

# EE/EC411: Microeconomic Analysis

## Continuous Strategy Nash Equilibrium

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## Games with Continuous Actions Spaces

- So far, we considered that players select one among a discrete list of available actions, e.g.,  $s_i \in \{Enter, NotEnter\}$ ,  $s_i \in \{x, y, z\}$ .
- But in some economic settings, agents can select among an infinite list of actions.
  - **Examples:** an output level  $q_i \in \mathbb{R}_+$  (as in the Cournot game of output competition),
  - A price level  $p_i \in \mathbb{R}_+$  (as in the Bertrand game of price competition),
  - Contribution  $c_i \in \mathbb{R}_+$  to a charity in a public good game,
  - Exploitation level  $x_i \in \mathbb{R}_+$  of a common pool resource, etc.

## Cournot Game of Output Competition

- We first assume that  $N = 2$  firms compete selling a homogenous product (no product differentiation).
  - Later on (maybe in a homework) you will analyze the case where firms sell differentiated products (easy! don't worry).
- Firm  $i$ 's total cost function is  $TC_i(q_i) = c_i q_i$ .
  - Note that this allows for firms to be symmetric in costs,  $c_i = c_j$ , or asymmetric,  $c_i > c_j$ .
- Inverse demand function is linear  $p(Q) = a - bQ$ , where  $Q = q_1 + q_2$  denotes the aggregate output,  $a > c$  and  $b > 0$ .

## Cournot Game of Output Competition

- Since  $p(Q) = a - bQ$ , where  $Q = q_1 + q_2$ , the profit maximization problem for firm 1 is therefore

$$\begin{aligned}\max_{q_1} \pi_1(q_1, q_2) &= [a - b(q_1 + q_2)]q_1 - c_1 q_1 \\ &= aq_1 - b(q_1 + q_2)q_1 - c_1 q_1 \\ &= aq_1 - bq_1^2 - bq_1 q_2 - c_1 q_1\end{aligned}$$

## Cournot Game of Output Competition

- Taking first-order conditions with respect to  $q_1$ ,

$$a - 2bq_1 - bq_2 - c_1 = 0$$

and solving for  $q_1$ , we obtain

$$q_1 = \frac{a - c_1}{2b} - \frac{1}{2}q_2$$

## Cournot Game of Output Competition

- Using  $q_1 = \frac{a - c_1}{2b} - \frac{1}{2}q_2$ , note that:
  - $q_1$  is positive when  $q_2 = 0$ , i.e.,  $q_1 = \frac{a - c_1}{2b}$ , but...
  - $q_1$  decreases in  $q_2$ , becoming zero when  $q_2$  is sufficiently large.  
In particular,  $q_1 = 0$ , when

$$0 = \frac{a - c_1}{2b} - \frac{1}{2}q_2 \quad \Leftrightarrow \quad \frac{a - c_1}{b} = q_2$$

## Cournot Game of Output Competition

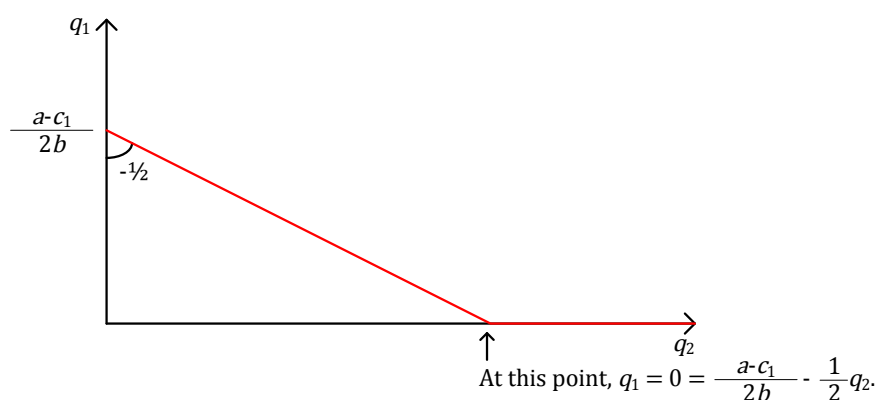
- We can hence, report firm 1's profit maximizing output as follows

$$q_1(q_2) = \begin{cases} \frac{a - c_1}{2b} - \frac{1}{2}q_2 & \text{if } q_2 \leq \frac{a - c_1}{b} \\ 0 & \text{if } q_2 > \frac{a - c_1}{b} \end{cases}$$

- This is firm 1's **best response function**: it tells firm 1 how many units to produce in order to maximize profits as a function of firm 2's output,  $q_2$  [See figure].

## Cournot Game of Output Competition

- Drawing a single BRF:  $q_1(q_2) = \begin{cases} \frac{a-c_1}{2b} - \frac{1}{2}q_2 & \text{if } q_2 \leq \frac{a-c_1}{b} \\ 0 & \text{if } q_2 > \frac{a-c_1}{b} \end{cases}$



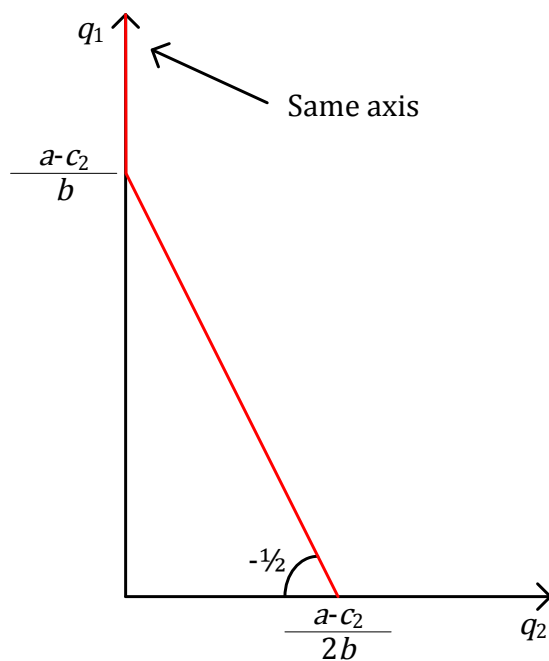
- In order to find the horizontal intercept, where  $q_1 = 0$ , we solve for  $q_2$ , as follows

$$0 = \frac{a-c_1}{2b} - \frac{1}{2}q_2 \implies \frac{a-c_1}{b} = q_2$$

- Hence, the horizontal intercept of  $BRF_1$  is  $q_2 = \frac{a-c_1}{b}$

## Cournot Game of Output Competition

- Similarly for  $BRF_2$ :  $q_2(q_1) = \begin{cases} \frac{a-c_2}{2b} - \frac{1}{2}q_1 & \text{if } q_1 \leq \frac{a-c_2}{b} \\ 0 & \text{if } q_1 > \frac{a-c_2}{b} \end{cases}$
- Note that we depict  $BRF_2$  using the same axis as for  $BRF_1$  in order to superimpose both BRFs later on.

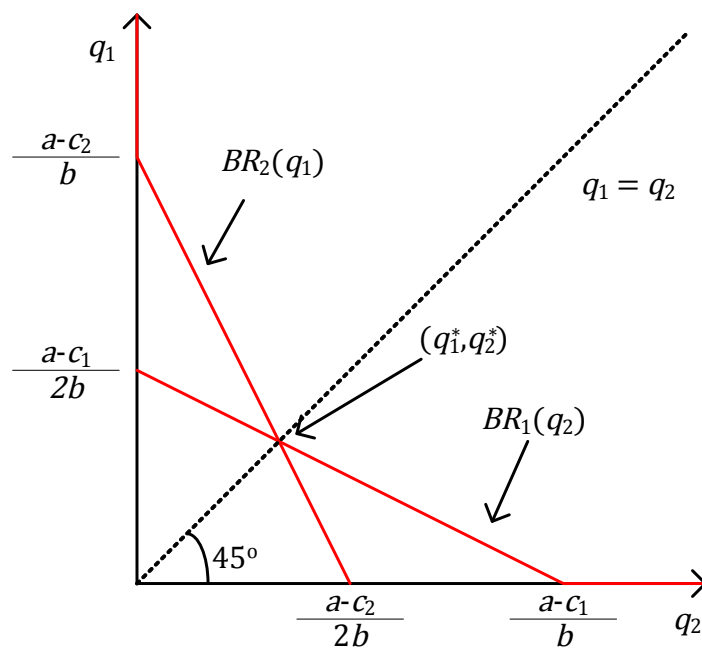


## Cournot Game of Output Competition

- Putting both firms' *BRF* together... we obtain two figures:
  - one for the case in which firms are symmetric in marginal costs,  $c_1 = c_2$ , and
  - another figure for the case in which firms are asymmetric,  $c_2 > c_1$ .

## Cournot Game of Output Competition

- If  $c_1 = c_2$ , (firms are symmetric in costs),



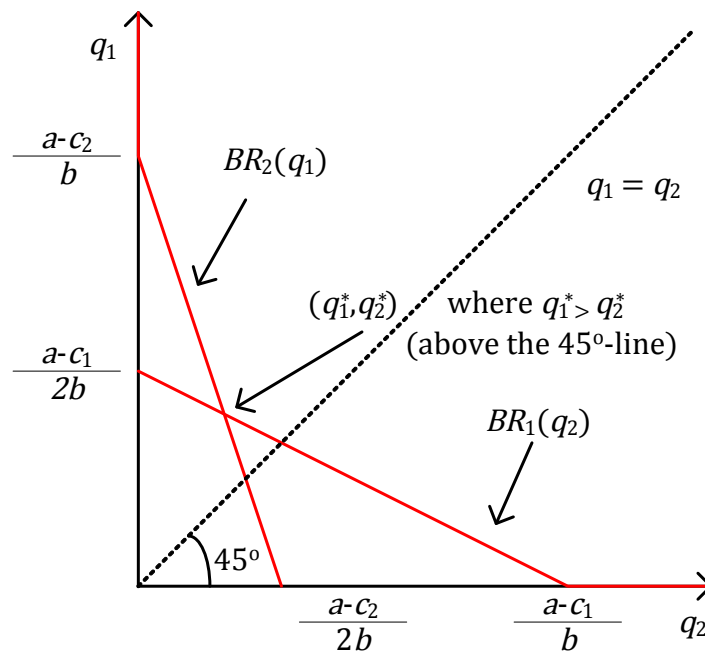
## Cournot Game of Output Competition

- Since  $c_1 = c_2$ , then

$$\frac{a - c_1}{2b} = \frac{a - c_2}{2b} \text{ (vertical intercepts)}$$
$$\frac{a - c_1}{b} = \frac{a - c_2}{b} \text{ (horizontal intercepts)}$$

# Cournot Game of Output Competition

- If  $c_2 > c_1$  (firm 1 is more competitive),



## Cournot Game of Output Competition

- Since  $c_2 > c_1$ ,

$$\frac{a - c_1}{2b} > \frac{a - c_2}{2b} \quad (\text{vertical intercepts})$$
$$\frac{a - c_1}{b} > \frac{a - c_2}{b} \quad (\text{horizontal intercepts})$$

## Cournot Game of Output Competition

- How can we find the NE of this game?
  - We know that each firm must be using its BRF in equilibrium.
  - We must then find the point where  $BRF_1$  and  $BRF_2$  cross each other.
  - Assuming an interior solution,

$$BRF_1 \text{ ' } q_1 = \frac{a - c_1}{2b} - \frac{1}{2}q_2 = \frac{a - c_1}{2b} - \frac{1}{2} \left( \underbrace{\frac{a - c_2}{2b} - \frac{1}{2}q_1}_{BRF_2} \right)$$

and solving for  $q_1$ ,

$$q_1 = \frac{a - 2c_1 + c_2}{3b}$$

Similarly for  $q_2$ ,

$$q_2 = \frac{a - 2c_2 + c_1}{3b}$$

## Cournot Game of Output Competition

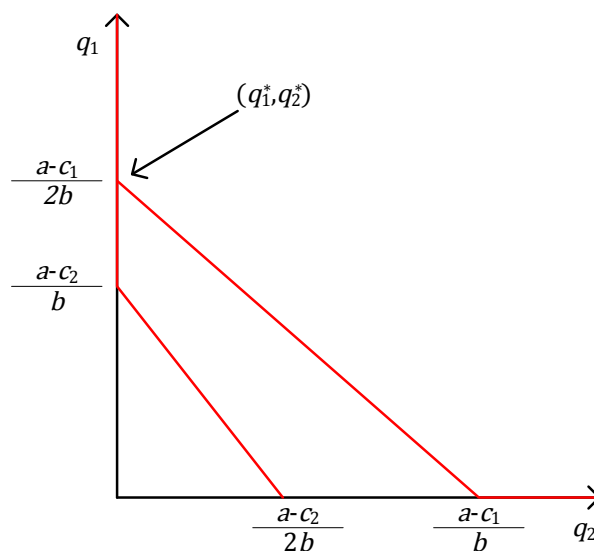
- What about Corner Solutions?
  - Using the figures, we can easily determine a condition for firm 2's equilibrium output,  $q_2$ , to be zero...
  - In particular, the horizontal intercept of firm 2's *BRF* lies below the vertical intercept of firm 1's *BRF*.
    - That is, if

$$\frac{a - c_2}{b} < \frac{a - c_1}{2b} \quad \text{or} \quad \frac{a + c_1}{2} < c_2$$

- As depicted in the next figure

## Cournot Game of Output Competition

- Corner Solution with only firm 1 producing



- Note that  $(q_1^{\leftarrow}, q_2^{\leftarrow})$  is the only crossing point between  $BRF_1$  and  $BRF_2$ , implying  $q_1^{\leftarrow} > 0$ , but  $q_2^{\leftarrow} = 0$ .

