollar’s not worth a dollar anymore”.
- We control for inflation by converting *nominal dollars* to *real dollars* (a.k.a. *constant dollars*).
- We usually use the consumer price index (CPI) deflator, but sometimes use the gross domestic product (GDP) deflator.

Thailand's CPI

Year	**Headline CPI (2011=100)**	**(% change)**
2000	74.50	1.60
2001	75.70	1.60
2002	76.20	0.70
2003	77.60	1.80
2004	79.80	2.70
2005	83.40	4.50
2006	87.30	4.70
2007	89.20	2.30
2008	94.10	5.50
2009	93.30	-0.90
2010	96.33	3.30
2011	100.00	3.81
2012	103.02	3.02
2013 p	105.27	2.18
2014 p	107.26	1.89

Source: http://www2.bot.or.th/statistics/ReportPage.aspx?reportID=409&language=eng

Problems with the CPI

- CPI tends to *overstate inflation* due to two reasons:

1) Commodity substitution effect: The CPI basket of goods does not accurately reflect consumers' purchases because people quickly switch to lower-priced substitutes as prices rise.
   - the "discount stores" effect
   - the "new goods" problem: e.g. new & cheaper generic drugs, "better quality" products such as iPads or Blackberrys.

2) Quality improvements: The CPI does not accurately reflect improvements in product quality to existing goods.
   - e.g. cars are more safe or reliable.

Analyzing Future Benefits and Costs in CBA (1)

- Analysts should either
  - measure the benefits and costs in real dollars and discount using a real discount rate, or
  - measure the benefits and costs in nominal dollars and discount using a nominal discount rate.
- Suppose the expected annual rate of inflation during the life of the project is denoted by *m*.
  - ➢ Benefits or costs that are given in nominal dollars may be converted to real dollars:

$$\text{Real cost or benefit}_t = \frac{\text{Nominal cost or benefitt}}{(1+m)^t}$$

Analyzing Future Benefits and Costs in CBA (2)

Example:

- If a city could invest \$10m for 5 years at 7%, the nominal future value would be:

$$\$10*(1+0.07)^5 = \$14.026 \text{ million}$$

- With an expected inflation rate of 4%, the real future value would be:

$$\$14.026/(1+0.04)^5 = \$11.53 \text{ million}$$

- Hence, the real interest rate, *r,* is defined by:

$$(1+r) = \frac{(1+i)}{(1+m)} \rightarrow r = \frac{(i-m)}{(1+m)}$$

- If *m* is small, then: $r \approx i - m$.

Example:
NPV of Investment in New Garbage Trucks

Year	*Event*	*Annual Benefits and Costs (in real dollars)*	*Annual Benefits and Costs (in nominal dollars)*
0	Truck purchase	−500,000	−500,000
1	Annual savings	100,000	104,000
2	Annual savings	100,000	108,160
3	Annual savings	100,000	112,486
4	Annual savings	100,000	116,986
4	Truck liquidation	200,000	233,972
	NPV	66,812[a]	66,812[b]

[a] *Using a real discount rate of 1.923 percent.*
[b] *Using a nominal discount rate of 6 percent.*

RELATIVE PRICE CHANGES

- Generally, we assume that the prices of all goods and services change at the same rate – the general rate of inflation.
- However, the price of a good may change faster or slower than this one. If so, it experiences a *relative price change*.
- If the NPV of a project depends importantly on the price of this item, then its price should be analyzed separately.
- Example:

A development project in BC is to supply coal to Japanese customers. Consider 2 situations:

1) The price of coal were to fall by 90% of the base price.
2) The price of coal were to fall by 90% and Japanese customers cut back their purchases to 90% of their expected orders.

CBA of North East Coal Development Project

	Benefits ($ million)	Costs ($ million)	Net Benefits: Base Case ($ million)	Net Benefits: 90% Base Price ($ million)	Net Benefits: 90% Base Price and Quantity ($ million)
Mining Sector	3,316	3,260*	56	−146	−240
Transport Sector					
Trucking	33	33	0	0	0
Canadian National Railway	504	358	146	146	121
B.C. Railway	216	202	15	15	6
Port Terminal	135	150	−15	−15	−23
Analysis and Survey	11	11	0	0	0
British Columbia**					
Royalties	77	0	77	69	62
Corporate Taxes	154	0	154	125	107
Producer Surplus (Labor)	25	0	25	25	25
Environment	10	5	5	5	5
Highways***	0	88	−88	−88	−88
Tumbler Ridge Branchline	91	267	−176	−176	−185
Canada					
Corporate Taxes	132	0	132	107	92
Highways, Port Navigation	0	26	−26	−26	−26
Producer Surplus (Labor)	25	0	25	25	25
Totals	4,729	4,400	330	66	−119

*Includes taxes and royalties.
**Excluding impacts included elsewhere.
***Highways, electric power, townsite.

Source: Based on W. G. Waters II, "A Reanalysis of the North East Coal Development," (undated), Tables 2 & 3. All figures in millions of 1980 dollars, assuming a real discount rate of 10 percent with the discounting period ending in 2003 and no terminal value.

