EE431: Economics of financial market and institutions

Semester 1/2017

Assignment 2 (due Sept, 28th 2017 at the BE office)

<u>Instruction:</u> Use PVF and PVIFA table in your calculation

1. Consider two assets with the structure of cash-flow payment given by

Case	Asset A	Asset B
Raining (P=1/3)	1	0
Not raining (P=2/3)	0	1

Indeed, asset A promises to deliver \$1 to the holder only if there will be rain tomorrow. But, when there is no rain tomorrow, the holder of asset A receives nothing in return. On the other hand, asset B will give out \$1 of cash flow if it won't rain tomorrow, and nothing if it rains. (Economically, both types of asset are considered as contingent asset whose cash-flow payments depend on some certain conditions or circumstances.)

Given that the price per unit of each asset is equal to 0.32 and 0.60 respectively. Consider the following questions. (Cash-flow given in the table is the amount of dollar received for each unit of asset.)

1.1) Calculate the market-implied discount rate for the two assets.

$$price = \frac{expected\ cash\ flow}{(1+r)}$$

$$A: \quad 0.32 = \frac{\frac{1}{3}}{(1+r)} \Rightarrow r = 4.16\% \qquad B: \quad 0.60 = \frac{\frac{2}{3}}{(1+r)} \Rightarrow r = 11.1\%$$

1.2) Suppose that a company is launching an IPO with the pattern of cash-flow structure given in the below table. (See the table in the next page.) That, for every single unit of stock purchased, the holder will be receiving $2 if raining occurs tomorrow, and $3 if there will be no rain.

Case	Stock cash-flow
Raining (P=1/3)	2
No raining (P=2/3)	3

Based on the information above (including what's given in 1.1), what should be the market price of this stock? Explain the reason for your answer.

The new IPO asset can be formed up by combining 2 units of Asset A and 3 units of Asset B. By the principle of equivalent asset, the new IPO asset should be equal to the price of new synthetic asset that is constructed from the two basic assets. Thus the price of new IPO should be equal to 2Pa + 3Pb = 2(0.32) + 3(0.60) = 0.64 + 1.8 = 2.44

1.3) What is the market-implied risk-free rate? What should be the price of risk-free asset? (Hint: An asset is considered to be a risk-free asset if the asset delivers the same fixed amount of cash-flow under any circumstances.)

Risk-free asset can be simply constructed by combining one unit of Asset A with one unit of Asset B. The new synthesized asset generates $1 cash-flow in every circumstance. Thus the price of risk-free asset is equal to $0.92. Given that the price of risk-free asset is 0.92, implied risk-free rate can be calculated by equating market price with its present value. We find that the market-implied risk-free rate is 8.69% (Setting 0.92 = 1/(1+r) and then solve for "r".)

2. Consider the following five zero-coupon bonds, each of which has the face value of $100.

Time to maturity	Spot rate (%)*	Price
1 year	1	A
2 year	2	B
3 year	3	C
4 year	4	D
5 year	5	E

*Financially, yield to maturity of a *zero-coupon bond* is called *spot rate*

Consider the following questions

2.1) Calculate the price of each of the five zero-coupon bonds

Prices should be equal to $99.00, $96.11, $91.51, $85.48, and $78.35

2.2) Suppose that an investment banker is now offering you a new investment asset with the asked price of $100. The structure of cash-flow of the new asset offered is given in the below table. Should your purchase this new asset? Why or why not?

Year	1	2	3	4	5
Cash-flow	4	4	4	4	104

The new asset can be constructed by combining 0.04 unit of each zero-coupon bond with maturity less than or equal to 4 years and 1.04 unit of 5-year zero coupon bond. Thus, the price of new asset should be equal to

0.04*99 + 0.04*96.11 + 0.04*91.51 + 0.04*85.48 + 1.04*78.35 = 96.37

Therefore, the asked price of $100 is too high.

2.3) Suppose that you hold that new asset to the maturity date, what would be the yield of maturity of your investment? (Hint: State the condition that is needed for the calculation, but you don't need to calculate the numerical value of YTM.)