## Cournot Game of Output Competition

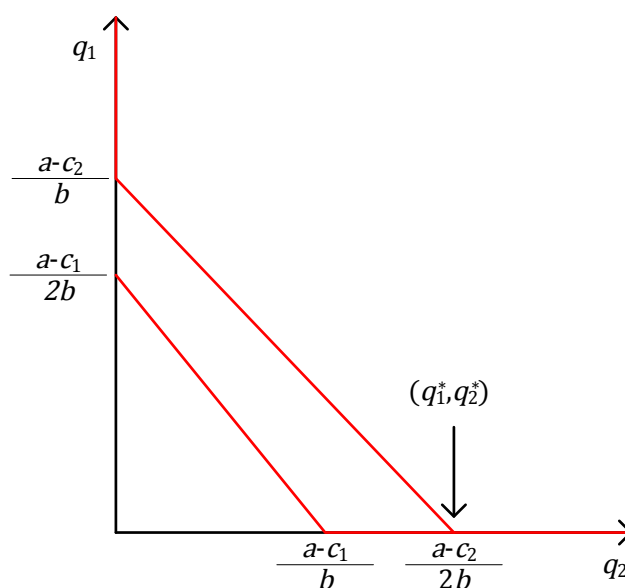
- This corner solution happens when

$$\frac{a - c_2}{b} < \frac{a - c_1}{2b} \quad \text{) } \Leftrightarrow \frac{a + c_1}{2} < c_2$$

- **Intuition:** Firm 1 is super-competitive (High  $c_2$ ).

## Cournot Game of Output Competition

- Another Corner Solution with only firm 2 producing:



- Note that  $(q_1^{(c)}, q_2^{(c)})$  is the only crossing point between  $BRF_1$  and  $BRF_2$ , implying  $q_2^{(c)} > 0$ , but  $q_1^{(c)} = 0$ .

## Cournot Game of Output Competition

- This corner solution happens when

$$\frac{a - c_2}{b} > \frac{a - c_1}{2b} \quad \Leftrightarrow \quad \frac{a + c_1}{2} > c_2$$

- **Intuition:** Firm 2 is super-competitive (Low  $c_2$ ).

## Cournot Game of Output Competition

- Hence, aggregate output (assuming interior solutions) is

$$Q = q_1 + q_2 = \frac{a - 2c_1 + c_2}{3b} + \frac{a - 2c_2 + c_1}{3b} = \frac{2a - c_1 - c_2}{3b}$$

and the equilibrium price is

$$p = a - bQ = a - b \left( \underbrace{\frac{2a - c_1 - c_2}{3b}}_Q \right) = \frac{a + c_1 + c_2}{3}.$$

- Assuming symmetry ( $c_1 = c_2 = c$ ), profits are

$$\pi_i = (p - c)q_i = \left( \frac{a + 2c}{3} - c \right) \frac{a - c}{3b} = \frac{(a - c)^2}{9b}$$

- Practice:** find profits *without symmetry*. If we assume that  $c_2 > c_1$ , which firm experiences the highest profit?

## Cournot Game of Output Competition

- This is very similar to the prisoner's dilemma!
- Indeed, if firms coordinate their production to lower production levels, they would maximize their joint profits.
  - Let us show how (for simplicity we assume symmetry in costs).
- First, note that firms would maximize their joint profits by choosing  $q_1$  and  $q_2$  such that

$$\begin{aligned}\max \quad \pi_1 + \pi_2 &= [(a - b(q_1 + q_2))q_1 - cq_1] \\ &\quad + [(a - b(q_1 + q_2))q_2 - cq_2] \\ &= (a - bQ)Q - cQ \\ &= aQ - bQ^2 - cQ\end{aligned}$$

## Cournot Game of Output Competition

- Taking first-order conditions with respect to  $Q$ , we obtain

$$a - 2bQ - c = 0$$

and solving for  $Q$ , we obtain the aggregate output level for the cartel

$$Q = \frac{a - c}{2b}$$

- Since firms are symmetric in costs, each produces half of this aggregate output level,

$$q_i = \frac{1}{2} \frac{a - c}{2b}$$

## Cournot Game of Output Competition

Hence, equilibrium price is

$$p = a - bQ = a - b \left( \frac{a + c}{2b} \right) = \frac{a + c}{2}$$

and profits for every firm  $i$  are

$$\pi_i = p^* q_i - cq_i = \frac{a + c}{2} \left( \frac{a + c}{2b} \right) - c \left( \frac{a + c}{4b} \right) = \frac{(a - c)^2}{8b}$$

which is higher than the individual profit for every firm under Cournot competition,  $\frac{(a - c)^2}{9b}$ .

## Cournot Game of Output Competition

- What if my firm deviates to Cournot output?

$$\begin{aligned}\pi_i &= pq_i - cq_i = \left[ a - b \left( \underbrace{\frac{a - c}{3b}}_{q_i^{\text{Cournot}}} + \underbrace{\frac{a - c}{4b}}_{q_j^{\text{Cartel}}} \right) \right] * \frac{a - c}{3b} \\ &= (a - c) \left( \frac{a - c}{3b} \right) \\ &= \frac{5(a - c)^2}{36b}\end{aligned}$$

(and Firm  $j$  makes a profit of  $\frac{5(a - c)^2}{48b}$ ).



- Hence, deviating to Cournot output levels is a best response for every firm regardless of whether its rival respects or violates the cartel agreement.
- In other words, deviating to Cournot output levels is a strictly dominant strategy for both firms, and thus constitutes the NE of this game.
- How can firms then collude effectively? By interacting for several periods. (We will come back to collusive practices in future chapters).

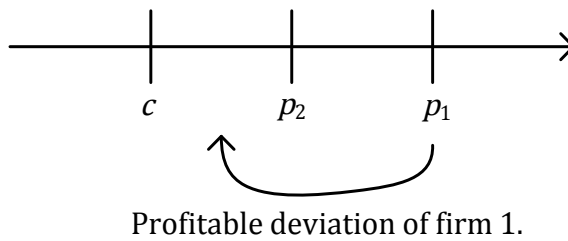
## Bertrand Game of Price Competition

**Competition in prices.** The firm with the lowest price attracts all consumers. If both firms charge the same price, they share consumers equally.

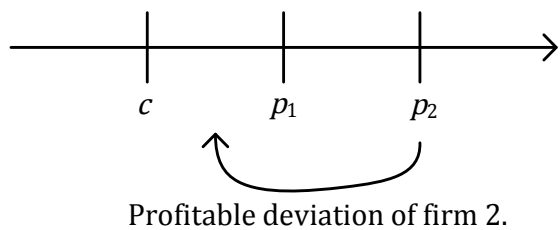
- Any  $p_i < c$  is strictly dominated by  $p_i + c$ .
- No *asymmetric* Nash equilibrium: (See Figures)
  - 1 If  $p_1 > p_2 > c$ , then firm 1 obtains no profit, and it can undercut firm 2's price to  $p_2 > p_1 > c$ . Hence, there exists a profitable deviation, which shows that  $p_1 > p_2 > c$  cannot be a psNE.
  - 2 If  $p_2 > p_1 > c$ . Similarly, firm 2 obtains no profit, but can undercut firm 1's price to  $p_1 > p_2 > c$ . Hence, there exists a profitable deviation, showing that  $p_2 > p_1 > c$  cannot be a psNE.
  - 3 If  $p_1 > p_2 = c$ , then firm 2 would want to raise its price (keeping it below  $p_1$ ). Hence, there is a profitable deviation for firm 2, and  $p_1 > p_2 = c$  cannot be a psNE.
  - 4 Similarly for  $p_2 > p_1 = c$ .

# Bertrand Game of Price Competition

①  $p_1 > p_2 > c$

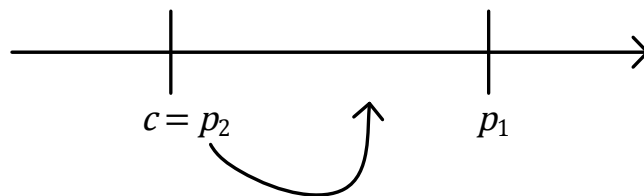


②  $p_2 > p_1 > c$

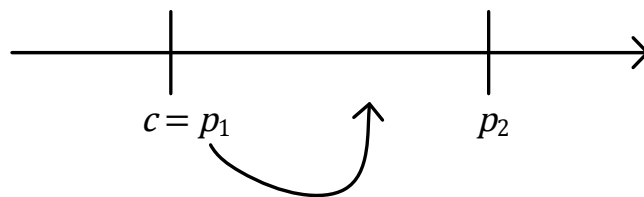


# Bertrand Game of Price Competition

1  $p_1 > p_2 = c$

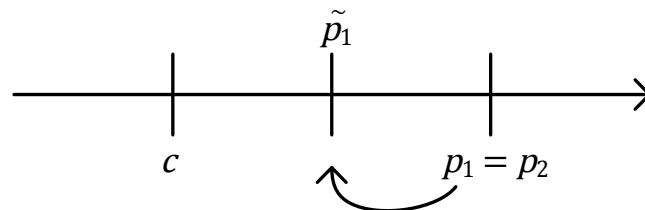


2  $p_2 > p_1 = c$



## Bertrand Game of Price Competition

- Therefore, it must be that the psNE is symmetric. If  $p_1 = p_2 > c$ , then both firms have incentives to deviate, undercutting each other's price (keeping it above  $c$ , e.g.,  $p_2 > \tilde{p}_1 > c$ ).



And similarly for firm 2

- Hence,  $p_1 = p_2 = c$  is the unique psNE.

## Bertrand Game of Price Competition

- The Bertrand model of price competition predicts intense competitive pressures until both firms set prices  $p_1 = p_2 = c$ .
- How can the "super-competitive" outcome where  $p_1 = p_2 = c$  be ameliorated? Two ways:
  - Offering price-matching guarantees.
  - Product differentiation

## Price-matching Guarantees in Bertrand

- These guarantees are relatively common in some industries
  - Walmart, Best Buy, Orbitz, etc.
- Under this guarantee, firm 1 gets
  - a price  $p_1$  for its products when  $p_1 \leq p_2$ , and...
  - a price  $p_2$  for its products when  $p_1 > p_2$  (the low-price guarantee kicks in)
- Hence, firm 1 sells its products at the lowest of the two prices, i.e.,  $\min\{p_1, p_2\}$ .

## Price-matching Guarantees in Bertrand

- Before analyzing the set of NEs under low-price guarantees, let's find the price that firm 1 would set under monopoly.
- If both firms' marginal costs are  $c = 10$ , and the demand function is  $q = 100 - p$ , firm 1's profits are

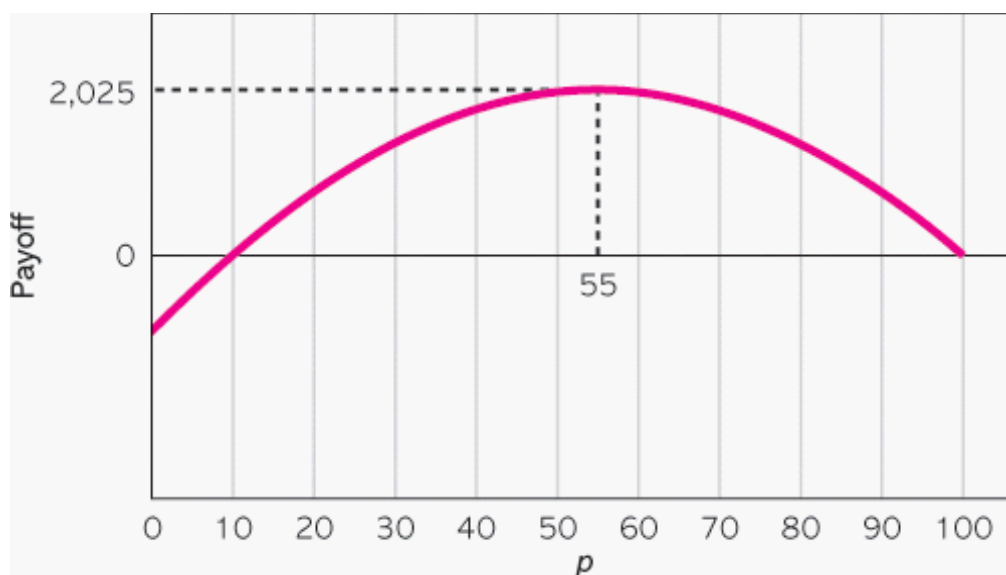
$$\pi = pq - cq = p(100 - p) - 10(100 - p) = (p - 10)(100 - p)$$

Taking FOCs with respect to  $p$ , we obtain  $100 - 2p + 10 = 0$  which implies  $p = \$55$ .