LONG LIVED PROJECTS AND TERMINAL VALUES

Long Lived Project (1)

- If benefits or costs occur indefinitely, then NPV can be calculated:

$$NPV = \sum_{t=0}^{\infty} \frac{NB_t}{(1+i)^t}$$

- If projects can be reasonably divided into two periods – a "near future"(the first *k* periods or *discounting horizon*), and a "far future" (the subsequent periods), then, the *NPV* can be computed:

$$NPV = \sum_{t=0}^{k} \frac{NB_t}{(1+i)^t} + PV(H_k)$$

where H_k *is* the *horizon value* – the NPV at the end of the discounting horizon of all subsequent impacts.

Long Lived Project (2)

Example

- Suppose the useful life of a highway is 20 years and the horizon value of the highway in 20 years is $100 million.
- Suppose further that the discount rate is 3.5%.

$$\rightarrow \; PV(H_k) = \frac{\$100\ million}{(1+0.035)^{20}} = \$50.26\ million$$

Alternative Methods for Estimating Horizon Values (1)

- **Horizon value based on simple projections** - Horizon value is estimated based on simple extrapolations of benefits and costs (or net benefits).

Example

- Consider a construction of a new dam. The annual net benefits have been calculated for 35 years. Suppose that the expected NB in the 36th year will be \$1 million, and it will grow at a constant rate 1.5% per annum indefinitely. (*i = 8%*)
- The PV of the horizon value can be calculated from:

$$PV(B) = \frac{B_1}{i - g} \text{ if } i > g.$$

$$\rightarrow PV(H_{35}) = \frac{\$1\, million}{0.08 - 0.015} = \$15.38\, million$$

Alternative Methods for Estimating Horizon Values (2)

- **Horizon value based on salvage value or liquidation value** - Horizon value is the scrap value of the assets of a project. This method is appropriate when:

  1) No other costs or benefits arise beyond the discounting period.

  2) There is a well functioning market in which to value the asset.

  3) The market values reflect social values (i.e., no externalities).

Example: Recall the library information system example. The NSB of the equipment liquidation is $20,000.

- If the equipment could be used in schools where its social value is more than $20,000, then a higher horizon value should be used.

Alternative Methods for Estimating Horizon Values (3)

- **Estimating horizon value based on depreciated value -** This method estimates the value of an asset by subtracting its depreciation from its initial value.
- The depreciation is calculated based on empirical market studies of similar assets.
- One should make adjustments where appropriate, for example, if the asset is used more or less intensely than average.
- Note: One never uses accounting depreciation in CBA.
  - Economic depreciation – the decline in the economic value of an asset over time.
  - Account depreciation (capital cost allowance: CCA) – determined by tax or reporting requirements.

Alternative Methods for Estimating Horizon Values (4)

- **Estimating horizon value based on the initial construction cost**
  - This method uses some arbitrary proportion of the initial construction cost as a horizon value.

Example: Suppose the construction cost of a highway is $338 million, and its horizon value is assumed to be 75% of the initial construction cost.

➔ Hk = 0.75*$338 = $253.3 million

- **Set the horizon value equal to zero -** This method chooses a fairly long discounting period and sets the present value of subsequent net benefits to zero.
  - If the discounting period is too short, this method may omit important impacts.

TIME-DECLINING DISCOUNTING

- Example: Consider a project that lasts for more than 300 years. The discount rates are as follows. [Assume continuous compounding ➔ PV = Y/(e^{in}).]

Year	Discount rate
0 - 50	3.5 percent
50-100	2.5 percent
100-200	1.5 percent
200-300	0.5 percent
Over 300	0.0 percent

- If the project has a cost of $1 million today and a benefit of $1 billion in 400 years.

➔ $PV(B) =$ $1 B*[($e^{-0.035*50}$) * ($e^{-0.025*(100-50)}$) * ($e^{-0.015*(200-100)}$) * ($e^{-0.005*(300-200)}$)] = $6.74 million

