BACHELOR of ECONOMICS
International Program
Thammasat University | Thailand

FN 312 Investments

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Optimal Portfolio With Many Risky Assets

Objectives

- Optimal diversification with N risky assets
- The risk aversion level
- Asset allocation in practice

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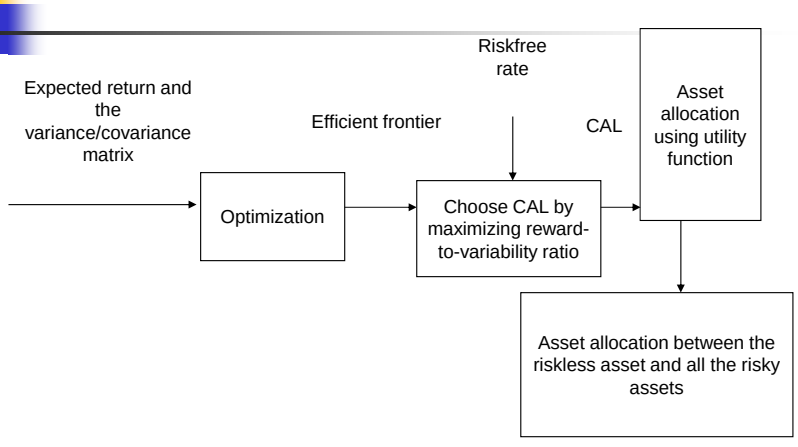
Markowitz Portfolio Selection Model



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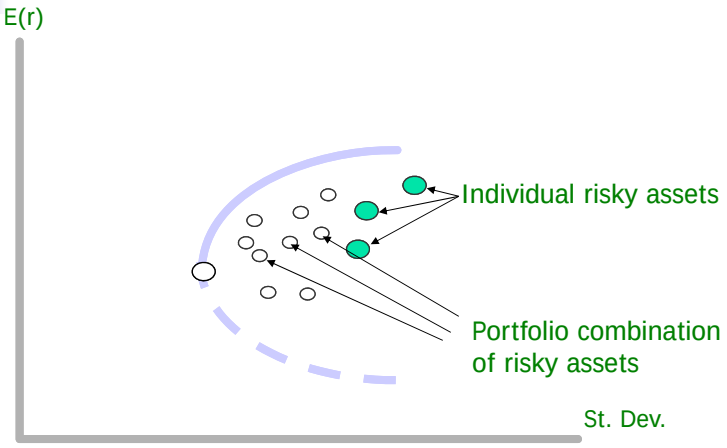
The case of N Risky Assets

The investment opportunity set of N Risky Assets



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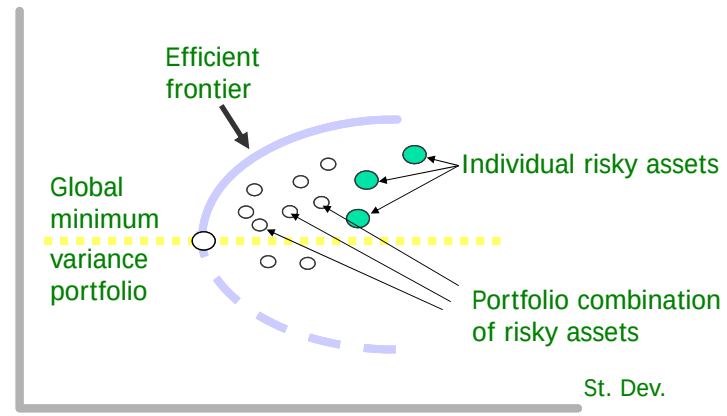

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Constructing the mean-variance frontier

Objective: find for each level of expected return the portfolio with minimum variance (or STD). How?

1. Estimate expected return, variances-covariance matrix
2. Choose a level of return and determine the weights of each asset that will result in a portfolio with the smallest variance

Expected return of a portfolio of N assets

$$E[\tilde{r}_P] = w_1 E[\tilde{r}_1] + w_2 E[\tilde{r}_2] + \ldots + w_N E[\tilde{r}_N]$$

Variance of a portfolio of N assets

$$\sigma_P^2 = \sum_{i=1}^{N} w_i^2 \sigma_i^2 + \sum_{i=1}^{N} \sum_{\substack{j=1 \\ i \neq j}}^{N} w_i w_j \sigma_i \sigma_j \rho_{i,j}$$