- At this price, monopoly profits become 2,025.

## Price-matching Guarantees in Bertrand

- Taking SOC with respect to  $p$ , we obtain  $\$2 < 0$ . Hence, the profit function is concave and is represented below.



## Price-matching Guarantees in Bertrand

- Let us now examine the case where instead firm 1 competes with firm 2 and both firms offer low-price guarantees.
- Firm 1's profits are

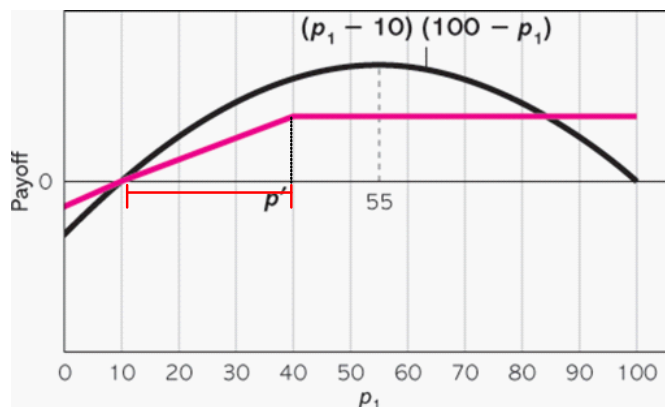
$$\underbrace{[\min\{p_1, p_2\} - \$10]}_{\text{lowest price}} \underbrace{\frac{1}{2} [100 - \min\{p_1, p_2\}]}_{\text{firm 1's sales}}$$

where the "1/2" is due to the fact that, under the low-price guarantee, both firms end up selling their products at the same price, namely, the lowest price in the market, and each firm gets 50% of the sales.

- Consider a symmetric strategy pair where both firms set  $p_1 = p_2 = p'$  such that  $p' \in [10, 55]$ , i.e., above marginal costs and below monopoly price.

## Price-matching Guarantees in Bertrand

- Firm 1's profits become



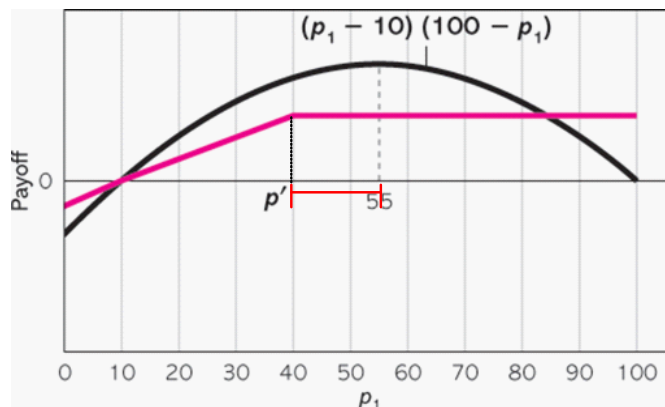
- When  $p_1$  &  $p'$ , firm 1 sells at  $p_1$ . This implies that its profits become

$$\underbrace{\min\{p_1, p_2\} - \$10}_{\text{lowest price}} \underbrace{\frac{1}{2}[100 - \min\{p_1, p_2\}]}_{\text{firm 1's sales}} = [p_1 - \$10] \frac{1}{2}[100 - p_1]$$

which firm 1 gets half of the total profit under monopoly.

## Price-matching Guarantees in Bertrand

- What if  $p_1 > p'$



- In that case, firm 1 sells at  $p'$  (because of the price-matching guarantee). This implies that its profits become

$$\underbrace{\min\{p_1, p_2\}}_{\text{lowest price}} - \$10 \underbrace{\frac{1}{2}[100 - \min\{p_1, p_2\}]}_{\text{firm 1's sales}} = [p' - \$10] \frac{1}{2}[100 - p']$$

which is constant (independent) in  $p_1$ , i.e., a flat line in the figure for all  $p_1 > p'$ .

## Price-matching Guarantees in Bertrand

- In order to find the NE of this game, consider any symmetric pricing profile  $p_1 = p_2 = p'$  such that  $p' \in [10, 55]$ .
- Can this be an equilibrium?
  - Let's check if firms have incentives to deviate from this equilibrium.
  - No! If you are firm 1, by undercutting firm 2's price (i.e., charging  $p_1 = p_2 - \epsilon$ ), you are not "stealing" customers.
  - Instead, you only sell your product at a lower price. Firm 2's customers are still with firm 2, since the low-price guarantee would imply that firm 2 charges firm 1's (lowest) price.
- A similar argument is applicable if you put yourself in the shoes of firm 2: You don't want to undercut your rival's price, since by doing so you sell the same number of units but at a lower price.

## Price-matching Guarantees in Bertrand

- Importantly, we were able to show that for any pricing profile  $p_1 = p_2 = p'$  such that  $p' \in [10, 55]$ .
- Hence, there is a continuum of symmetric NEs, one for each price level from  $p' = 10$  to  $p' = 55$ .
- Wow!
  - Low-price guarantees destroy the incentive to undercut a rival's price and allows firms to sustain higher prices.
  - *What appears to enhance competition actually destroys it!*

## Price-matching Guarantees in Bertrand

- **Empirical evidence:**
- We first need to set a testable hypothesis from our model
  - According to our theoretical results, products that are suddenly subject to price-matching guarantees should experience a larger increase in prices than products which were not subject to the price-matching guarantee.
- That is,  $\Delta P_{PM} > \Delta P_{NPM}$
- Lets go to the data now.

- This has been empirically shown for a group of supermarkets in North Carolina.
- In 1985, Big Star announced price-matching guarantees for a weekly list of products.
  - These products experienced a larger price increase than the products that were not included in the list.

## Price-matching Guarantees in Bertrand

- *Example:*
  - Before introducing price-matching guarantees, Maxwell House Coffee sold for \$2.19 at Food Lion, \$2.29 at Winn-Dixie (two other groceries), and \$2.33 at Big Star.
  - After announcing the price-matching guarantee, it sold for exactly the same (higher) price, \$2.89 in all three supermarkets.
  - Price-matching guarantees hence allowed these groceries to ameliorate price competition, and to coordinate on higher prices.

## Price Competition with Differentiated Products

- Another variation of the standard Bertrand model of price competition is to allow for product differentiation:
  - In the standard Bertrand model, firms sell a homogeneous (undifferentiated) product, e.g., wheat.
  - We will now see what happens if firms sell heterogeneous (differentiated) products, e.g., Coke and Pepsi.
- Let's consider the following example from Harrington (pp. 160-164) analyzing the competition between Dell and HP.

## Price Competition with Differentiated Products

- Demand for Dell computers

$$q_{Dell}(p_{Dell}, p_{HP}) = 100 - 2p_{Dell} + p_{HP}$$

so that an increase in  $p_{Dell}$  *reduces* the demand for Dell computers (own-price effect), but an increase in  $p_{HP}$  actually *increases* the demand for Dell computers (cross-price effect).

- Similarly for HP,

$$q_{HP}(p_{HP}, p_{Dell}) = 100 - 2p_{HP} + p_{Dell}$$

- Hence, profits for Dell are

$$\pi_{Dell}(p_{Dell}, p_{HP}) = \underbrace{[p_{Dell} - 10]}_{\text{Profits per unit}} \underbrace{(100 - 2p_{Dell} + p_{HP})}_{q_{Dell}}$$

or, expanding it,

$$100p_{Dell} - 2p_{Dell}^2 + p_{HP}p_{Dell} - 1000 + 20p_{Dell} - 10p_{HP}$$

## Price Competition with Differentiated Products

- Taking FOCs with respect to  $p_{Dell}$  (the only choice variable for Dell), we obtain

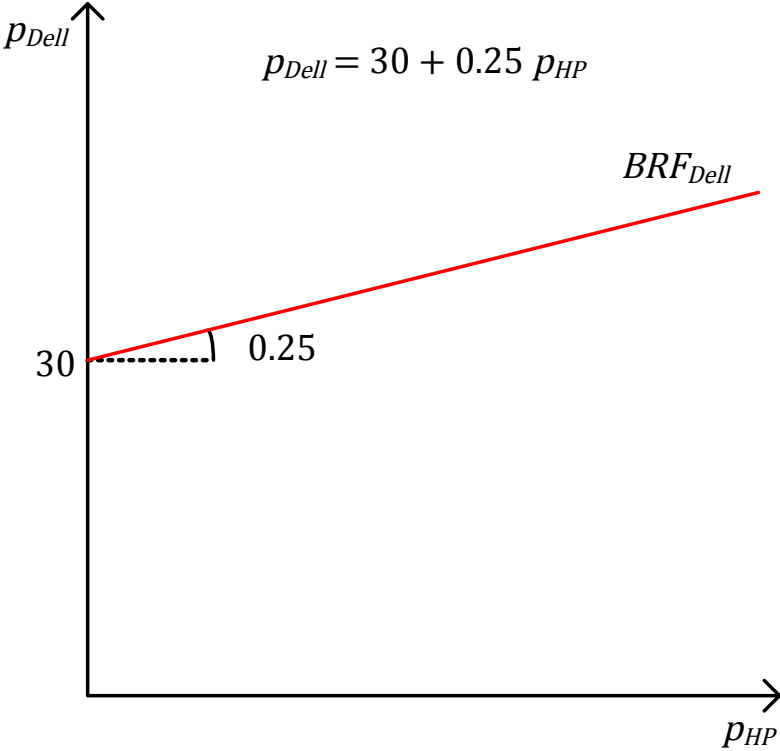
$$\frac{\partial \pi_{Dell}(p_{Dell}, p_{HP})}{\partial p_{Dell}} = 100 - 4p_{Dell} + p_{HP} + 20 = 0$$

and solving for  $p_{Dell}$  we find

$$p_{Dell} = \frac{120 + p_{HP}}{4} = 30 + 0.25p_{HP} \quad (BRF_{Dell})$$

- (See figure).

# Price Competition with Differentiated Products



- Note the difference with the Cournot model of price competition:  $BRF$  is positively (not negatively) sloped.
  - *Intuition*: strategic complementarity vs. strategic substitutability.

## Price Competition with Differentiated Products

- Similarly operating with HP (where marginal costs are  $c = 30$ ), we have

$$\pi_{HP}(p_{HP}, p_{Dell}) = \underbrace{[p_{HP} - 30]}_{\text{Profits per unit}} \underbrace{(100 - 2p_{HP} + p_{Dell})}_{q_{HP}}$$

- Taking FOCs with respect to  $p_{HP}$  (the only choice variable for HP), we obtain

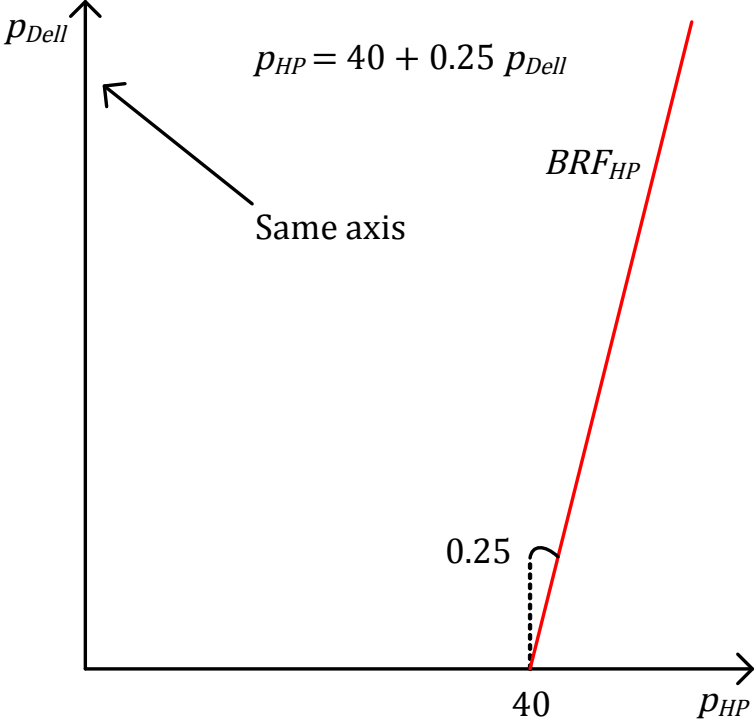
$$\frac{\partial \pi_{HP}(p_{HP}, p_{Dell})}{\partial p_{HP}} = 100 - 4p_{HP} + p_{Dell} + 60 = 0$$

and solving for  $p_{HP}$  we find

$$p_{HP} = \frac{160 + p_{Dell}}{4} = 40 + 0.25p_{Dell} \quad (BRF_{HP})$$

- (See figure).