➔ NPV = $5.74 million

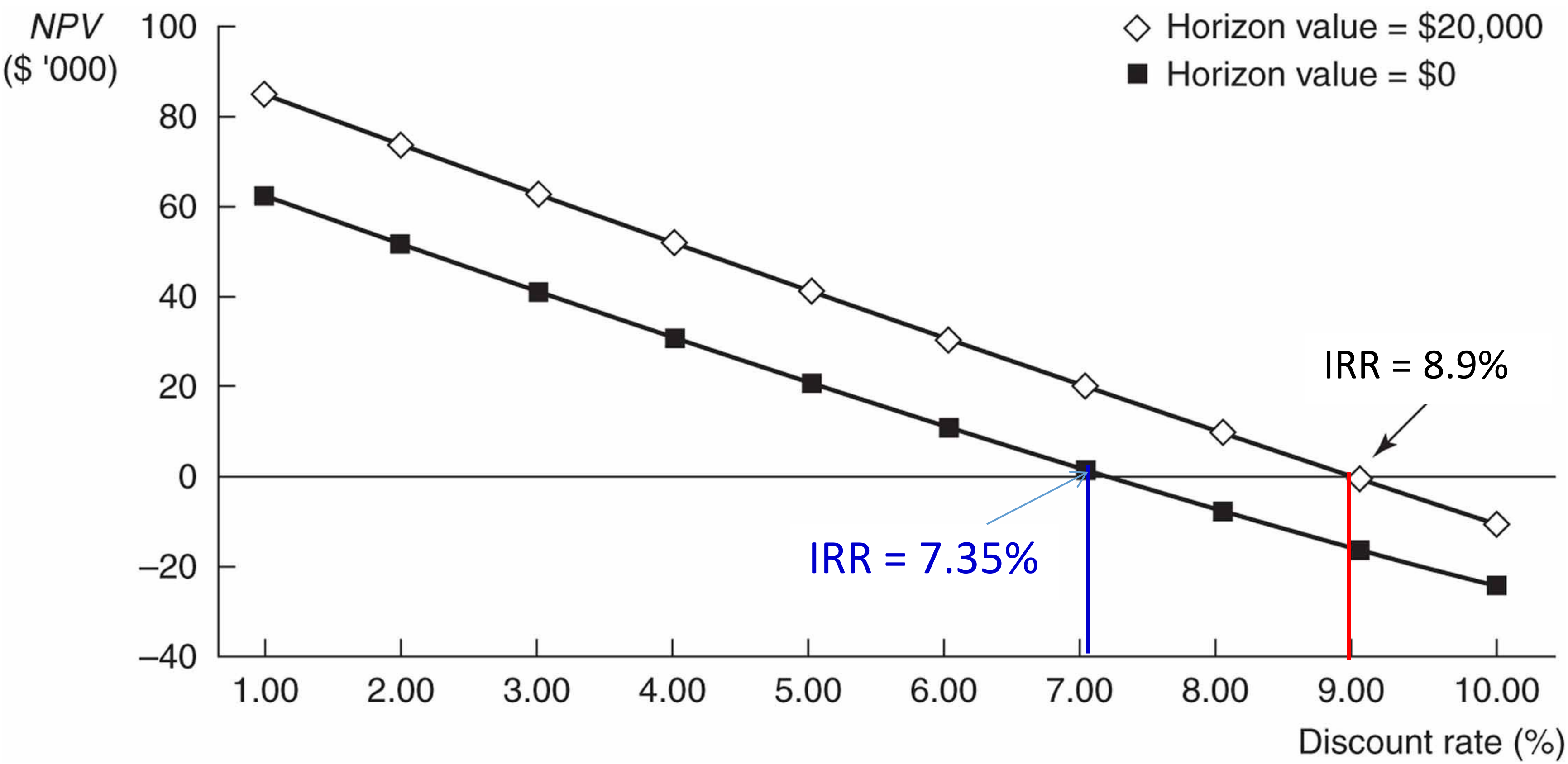
SENSITIVITY ANALYSIS IN DISCOUNTING

Varying the Discount Rate and Horizon Value

- There is often some disagreement remains about the appropriate discounting method and the value of the social discount rate.
- Consequently, sensitivity analysis should usually be conducted on the discount rate.
- The horizon value is also a target for sensitivity analysis.
- It is helpful and easy to plot the NPV of a project against the discount rate for one or two estimates of the horizon value.
  - Example:

Recall the library information system. Consider two different horizon values: $20,000 and $0.

Sensitivity Analysis: NPV of the Library Information System as a Function of the Discount Rate and the Horizon Value



Internal Rate of Return (IRR)

- The IRR (or *breakeven discount rate*) of a project equals the discount rate at which the project's NPV would equal 0.
- Example:
  - If horizon value = \$20,000, IRR = 8.9%

$$NPV = 0 = -325{,}000 + \sum_{t=1}^{5} \frac{100000}{(1+IRR)^t} + \frac{20000}{(1+IRR)^5} \Rightarrow IRR = 0.089$$

  - If horizon value = \$0, IRR = 7.35%

$$NPV = 0 = -325{,}000 + \sum_{t=1}^{5} \frac{100000}{(1+IRR)^t} + \frac{0}{(1+IRR)^5} \Rightarrow IRR = 0.0735$$

- *Breakeven horizon value* – horizon value at which NPV = 0.

$$NPV = 0 = -325{,}000 + \sum_{t=1}^{5} \frac{100000}{(1+0.07)^t} + \frac{H_{BE}}{(1+0.07)^5} \Rightarrow H_{BE} = -4{,}230$$

Using IRR as a Decision Rule

- IRR indicates the *effective annual rate of return* on the project.
- *When there is only one alternative to the status quo*, one should invest in the project if its IRR > social discount rate. Otherwise, do not proceed.
- However, there are some problems using the IRR as a decision rule.
  - There may be more than one discount rate at which NPV = 0.
  - IRRs are percentages, not dollar values.
- Hence, although the IRR conveys useful information about how sensitive the results are to the discount rate, the NPV rule for decision making is still preferred.