The new asset looks similar to a 5-year coupon bond with its face value of $100 and 4% coupon rate paid annually. IN the equilibrium, market price is supposed to be $96.37. Since market price is lower than $100, yield is supposed to be greater than coupon rate. Upon the calculation, YTM is approximately equal to 4.84%, i.e. around 5%

3. Consider the following table

Bond	Time to maturity (years)	Payments per year	coupon (% pa)	yield (% pa)	price ($)
A	3	1	3%	2%	**F**
B	5	2	4%	**G**	918.24332
C	7	3	**H**	5%	1000
D	10	4	4%	8%	**J**
E	10	0	**K**	5%	**L**

Suppose that each of the five bonds has its face value of $1,000. Consider the following questions.

3.1) Complete the table by filling in the value of F, G, H J, K and L. Show your work and explain everything step-by-step.

3.2) Calculate the Macaulay Duration, Modified Duration, and % change the price of each of the five bonds. Which one of them has the greatest interest rate risk? Is it the one with longest remaining time to maturity?

- To find "F", we note that:

$$F = \left(\frac{3}{100} * 1000\right) * PVIFA(2\%, 3) + 1000 * PVF(2\%, 3)$$

$$= 30*(2.88388) + 1000*(0.94232) = 1028.836$$

To find G, we note first that yield must be greater than coupon rate. Using the bond pricing equation, we obtain that

$$918.24332 = \left(\frac{2}{100} * 1000\right) * PVIFA(r, 10) + 1000 * PVF(r, 10)$$

Having looked at the table "r" should be roughly 3% per semi-annual; yield to maturity is then 6% per annual.

- This might be the easiest one. To find H, it is obvious that H = 5% because price is equal to face value.

- To find J, use the bond pricing formula:

$$J = \left(\frac{1}{100} * 1000\right) * PVIFA(2\%, 40) + 1000 * PVF(2\%, 40)$$

$$= 10*(27.35548) + 1000*(0.45289) = 726.2455$$

- To find K and L, note that there is no coupon payment in each year. Thus, this bond is called zero-coupon bond; K = 0%. Price of the discount bond is then equal to:

  L = $1000 * PVF(5\%, 10) = 1000 * 0.61391 = 613.91$

1	2	3	4	5	6	7	8	9
Bond	Time to maturity (years)	Payments per year	coupon (% pa)	yield (% pa)	price ($)	MacD (period)	ModD =Mac/(1+YTM/N)	% change in price
A	3	1	3%	2%	F= 1028.836	2.91	$\frac{2.91}{1.02}$	$\frac{2.91}{1.02} = 2.85$
B	5	2	4%	G=6%	918.24332	8.84	$\frac{8.84}{1.03}$	$\frac{9.12}{1.03} * \frac{1}{2} = 4.42$
C	7	3	H=5%	5%	1000	17.88	$\frac{17.9}{1.016}$	$\frac{17.9}{1.016} * \frac{1}{3} = 5.87$
D	10	4	4%	8%	J = 726.2455	31.67	$\frac{31.67}{1.02}$	$\frac{31.67}{1.02} * \frac{1}{4} = 7.76$
E	10	0	K = 0%	5%	$L = 613.91$	10	$\frac{10}{1.05}$	$\frac{10}{1.05} = 9.52$

- Note that you can't just look at the figures in column#7 and column#8, and conclude that which one is more risky. The reason is because each figure is measured in different unit of period of time. For example, duration of E is measured in year whereas duration of D is measured in quarterly. (10 is less than 31.67; but that 31.67 is in fact equal to 7.91 year.)

4. A 2-year government bond with its face value of $1,000 offers the holder $100 as the coupon payment which is given out to the investor at the end of each year. Suppose that the current market price is $1,000, consider the following problems.

4.1) What is the coupon rate of this 2-year bond?

Coupon rate =10% (=100/1000)

4.2) What is the yield to maturity of this bond?

As price is equal to face value, yield to maturity must be equal to coupon rate.

4.3) The below table represents the investors' survey over that outlook of market interest next year. Calculate the 1-year expected rate of return on the 2-year bond purchased today. Suppose that you can sell off this bond after you've collected bond's coupon next year.