# Price Competition with Differentiated Products



## Price Competition with Differentiated Products

- As a side, note that the SOC's for a max are also satisfied since:

$$\frac{\partial^2 \pi_{Dell}(p_{Dell}, p_{HP})}{\partial p_{Dell}^2} = -4 < 0 \text{ (Dell's profit function is concave),}$$

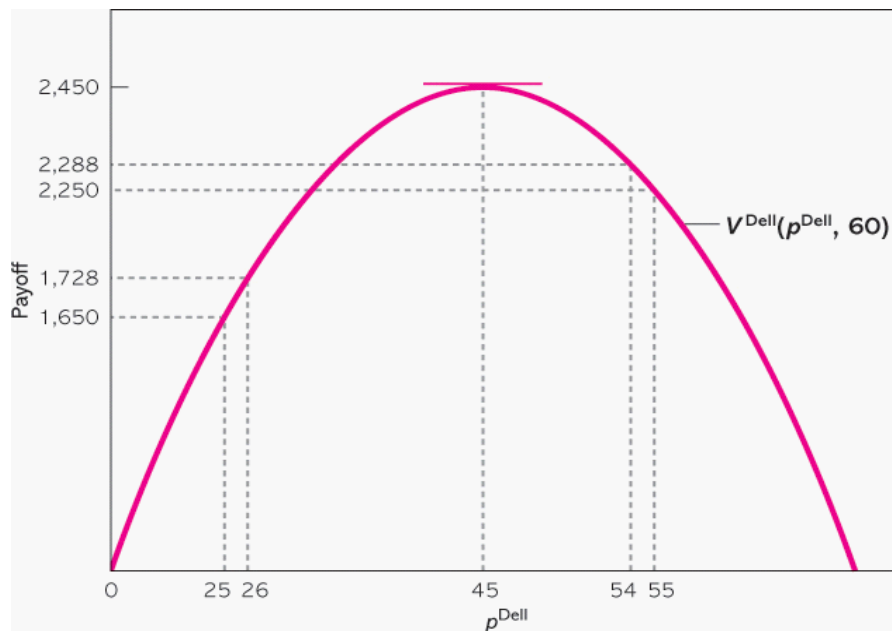
$$\frac{\partial^2 \pi_{HP}(p_{HP}, p_{Dell})}{\partial p_{HP}^2} = -4 < 0 \text{ (HP's profit function is concave)}$$

## Price Competition with Differentiated Products

- Indeed, if we graphically represent Dell's profit function for  $p_{HP} = \$60$ , that is

$$(p_{Dell} - 10)(100 - 2p_{Dell} + 60) = 180p - 2p^2 - 1600$$

we obtain the following concave profit function:



## Price Competition with Differentiated Products

- Hence, both firms'  $BRF$ s cross at

$$p_{Dell} = 30 + 0.25 \underbrace{(40 + 0.25p_{Dell})}_{p_{HP}} = 30 + 10 + 0.625p_{Dell}$$

and solving for  $p_{Dell}$  (the only unknown), we obtain

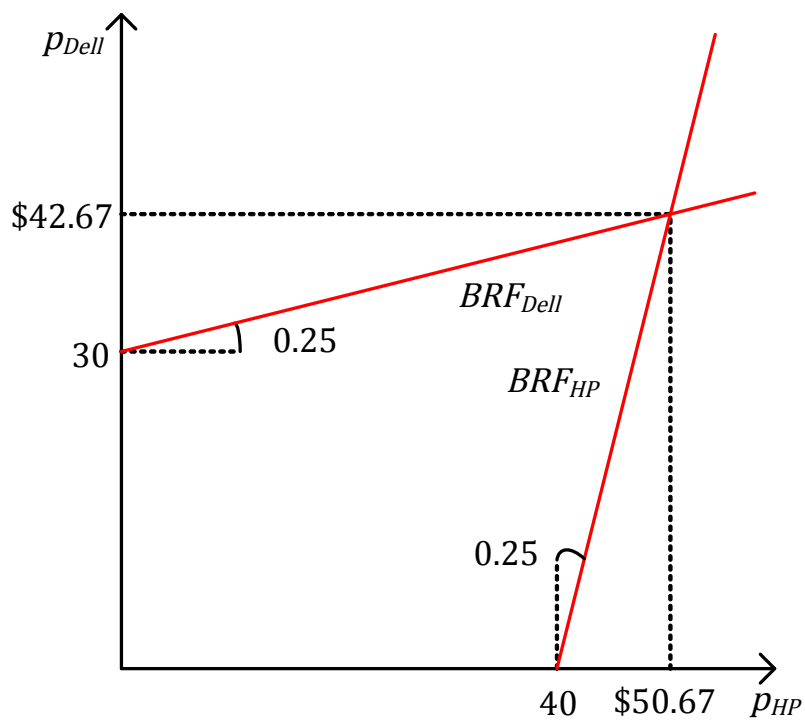
$$p_{Dell} = \$42.67.$$

- We can now find  $p_{HP}$  by just plugging  $p_{Dell} = \$42.67$  into  $BRF_{HP}$ , as follows

$$p_{HP} = 40 + 0.25p_{Dell} = 40.25 + 0.25 (42.67) = \$50.67$$

# Price Competition with Differentiated Products

- Putting  $BRF_{Dell}$  and  $BRF_{HP}$  together



## More Problems that Include Continuum Strategy Spaces

- Let's move outside the realm of industrial organization. There are still several games where players select an action among a continuum of possible actions.
- What's ahead...
- **Tragedy of the commons:** how much effort to exert in fishing, exploiting a forest, etc, incentives to overexploit the resource.
- **Tariff setting by two countries:** what precise tariff to set.
- **Charitable giving:** how many dollars to give to charity.
- **Electoral competition:** political candidates locate their platforms along the line (left-right, more or less spending, more or less security, etc.)
- **Accident law:** how much care a victim and an injurer exert, given different legal rules.

# Tragedy of the Commons

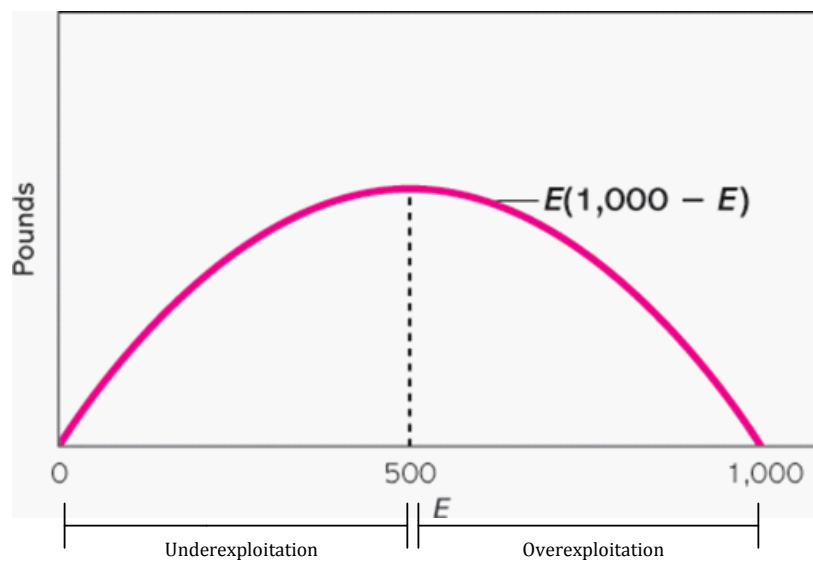
- *Reading*: Harrington pp. 164-169.



- $n$  hunters, each deciding how much effort  $e_i$  to exert, where

$$e_1 + e_2 + \dots + e_n = E$$

- Every hunter  $i$ 's payoff is a function of the total pounds of mammoth killed  $Pounds = E(1000 - E)$



## Tragedy of the Commons

- From the total pounds of mammoth killed, hunter  $i$  obtains a share that depends on how much effort he contributed relative to the entire group, i.e.,  $\frac{e_i}{E}$ .
- Effort, however, is costly for hunter  $i$ , at a rate of 100 per unit (opportunity cost of one hour of effort = gathering fruit?).
- Hence, every hunter  $i$ 's payoff is given by

$$u_i(e_i, e_{-i}) = \underbrace{\frac{e_i}{E}}_{\text{share}} \underbrace{E(1000 - E)}_{\text{total pounds}} - \underbrace{100e_i}_{\text{cost}}$$

cancelling  $E$  and rearranging, we obtain

$$e_i \left[ 1000 - \underbrace{(e_1 + e_2 + \dots + e_n)}_E \right] - 100e_i$$

## Tragedy of the Commons

- Taking FOCs with respect to  $e_i$ ,

$$\frac{\partial u_i(e_i, e_{-i})}{\partial e_i} = 1000 - (e_1 + e_2 + \dots + e_n) - e_i - 100 = 0$$

and noting that

$e_1 + e_2 + \dots + e_n = (e_1 + e_2 + e_{i+1} + \dots + e_n) + e_i$ ,  
we can rewrite the above FOC as

$$900 - (e_1 + e_2 + e_{i+1} + \dots + e_n) - 2e_i = 0$$

(SOCs are also satisfied and equal to -2)

- Solving for  $e_i$ ,

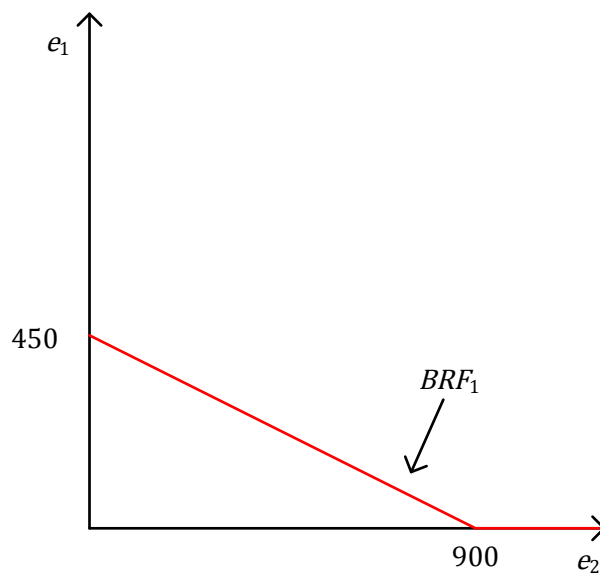
$$e_i = 450 \$ \frac{e_1 + e_2 + e_i + e_{i+1} + \dots + e_n}{2} \quad (BRF_i)$$

- Intuitively, there exists a strategic substitutability between efforts:
  - the more you hunt, the less prey is left for me.

# Tragedy of the Commons

- Note that for the case of only two hunters,

$$e_1 = 450 - \frac{e_2}{2}$$



## Tragedy of the Commons

- A similar maximization problem (and resulting *BRF*) can be found for all hunters, since they are all symmetric.
- Hence,  $e_1^C = e_2^C = \dots = e_n^C = e^C$  (symmetric equilibrium) implying that  $e_1^C + e_2^C + e_{i+1}^C + e_{i+1}^C + \dots + e_n^C = (n + 1)e^C$ .
- Putting this information into the *BRF* yields

$$e^C = 450 \$ \frac{e_1^C + e_2^C + e_{i+1}^C + e_{i+1}^C + \dots + e_n^C}{2} = 450 \$ \frac{(n + 1)e^C}{2}$$

and solving for  $e^C$ , we obtain

$$e^C = \frac{900}{n + 1}$$

## Tragedy of the Commons

- **Comparative statics on the above result:**
- First, note that individual equilibrium effort,  $e^c$ , is decreasing in  $n$  since

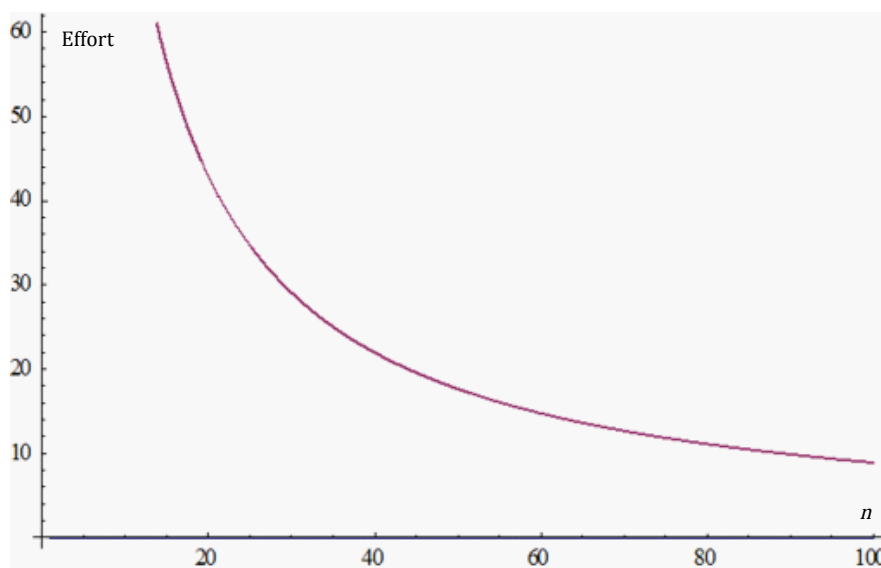
$$\frac{\partial e^c}{\partial n} = -\frac{900}{(n+1)^2} < 0$$

- Intuitively, this implies that an increase in the number of potential hunters reduces every hunter's individual effort, since more hunters are chasing the same set of mammoths. (Why not gather some fruit instead?)