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Variance of a portfolio of 3 assets

	w1	w2	w3
w1	σ_1^2	$\rho_{12}\sigma_1\sigma_2$	$\rho_{13}\sigma_1\sigma_3$
w2	$\rho_{12}\sigma_1\sigma_2$	σ_2^2	$\rho_{23}\sigma_2\sigma_3$
w3	$\rho_{13}\sigma_1\sigma_3$	$\rho_{23}\sigma_2\sigma_3$	σ_3^2

$w_1^2\sigma_1^2$	$w_1 w_2 \rho_{12}\sigma_1\sigma_2$	$w_1 w_3 \rho_{13}\sigma_1\sigma_3$
$w_1 w_2 \rho_{12}\sigma_1\sigma_2$	$w_2^2\sigma_2^2$	$w_2 w_3 \rho_{23}\sigma_2\sigma_3$
$w_1 w_3 \rho_{13}\sigma_1\sigma_3$	$w_2 w_3 \rho_{23}\sigma_2\sigma_3$	$w_3^2\sigma_3^2$

Sum all cells in the second matrix = Variance of the portfolio

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Constructing the efficient frontier

Minimizing portfolio variance

Subject to : 1) the proportion of investment sum to one

2) investment in each asset being more than or equal to 0

3) a specific level of expected return

3. The weights are determined by quadratic programming optimization with constraints
4. Trace out the frontier by repeating the optimization for all levels of expected return
5. Graph out the efficient frontier

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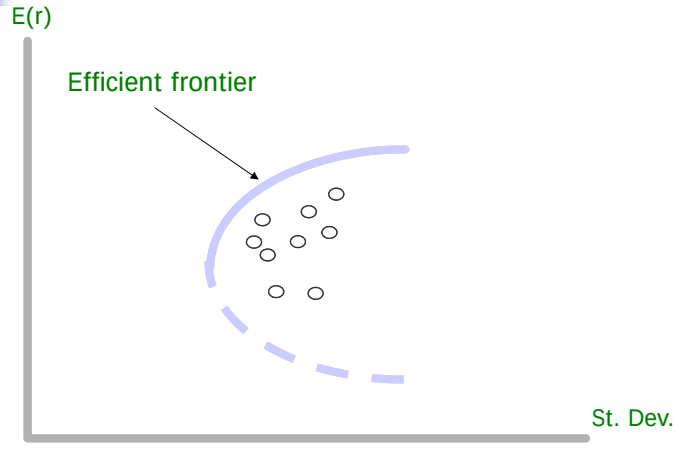
The case of N Risky Assets



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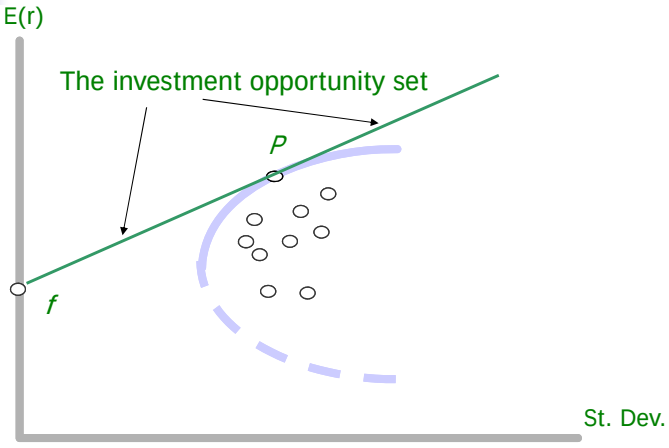
The case of N Risky Assets

The investment opportunity set of N Risky Assets and the riskfree asset



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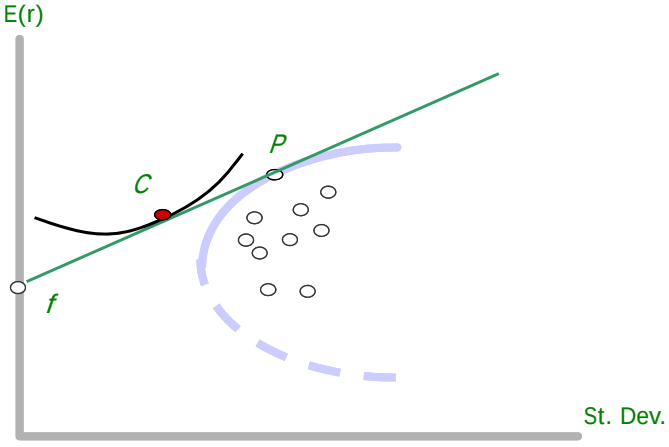
The case of N Risky Assets

Find the optimal portfolio for a specific investor using his utility function



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Summary

- Optimal diversification of many risky assets
- Application using Excel

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