Rate	8%	10%	12%
Probability	1/2	1/4	¼

Under the three cases, current yield is equal to 10%.

- When market interest rate drops to 8%, future price will be 1018.519. Hence, we earn a 1.8% of capital gain.
- When market interest rate remains at 10%, there will no gain/loss because future price would remain constant at 1,000.
- When market interest rate increases to 12%, future price will be 982.1429. That is we have -1.7% of capital loss

Rate	8%	10%	12%
Probability	1/2	1/4	¼
Return	**11.8%**	**10.00%**	**9.3%**

Expected rate of return = ½*(11.8%) + ¼ *(10.00%) + ¼ *(9.3%)

Your expected return is the weighted average return under the three scenarios of future market interest rate.

5. Consider 3-year and 5-year bond with the face value of $1,000. Each type of the two bonds pays the holder of 2% and 5% coupon rate. The payment is given once at the end of each year. Following the information in the bond market,

current price of each of the two bonds is $971.71388 and $1,044.51822. Consider the following problems.

5.1) which one of the two bonds offers the investor higher yield? (Hint: your answer may be an approximate to the exact solution!)

Approximately 3% and 4%, respectively.

5.2) calculate the Macaulay Duration and the Modified Duration of each bond.

Macaulay Duration: 2.94 year and 4.56 year
Modified Duration: 2.88 and 4.34

5.3) which one of the two bonds has a greater degree of risk? Based on your calculation, is it true that long-term bond is riskier than short-term bond?

5-year bond is more risky because % change is price is 4.34 for a 100 basis point change in market interest rate.

6. Consider two zero-coupon bonds with remaining time to maturity of 10 years. The first one is issued by a private company named "The face". The second one is issued by the Thai government. Each bond has its yield to maturity of 6% and 3%, respectively. Suppose that the face value of each of the two bonds is $1,000. (Don't get surprised why the two domestic bonds have their face value in USD. This is an example of what is called foreign-currency denominated bond. That is, a domestic entity promises investors to repay coupon and principal amount in terms of foreign currency. In this example, the payment is made inUS dollar.) Form the information, consider the following problems.

6.1) calculate the current price of each of the two bonds.

6.2) using the exact bond pricing formula, calculate the percentage change in price of the two bonds if market interest unexpectedly increases by 1% right after the investor had purchased the two bonds.

6.3) compare the percentage change in prices implied by the duration concept. How big is the approximation error? Would the approximate error be bigger under the high-yield bond than the low-yield bond, or vice versa?

- Macaulay Duration of the two discount bonds is 10 year.
- However, Modified duration of government bond is 9.70 (=10/(1.03)); meanwhile, modified duration of "the Face" is 9.43 (=10/(1.06)). The figures implies that % change in price of "The face" bond is smaller than that of government bond. This follows the convexity relationship.
- To obtain the exact % change in price of the two bonds, one needs to recalculate the new price using the new interest rate, i.e. 7% and 4%.
- At the initial level of YTM (6% and 3%), price of the two bonds are 1000*PVF(6%,10) = 558.39 (The face) and 1000*PVF(3%,10) = 744.09 (Government bond)
- After the increase in yield to 7% and 4% for each of the two bonds, new price should be 1000*PVF(7%,10) = 508.35 (The face) and 1000*PVF(4%,10) = 675.56 (Government bond)
- In terms of its percentage change, price of the two bonds drop by 8.4% and 9.2%. Consistent with what has been discussed in class, modified duration typically overestimate the size of downside risk of bond returns.

7. As a CFO of an invest bank, you are presented with daily fact sheet that summarizes the activity in bond market. Below is an example of your fact sheet in table.

Bond	Price	Coupon rate (%)	YTM (%)
U	90	6	9
V	96	9	8
W	110	8	6
X	105	0	5
Y	107	7	9
Z	100	6	6

Do you find anything wrong with the fact sheet? If yes, please state out what they are?

V; yield is lower than coupon, but price is also lower than its face value of $100.
X; Coupon = 0, price must be lower than its face value.
Y; Yield is higher than coupon, but we have that price is higher than its face value.