# Tragedy of the Commons

- Individual effort in equilibrium

$$e(n) = \frac{900}{n+1}$$

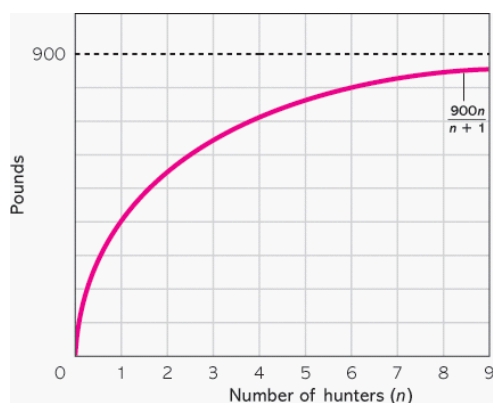


## Tragedy of the Commons

- Comparative statics on the above result:
- Second, note that aggregate equilibrium effort,  $ne^c$ , is increasing in  $n$  since

$$\frac{\partial (ne^c)}{\partial n} = \frac{900(n+1) - 900n}{(n+1)^2} = \frac{900}{(n+1)^2} > 0$$

- Although each hunter hunts less when there are more hunters, the addition of another hunter offsets that effect, so the total effort put into hunting goes up.



## Tragedy of the Commons

- Finally, what about overexploitation?
  - We know that overexploitation occurs if  $E > 500$  (the point at which aggregate meat production is maximized).
  - Total effort exceeds 500 if  $n \frac{900}{n+1} > 500$ , or  $n > 1.2$ .
  - That is, as long as there are 2 or more hunters, the resource will be overexploited.

## Tragedy of the Commons

- The exploitation of a common pool resource (fishing grounds, forests, aquifers, etc.) to a level beyond the level that is socially optimal is referred to as the "**tragedy of the commons.**"
  - Why does this "tragedy" occur?
  - Because when an agent exploits the resource he does not take into account the negative effect that his action has on the well-being of other agents exploiting the resource (who now find a more depleted resource).
  - Or more compactly, because every agent does not take into account the negative externality that his actions impose on other agents.

## Tariff Setting by Two Countries

- *Reading*: Watson pp. 111-112
- Players: two countries  $i = 1, 2$ , e.g., US and EU.
- Each country  $i$  simultaneously selects a tariff  $x_i \in [0, 100]$ .
- Country  $i$ 's payoff is

$$V_i(x_i, x_j) = 2000 + 60x_i + x_i x_j - x_i^2 - 90x_j$$

## Tariff Setting by Two Countries

- Let's put ourselves in the shoes of country  $i$ .
- Taking FOCs with respect to  $x_i$  we obtain

$$60 + x_j - 2x_i = 0$$

and solving for  $x_i$ , we have

$$x_i = 30 + \frac{1}{2}x_j$$

- We can check that SOC's are satisfied, by differentiating with respect to  $x_i$  again

$$\frac{\partial^2 V_i(x_i, x_j)}{\partial x_i^2} = -2 < 0$$

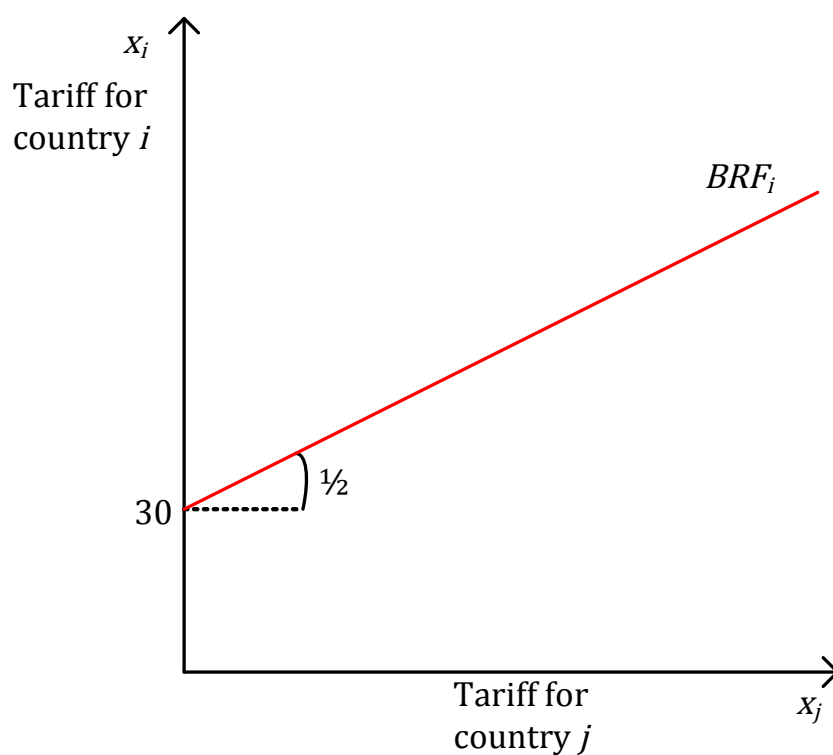
showing that country  $i$ 's payoff function is concave in  $x_i$ .

## Tariff Setting by Two Countries

- We can depict country  $i$ 's *BRF*,  $x_i = 30 + \frac{1}{2}x_j$  (see next page)
  - The figure indicates that country  $i$ 's and  $j$ 's tariffs are strategic complements: an increase in tariffs by the EU is responded by an increase in tariffs in the US.

## Tariff Setting by Two Countries

- Country  $i$ 's  $BRF$ :  $x_i = 30 + \frac{1}{2}x_j$



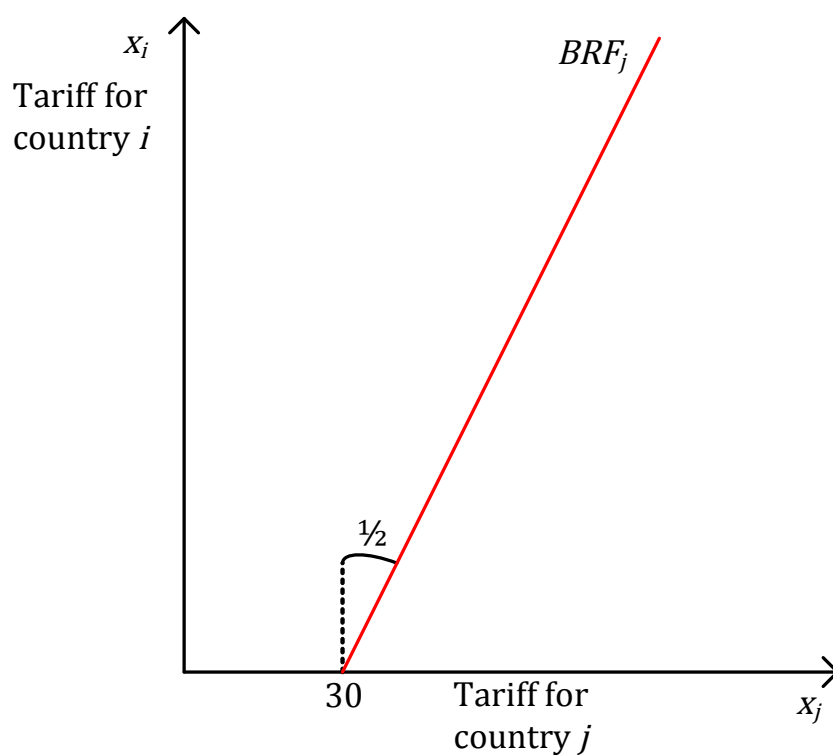
- By symmetry, country  $j$ 's *BRF* is

$$x_j = 30 + \frac{1}{2}x_i$$

See figure in the next slide

## Tariff Setting by Two Countries

- Country  $j$ 's  $BRF$ :  $x_j = 30 + \frac{1}{2}x_i$



## Tariff Setting by Two Countries

- Putting both countries' *BRF* together (see figure on the next page), we obtain

$$x_i = 30 + \frac{1}{2} \underbrace{\left( 30 + \frac{1}{2} x_i \right)}_{x_j}$$

simplifying

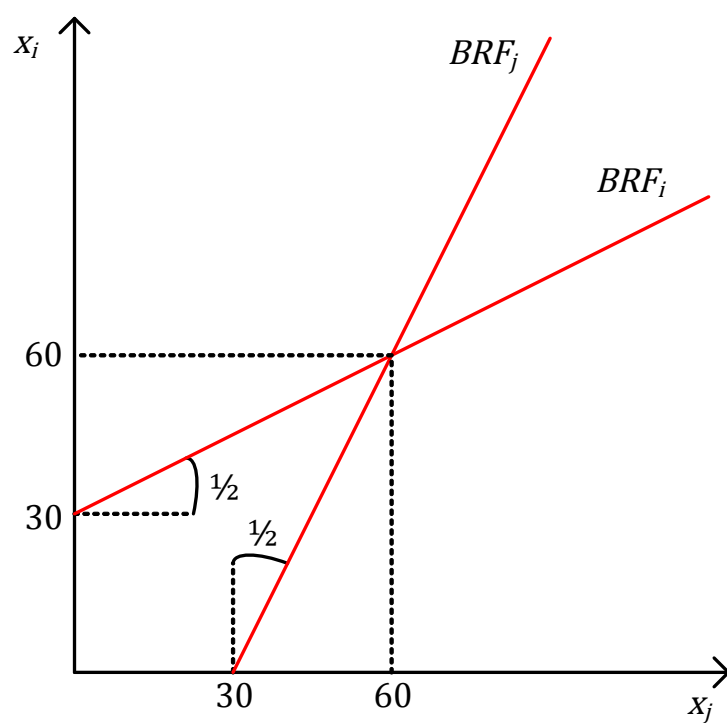
$$x_i = 30 + 15 + \frac{1}{4} x_i$$

and solving for  $x_i$  we obtain  $x_i^{(c)} = 60$ .

- Therefore, the psNE of this tariff setting game is  $(x_i^{(c)}, x_j^{(c)}) = (60, 60)$ .

## Tariff Setting by Two Countries

- Both countries' *BRF* together:



## Charitable Giving

- *Reading*: Harrington pp. 169-174.
- Consider a set of  $N$  donors to a charity.
- Each donor  $i$  contributes an amount of  $s_i$  dollars.
- Donor  $i$  benefits from the donations from all contributors  $\sum_{i=1}^N s_i$  (which includes his own contribution), obtaining a benefit of  $\sqrt{\sum_{i=1}^N s_i}$ , i.e.,  $\frac{1}{\sqrt{N}} \bar{S}$  where  $S = \sum_{i=1}^N s_i$ .
- Finally, the marginal cost of giving one more dollar to the charity for player  $i$  is  $k_i$  (alternative uses of that dollar)

## Charitable Giving

- Therefore, donor  $i$ 's utility is

$$u_i(s_i, s_{-i}) = \underbrace{\sqrt{\sum_{i=1}^N s_i}}_{\text{Public benefit from all donations}} \underbrace{\$}_{\text{Private cost of my donation}} \underbrace{k_i s_i}_{\text{Private cost of my donation}}$$

- Taking FOCs with respect to  $s_i$  we obtain

$$\frac{1}{2} \left( \sum_{i=1}^N s_i \right)^{\frac{1}{2}} - k_i = 0$$

which in the case on  $N = 2$  players reduce to

$$\frac{1}{2} (s_1 + s_2)^{\frac{1}{2}} - k_i = 0$$

- SOCs are  $-\frac{1}{4} \left( \sum_{i=1}^N s_i \right)^{\frac{3}{2}} < 0$  thus guaranteeing concavity.

## Charitable Giving

- In the case of  $N = 2$ , we obtain the following FOC:

$$\frac{1}{2}(s_i + s_j)^{\frac{1}{2}} = k_i$$

or alternatively

$$\frac{1}{s_i + s_j} = 4k_i^2$$

Hence, we can solve for  $s_i$ , obtaining donor  $i$ 's *BRF*

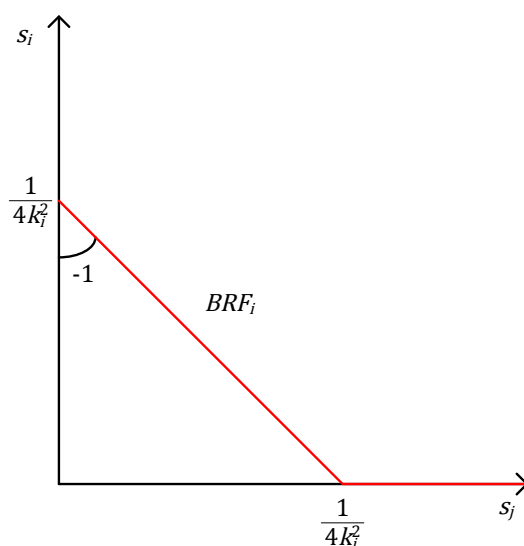
$$s_i(s_j) = \frac{1}{4k_i^2} s_j$$

- Graphical representation of players' *BRF*:
  - when  $k_j = k_i$  (continuum of solutions), and
  - when  $k_j < k_i$ , where  $s_i = 0$  and  $s_j > 0$ .

## Charitable Giving

- Donor  $i$ 's  $BRF$ :

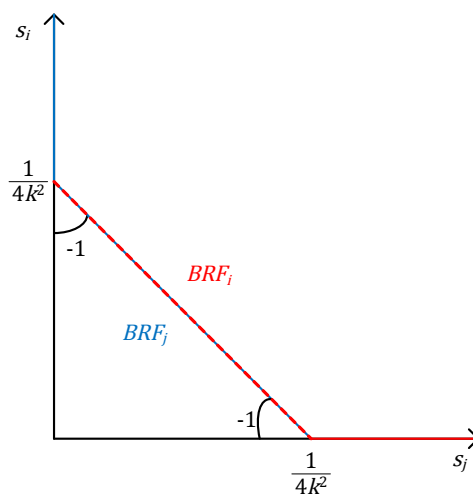
$$s_i(s_j) = \frac{1}{4k_i^2} \$ s_j$$



- But how can we depict  $BRF_j$  using the same axes?

## Charitable Giving

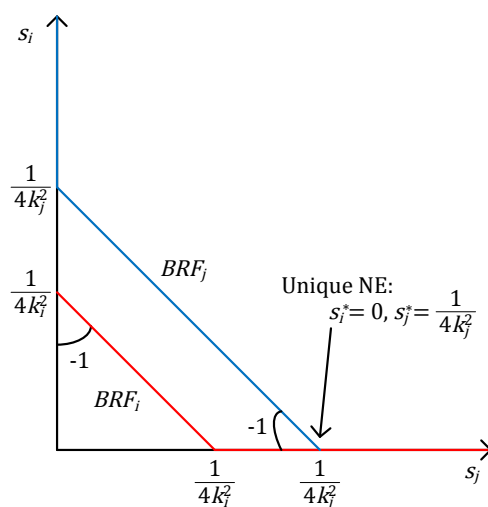
- Case 1:  $k_i = k_j = k$



- "Total overlap" of  $BRF_i$  and  $BRF_j$ : any combination of  $(s_i, s_j)$  on the overlap is a NE.

## Charitable Giving

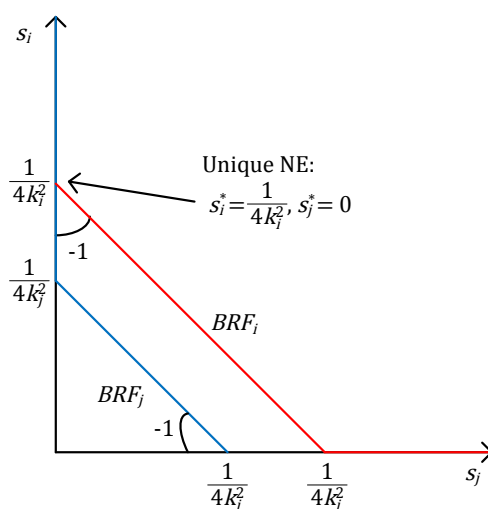
- Case 2:  $k_i > k_j \iff \frac{1}{4k_i^2} < \frac{1}{4k_j^2}$



- *Intuition:* Donor  $i$ 's private cost from donating money to the charity,  $k_i$ , is higher than that of donor  $j$ ,  $k_j$ , leading the former to donate zero and the latter to bear the burden of all contributions.

## Charitable Giving

- Case 3:  $k_i < k_j \Rightarrow \frac{1}{4k_i^2} > \frac{1}{4k_j^2}$



- *Intuition:* Donor  $j$ 's private cost from donating money to the charity,  $k_j$ , is higher than that of donor  $i$ ,  $k_i$ , leading the former to donate zero and the latter to bear the burden of all contributions.

## Charitable Giving

- **Another example:** In Harrington, you have another example of charitable giving where

$$u_i(s_i, s_j) = \frac{1}{5}(s_i + s_{\$i}) \underbrace{\$}_{k_i=1} s_i, \text{ where } s_{\$i} = \sum_{j=i} s_j$$

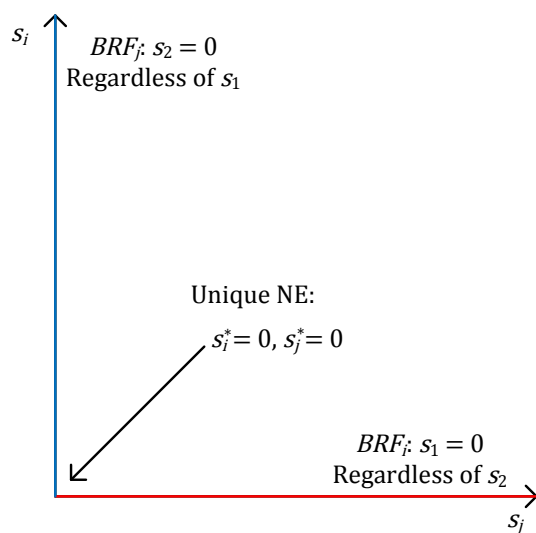
clearly, from FOCs,

$$\frac{\partial u_i(s_i, s_{\$i})}{\partial s_i} = \frac{1}{5} \$ 1 = \$ \frac{4}{5} \text{ for all } s_i \text{ and } s_j$$

which implies that  $s_i^{\ast} = 0$  for all players.

# Charitable Giving


- How can we represent that result using *BRFs*?



## Charitable Giving

- **What if we add a matching grant,  $\bar{s}$ ?** This is indeed commonly observed (NPR, Warren Buffet, CAHNRS, etc).
- In this setting, every donor  $i$ 's utility function
  - remains being  $\frac{1}{5}(s_i + s_{\$i})$  \$  $s_i$  if total contributions do not exceed the matching grant from the philanthropist (i.e., if  $s_i + s_{\$i} < \bar{s}$ ), but...
  - increases to  $\frac{1}{5}(s_i + s_{\$i} + \bar{s})$  \$  $s_i$  if total contributions exceed the matching grant from the philanthropist (i.e., if  $s_i + s_{\$i} + \bar{s}$ ).

$$u_i(s_i, s_{\$i}) = \begin{cases} \frac{1}{5}(s_i + s_{\$i}) & \$ s_i \text{ if } s_i + s_j < \bar{s} \\ \frac{1}{5}(s_i + s_{\$i} + \bar{s}) & \$ s_i \text{ if } s_i + s_j + \bar{s} \end{cases}$$

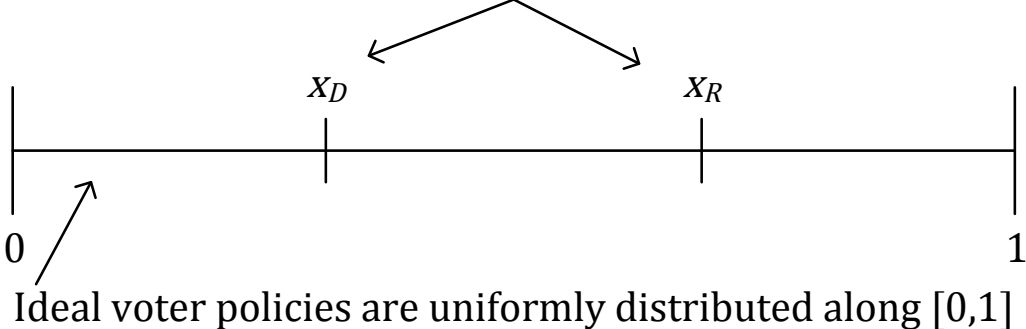
- Note that this implies that donor  $i$ 's utility function is not continuous at  $\bar{s}$ , so you cannot start taking derivatives right away.
- *Practice* this exercise (all answers are in Harrington, pages 169-174). Good news: equilibrium donations increase! 

## Electoral Competition

- Candidates running for elected office compete in different dimensions
  - advertising, endorsements, looks, etc.
- Nonetheless, the most important dimension of competition lies on their positions on certain policies
  - Gov't spending on social programs, on defense, etc.
- Let's represent the position of candidates along a line  $[0, 1]$ .
  - Candidates only care about winning the election: payoff of 2 if winning, 1 if tying and 0 if losing.
  - Candidates cannot renege from their promises.
- Each voter has an ideal position on the line  $[0, 1]$ .
  - Voters are uniformly distributed along the line  $[0, 1]$ .
  - Non-strategic voters: they simply vote for the candidate whose policy is closest to their ideal.

# Electoral Competition

Candidate D and R positions (Political promises)

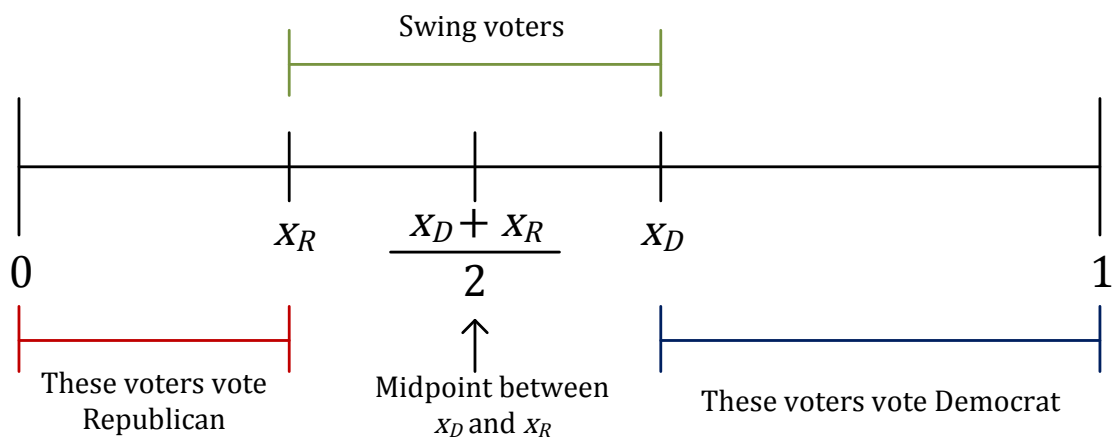


## Electoral Competition

- Let us consider two candidates for election: Democrat (D) and Republican (R).
- In order to better understand under which cases each candidate wins (accumulates the majority of votes), let us consider:
  - First, the case in which  $x_D > x_R$ . (See figures in next slide)
  - Second, the case where  $x_D < x_R$ . (See figures two slides ahead)

# Electoral Competition

- If  $x_D > x_R$ , then:

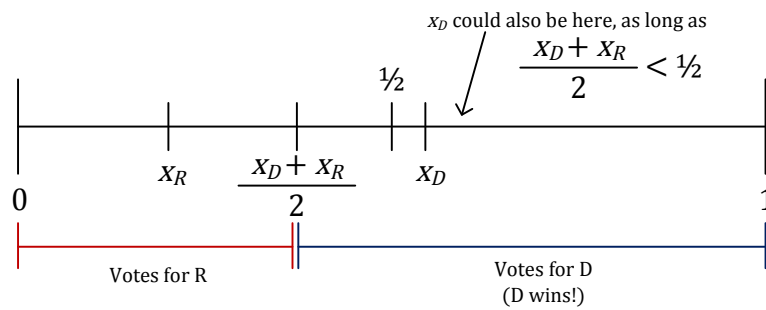


- Which party wins? It depends: we have two cases \$'