8. A 6% coupon bond paying interest annually has a modified duration of 10 years, sells for $800, and is priced at a yield to maturity of 8%. If the YTM increases to 9%, what is the predicted change in price using the duration concept?

dp/p = - ModD * (change in r) =
= - (10) * (0.09 – 0.08)
= - (10)*(0.01) = -0.1
That is, price would drop by 10% when interest rate increases by 1%.
Price will decline from $800 to $720. Hence, price would decrease by $80.

9. A 6% coupon bond with semiannual coupons has a convexity (in years) of 120, sells for 80% of par, and is priced at a yield to maturity of 8%. If the YTM increases to 9.5%, what is the predicted contribution to the percentage change in price due to convexity?

You may skip because this is beyond the scope of our numerical exercise.

10. A bond with annual coupon payments has a coupon rate of 8%, yield to maturity of 10%, and Macaulay duration of 9 years. What is the bond's modified duration?

Modified duration = MacD / (1+r) = 9 / (1+0.1) = 9 / 1.1 = 8.18

11. When interest rates decline, the duration of a 30-year bond selling at
    a. premium:
    b. Increases.
    c. Decreases.

d. Remains the same.
e. Increases at first, then declines.

When interest rate declines, price will increase. Your modified duration would increase when the interest rate decrease.

12. Which bond has the longest duration?
a. 8-year maturity, 6% coupon.
b. 8-year maturity, 11% coupon.
c. 15-year maturity, 6% coupon.
d. 15-year maturity, 11% coupon.

Let's assume that the coupon is equal to yield, so each type of the bond is getting sold at part. This is to allow us to continue doing this problem.

First, note that Modified duration increase with the lowered yield. Thus, based on the pairwise comparison, you will find that duration of "a" is longer than duration of "b". At the same time, duration of "c" is longer duration of "d".

Second, modified duration increases with time to maturity. From the first point mentioned above, this implies that "c" would have longer modified duration than "a" because MacD of "c" is larger than MacD of "a".

So, the answer is "c" that is the longest modified duration.

13. A newly issued bond has the following characteristics:

Coupon	Yield	Maturity	Duration
8%	8%	15 years	10 years

a. Calculate modified duration using the information above.
Bond sold at part. MacD = 10. Then, ModD = 10/(1.08) = 9.25

b. Explain why modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates.

We have shown in class that Modified duration is the elasticity of bond price with respect to yield. Modfied duration requires that we know both MacD and the initial level of yield. Using the MacD could give us a misled conlusion over the interst rate risk of bond. It might be possible that longer bond with larger value of MacD may have smaller value of ModD than the shorter bond that has small value of MacD. Both MacD and the level of yield affect how price is sensitive to the change in the interest rate.

c. Identify the direction of change in modified duration if:

c.1) The coupon of the bond were 4%, not 8%.

As bond is newly issued, bond rate (yield) will always be equal to coupon. Bond will be sold at par. If coupon were lowered, ModD would rise. Remember ModD would rise/fall if the initial level of yield varies, i.e. ModD = MacD/(1+r).

c.2) The maturity of the bond were 7 years, not 15 years.

If bond gets its maturity shorten, modified duration will drop. That's because MacD would drop as bonds have short maturity.

d. Define convexity and explain how modified duration and convexity are used to approximate the bond's percentage change in price, given a change in interest rates.

You may skip the convexity.