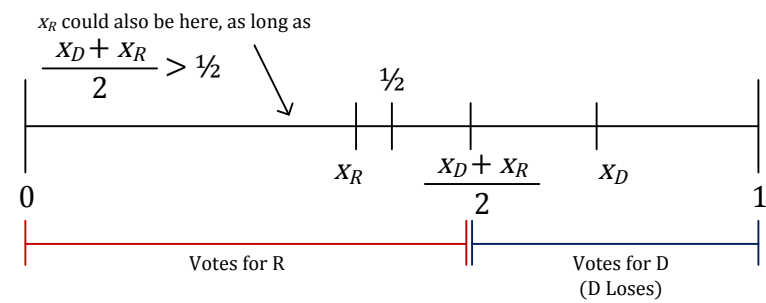
# Electoral Competition

- If  $x_D > x_R$ , then:

①  $\frac{x_D + x_R}{2} < \frac{1}{2}$



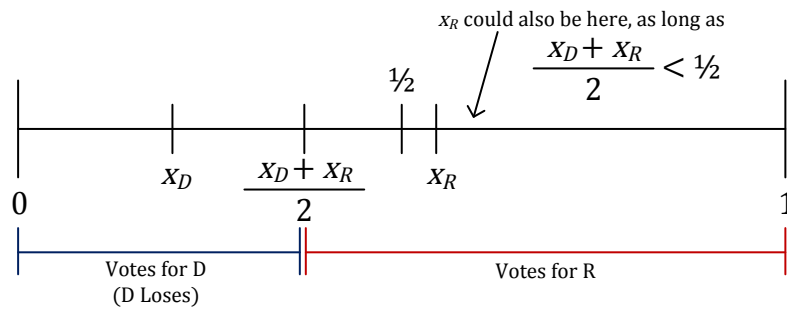
②  $\frac{x_D + x_R}{2} > \frac{1}{2}$



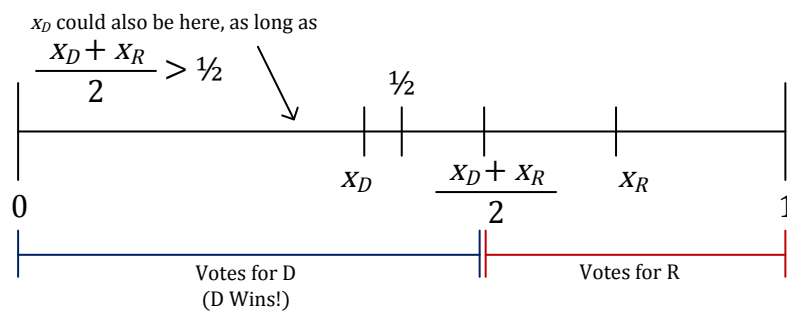
# Electoral Competition

- If  $x_D < x_R$ , then:

①  $\frac{x_D + x_R}{2} < \frac{1}{2}$



②  $\frac{x_D + x_R}{2} > \frac{1}{2}$



## Electoral Competition

- Hence, if  $x_D < x_R$ , candidate D wins if he selects a  $x_D$  that satisfies

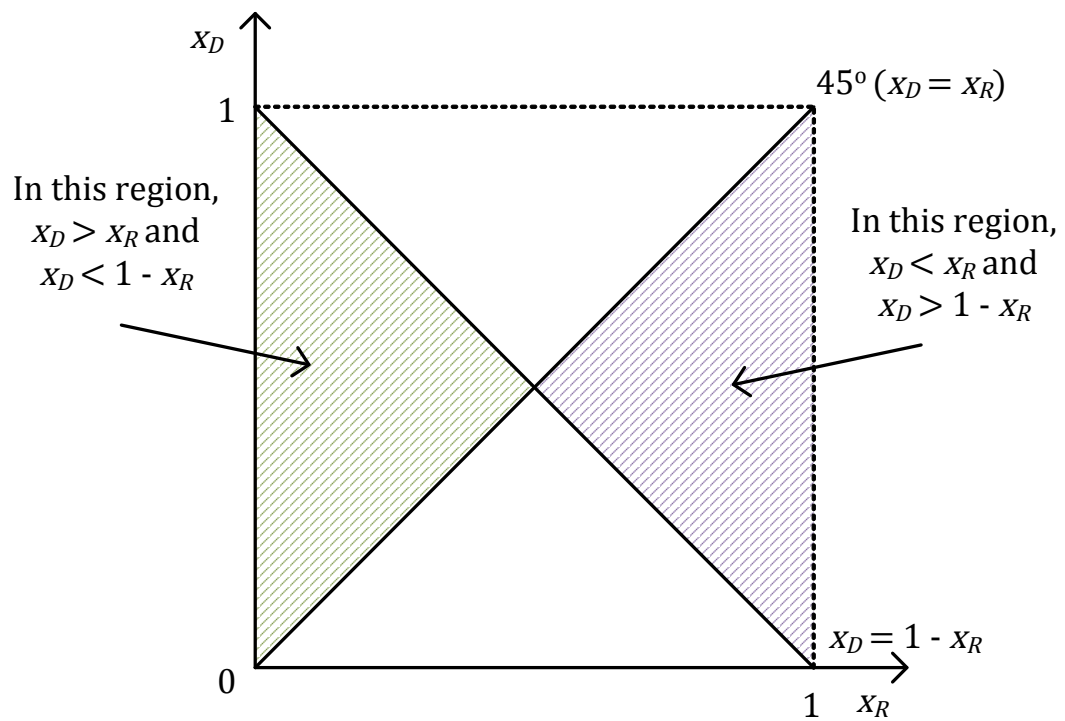
$$\frac{x_D + x_R}{2} > \frac{1}{2} \quad ) \quad \% \quad x_D + x_R > 1 \quad ) \quad \% \quad x_D > 1 - x_R$$

- And if  $x_D > x_R$ , candidate D wins if he selects a  $x_D$  that satisfies

$$\frac{x_D + x_R}{2} < \frac{1}{2} \quad ) \quad \% \quad x_D + x_R < 1 \quad ) \quad \% \quad x_D < 1 - x_R$$

# Electoral Competition

- Candidate D's best response "set"



## Electoral Competition

- Similarly for candidate R,

- If  $x_D < x_R$ , candidate R selects  $x_R$  that satisfies

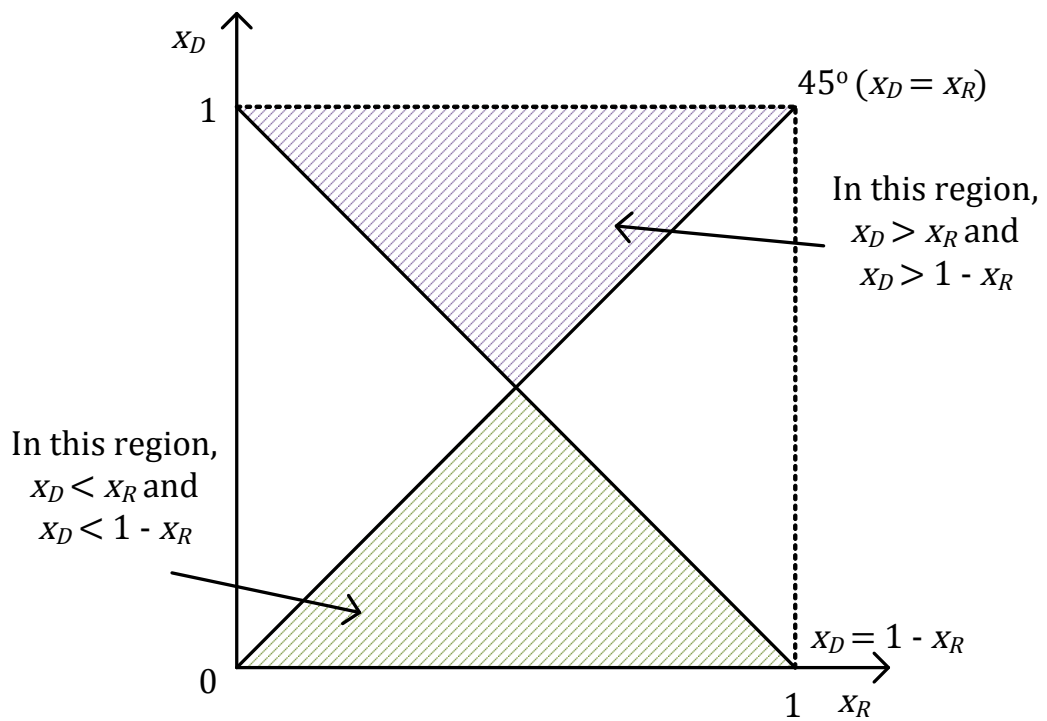
$$\frac{x_D + x_R}{2} < \frac{1}{2} \quad ) \quad \% \quad x_D + x_R < 1 \quad ) \quad \% \quad x_D < 1 \quad \$ \quad x_R$$

- If  $x_D > x_R$ , candidate R selects  $x_R$  that satisfies

$$\frac{x_D + x_R}{2} > \frac{1}{2} \quad ) \quad \% \quad x_D + x_R > 1 \quad ) \quad \% \quad x_D > 1 \quad \$ \quad x_R$$

# Electoral Competition

- Candidate R's best response "set"

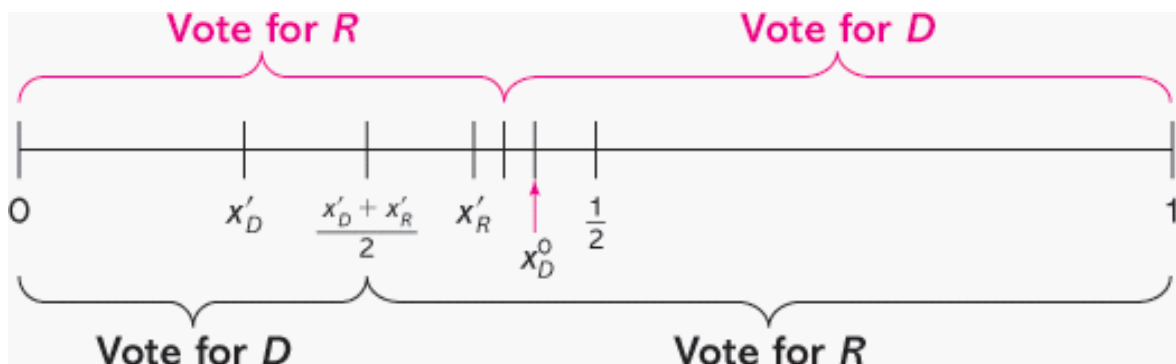


## Electoral Competition

- Superimposing both candidate's best response "sets" ...
  - We can see that the only point where they cross (or overlap) each other is...
  - in the policy pair  $(x_D, x_R) = (\frac{1}{2}, \frac{1}{2})$ .
- This is the NE of this electoral competition game:
  - Both candidates select the same policy  $(\frac{1}{2})$ , which coincides with the ideal policy for the median voter.
  - Political convergence among candidates.

## Electoral Competition

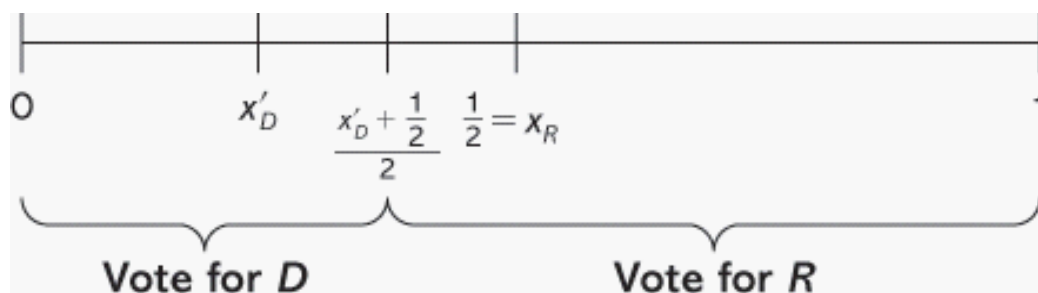
- We can alternatively show that this must be an equilibrium by starting from any other strategy pair  $(x'_D, x'_R)$  different from  $\frac{1}{2}$ .



- Where the black lettering represents the votes before D's deviation and the pink represents the votes after D's deviation.
- Candidate R would win.
- However, candidate D can instead take a position  $x_D^0$  between  $x'_R$  and  $\frac{1}{2}$ , which leads him to win.
- Thus,  $(x'_D, x'_R)$  cannot be an equilibrium.

## Electoral Competition

- We can extend the same argument to any other strategy pair where one candidate takes a position different from  $\frac{1}{2}$ .
  - Indeed, the other candidate can take a position between  $\frac{1}{2}$  and his rival's position, which guarantees him winning the election.
- Therefore, no candidate can be located away from  $\frac{1}{2}$ ,
  - Hence, the only psNE is  $(x_D, x_R) = (\frac{1}{2}, \frac{1}{2})$ .
- Finally, note that a unilateral deviation from this strategy pair cannot be profitable (see figure where D deviates)



# Electoral Competition

- We considered some simplifying assumptions:
- **Candidates:**
  - Only two candidates (ok in the US, not for EU).
  - Candidates could not renege from their promises.
  - Candidate did not have political preferences (they only cared about winning!)
- **Voters:**
  - Voters' preferences were uniformly distributed over the policy space  $[0, 1]$ . (If they are not, results are not so much affected; see Osborne)
  - Voters were not strategic: they simply voted for the candidate whose policy was closest to their ideal.
  - Voters might be asymmetric about how much they care about the distance between a policy and his ideal.

## Accident Law

- Reading: Osborne pp. 91-96.
- Injurer (player 1) and Victim (player 2)
- The loss that the victim suffers is represented by  $L(a_1, a_2)$ .
  - which decreases in the amount of care taken by the injurer  $a_1$  and the victim  $a_2$ .
  - It can be understood as the expected loss over many occurrences.
  - Example :  $L(a_1, a_2) = 2 \$ \alpha(a_1)^2 \$ \beta(a_2)^2$
- **Legal rule:** it determines the fraction of the loss suffered by the victim that must be borne by the injurer.
  - $\rho(a_1, a_2) \in [0, 1]$  (Think about what it means if it is zero or one).
    - = 0 / \$ The victim bears the entire loss
    - = 1 / \$ The injurer bears the entire loss

## Accident Law

- Injurer's payoff is

$$a_1 - \rho(a_1, a_2)L(a_1, a_2)$$

which decreases in the amount of care he takes  $a_1$  (which is costly) and in the share of the loss that the injurer must bear.

- The victim's payoff is

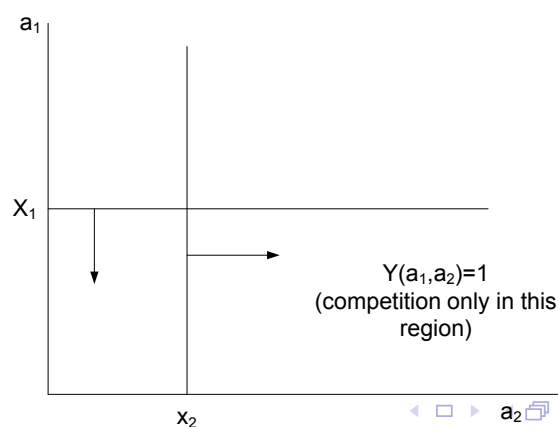
$$a_2 - [1 - \rho(a_1, a_2)]L(a_1, a_2)$$

which decreases in the amount of care taken by the injurer  $a_1$  and the victim  $a_2$ .

- Players simultaneously and independently select an amount of care  $a_1$  and  $a_2$ .

## Accident Law

- For simplicity, we will focus on a particular class of legal rules known as "negligence with contributory negligence."
- Each rule in this class requires the injurer to fully compensate the victim for his loss,  $\rho(a_1, a_2) = 1$ , if and only if:
  - The injurer is sufficiently careless (i.e.,  $a_1 < X_1$ ), and
  - The victim is sufficiently careful (i.e.,  $a_2 > X_2$ ).
  - (Otherwise, the injurer does not have to pay anything, i.e.,  $\rho(a_1, a_2) = 0$ ).



- Note that, included in this class of rules, are those in which:
  - **Pure negligence:**  $X_1 > 0$ , but  $X_2 = 0$  (the injurer has to pay if she is sufficiently careless, even if the victim did not take care at all).
  - **Strict liability:**  $X_1 = +\infty$ , but  $X_2 = 0$  (the injurer has to pay regardless of how careful she is and how careless the victim is).

## Accident Law

- Consider we find out that standards of care  $X_1 = \hat{a}_1$  and  $X_2 = \hat{a}_2$  are socially desirable.
  - That is,  $(\hat{a}_1, \hat{a}_2)$  maximizes the sum of the players' payoffs

$$\begin{aligned} & [\$a_1 \$ \rho(a_1, a_2)L(a_1, a_2)] + [\$a_2 \$ [1 \$ \rho(a_1, a_2)]L(a_1, a_2)] \\ = & \$a_1 \$ a_2 \$ L(a_1, a_2) \\ = & \$a_1 \$ a_2 \$ (2 \$ \alpha(a_1)^2 \$ \beta(a_2)^2) \end{aligned}$$

## Accident Law

- Next, we want to show that the unique NE of the game that arises when we set standards of care  $X_1 = \hat{a}_1 = \frac{1}{2\alpha}$  and  $X_2 = \hat{a}_2 = \frac{\beta^2}{4}$  is exactly

$$(a_1, a_2) = (\hat{a}_1, \hat{a}_2) = \left( \frac{1}{2\alpha}, \frac{\beta^2}{4} \right).$$

- Note that by showing that, we would demonstrate that such a legal rule induces players to voluntarily behave in a socially desirable way.

## Accident Law

- **Injurer:**

- Given that the victim's action is  $a_2 = \hat{a}_2 = \frac{\beta^2}{4}$  in equilibrium, the injurer's payoff is

$$u_1(a_1, a_2) = \begin{cases} \$a_1 - (2 - \alpha(a_1)^2 - \beta(\frac{\beta^2}{4})^2) & \text{if } a_1 < \frac{1}{2\alpha} \\ \$a_1 & \text{if } a_1 > \frac{1}{2\alpha} \end{cases}$$

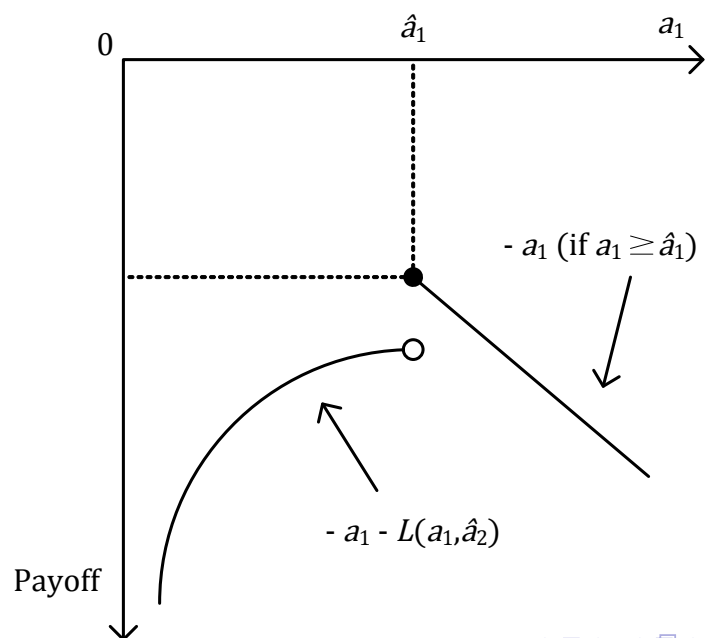
That is, the injurer only has to pay compensation when he is sufficiently careless.

- Let us depict this payoff in a figure.

## Accident Law

- Injurer's payoff function

$$u_1(a_1, a_2) = \begin{cases} a_1 - (2\alpha(a_1)^2 + \beta(\frac{\beta^2}{4})^2) & \text{if } a_1 < \frac{1}{2\alpha} \\ a_1 & \text{if } a_1 > \frac{1}{2\alpha} \end{cases}$$



## Accident Law

- **Injurer:**

- Another way to see that  $\hat{a}_1 = \frac{1}{2\alpha}$  maximizes  $u_1(a_1, \hat{a}_2)$  is by noticing that:

- by definition we know that  $(\hat{a}_1, \hat{a}_2)$  maximizes

$$u_1(a_1, a_2) = \alpha(a_1)^2 + \beta(a_2)^2$$

Hence, given  $\hat{a}_2 = \frac{\beta^2}{4}$ ,  $\hat{a}_1$  maximizes

$$u_1(a_1, \frac{\beta^2}{4}) = \alpha(a_1)^2 + \beta(\frac{\beta^2}{4})^2$$

## Accident Law

- **Injurer: cont'd..**

And because  $\hat{a}_2 = \frac{\beta^2}{4}$  is a constant, then

$$\hat{a}_1 \text{ maximizes } \underbrace{\$ a_1 \$ (2 \$ \alpha (a_1)^2 \$ \beta (\frac{\beta^2}{4})^2)}_{\text{which coincides with the injurer's payoff when } a_1 < \hat{a}_1 \text{ and } a_2 = \hat{a}_2.}$$

- Hence, action  $a_1 = \hat{a}_1 = \frac{1}{2\alpha}$  is a *BR* of the injurer to action  $\hat{a}_2 = (\frac{\beta^2}{4})^2$  by the victim. (This was visually detected in our previous figure).

## Accident Law

- **Victim:** Since we just showed that when the injurer's action is  $a_1 = \hat{a}_1 = \frac{1}{2\alpha}$ , the victim never receives compensation, yielding a payoff of

$$u_2(\hat{a}_1, a_2) = \$ a_2 \$ (2 \$ \alpha (\frac{1}{2\alpha})^2 \$ \beta (a_2)^2)$$

- We can use a similar argument as above: by definition we know that

$$(\hat{a}_1, \hat{a}_2) \text{ maximizes } \$ a_1 \$ a_2 \$ (2 \$ \alpha (a_1)^2 \$ \beta (a_2)^2)$$

Hence, given  $\hat{a}_1 = \frac{1}{2\alpha}$ ,

$$\hat{a}_2 \text{ maximizes } \$ (\frac{1}{2\alpha}) \$ a_2 \$ (2 \$ \alpha (\frac{1}{2\alpha})^2 \$ \beta (a_2)^2)$$

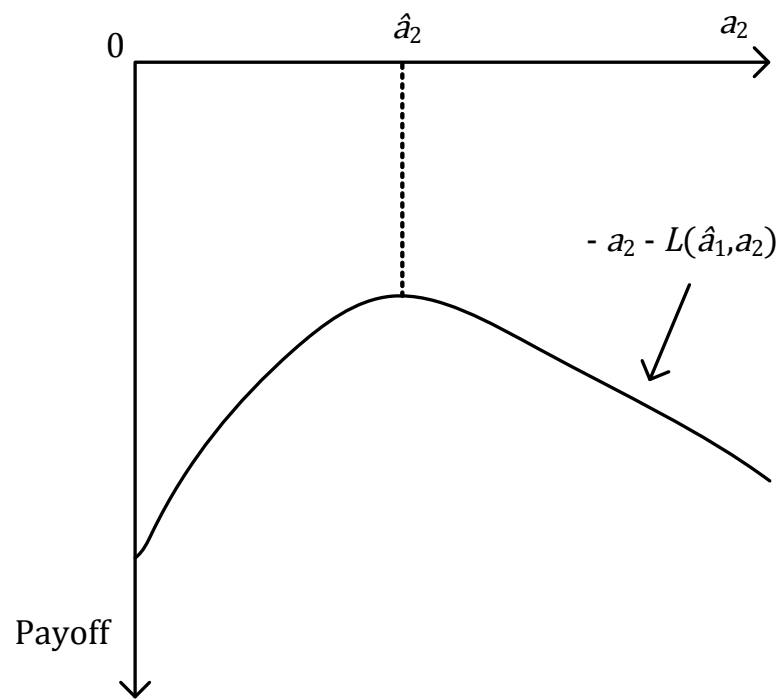
And because  $\hat{a}_1 = \frac{1}{2\alpha}$  is a constant, then

$$\hat{a}_2 \text{ maximizes } \underbrace{\$ a_2 \$ (2 \$ \alpha (\frac{1}{2\alpha})^2 \$ \beta (a_2)^2)}$$

which coincides with the victim's payoff. 

## Accident Law

- Victim's payoff when the injurer selects  $a_1 = \hat{a}_1 = \frac{1}{2\alpha}$



## Accident Law

- Hence, action  $a_2 = \hat{a}_2 = \frac{\beta^2}{4}$  is a *BR* of the victim to action  $\hat{a}_1 = \frac{1}{2\alpha}$  by the injurer.
- Therefore, for standards of care  $X_1 = \hat{a}_1 = \frac{1}{2\alpha}$  and  $X_2 = \hat{a}_2 = \frac{\beta^2}{4}$  the NE of the game is exactly  $(a_1, a_2) = (\hat{a}_1, \hat{a}_2) = (\frac{1}{2\alpha}, \frac{\beta^2}{4})$ .
  - It is easy to check that there are no other equilibria in this game (see Osborne pp 95-96).
- Hence, if legislators can determine the values of  $\hat{a}_1$  and  $\hat{a}_2$  that maximize aggregate welfare...
  - then by writing these levels into law they will induce a game that has as its unique NE these socially optimal actions.

## Accident Law

- **Reading recommendations.** If you are more interested in extensions on these topics you will enjoy these two books on law and economics:
  - *Economics of the Law: Torts, Contracts, Property and Litigation.* Thomas Miceli. Oxford University Press.
  - *The failure of judges and the rise of regulators.* Andrei Shleifer. MIT Press.