PRESENT VALUE OF $1

Periods	RATE PER PERIOD																
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	0.99502	0.99007	0.98517	0.98030	0.97066	0.96117	0.95181	0.94260	0.92456	0.90703	0.89000	0.87344	0.85734	0.84168	0.82645	0.81162	0.79719
3	0.99254	0.98515	0.97783	0.97059	0.95632	0.94232	0.92860	0.91514	0.88900	0.86384	0.83962	0.81630	0.79383	0.77218	0.75131	0.73119	0.71178
4	0.99006	0.98025	0.97055	0.96098	0.94218	0.92385	0.90595	0.88849	0.85480	0.82270	0.79209	0.76290	0.73503	0.70843	0.68301	0.65873	0.63552
5	0.98759	0.97537	0.96333	0.95147	0.92826	0.90573	0.88385	0.86261	0.82193	0.78353	0.74726	0.71299	0.68058	0.64993	0.62092	0.59345	0.56743
6	0.98513	0.97052	0.95616	0.94205	0.91454	0.88797	0.86230	0.83748	0.79031	0.74622	0.70496	0.66634	0.63017	0.59627	0.56447	0.53464	0.50663
7	0.98267	0.96569	0.94904	0.93272	0.90103	0.87056	0.84127	0.81309	0.75992	0.71068	0.66506	0.62275	0.58349	0.54703	0.51316	0.48166	0.45235
8	0.98022	0.96089	0.94198	0.92348	0.88771	0.85349	0.82075	0.78941	0.73069	0.67684	0.62741	0.58201	0.54027	0.50187	0.46651	0.43393	0.40388
9	0.97778	0.95610	0.93496	0.91434	0.87459	0.83676	0.80073	0.76642	0.70259	0.64461	0.59190	0.54393	0.50025	0.46043	0.42410	0.39092	0.36061
10	0.97534	0.95135	0.92800	0.90529	0.86167	0.82035	0.78120	0.74409	0.67556	0.61391	0.55839	0.50835	0.46319	0.42241	0.38554	0.35218	0.32197
11	0.97291	0.94661	0.92109	0.89632	0.84893	0.80426	0.76214	0.72242	0.64958	0.58468	0.52679	0.47509	0.42888	0.38753	0.35049	0.31728	0.28748
12	0.97048	0.94191	0.91424	0.88745	0.83639	0.78849	0.74356	0.70138	0.62460	0.55684	0.49697	0.44401	0.39711	0.35553	0.31863	0.28584	0.25668
13	0.96806	0.93722	0.90743	0.87866	0.82403	0.77303	0.72542	0.68095	0.60057	0.53032	0.46884	0.41496	0.36770	0.32618	0.28966	0.25751	0.22917
14	0.96565	0.93256	0.90068	0.86996	0.81185	0.75788	0.70773	0.66112	0.57748	0.50507	0.44230	0.38782	0.34046	0.29925	0.26333	0.23199	0.20462
15	0.96324	0.92792	0.89397	0.86135	0.79985	0.74301	0.69047	0.64186	0.55526	0.48102	0.41727	0.36245	0.31524	0.27454	0.23939	0.20900	0.18270
16	0.96084	0.92330	0.88732	0.85282	0.78803	0.72845	0.67362	0.62317	0.53391	0.45811	0.39365	0.33873	0.29189	0.25187	0.21763	0.18829	0.16312
17	0.95844	0.91871	0.88071	0.84438	0.77639	0.71416	0.65720	0.60502	0.51337	0.43630	0.37136	0.31657	0.27027	0.23107	0.19784	0.16963	0.14564
18	0.95605	0.91414	0.87416	0.83602	0.76491	0.70016	0.64117	0.58739	0.49363	0.41552	0.35034	0.29586	0.25025	0.21199	0.17986	0.15282	0.13004
19	0.95367	0.90959	0.86765	0.82774	0.75361	0.68643	0.62553	0.57029	0.47464	0.39573	0.33051	0.27651	0.23171	0.19449	0.16351	0.13768	0.11611
20	0.95129	0.90506	0.86119	0.81954	0.74247	0.67297	0.61027	0.55368	0.45639	0.37689	0.31180	0.25842	0.21455	0.17843	0.14864	0.12403	0.10367
21	0.94892	0.90056	0.85478	0.81143	0.73150	0.65978	0.59539	0.53755	0.43883	0.35894	0.29416	0.24151	0.19866	0.16370	0.13513	0.11174	0.09256
22	0.94655	0.89608	0.84842	0.80340	0.72069	0.64684	0.58086	0.52189	0.42196	0.34185	0.27751	0.22571	0.18394	0.15018	0.12285	0.10067	0.08264
23	0.94419	0.89162	0.84210	0.79544	0.71004	0.63416	0.56670	0.50669	0.40573	0.32557	0.26180	0.21095	0.17032	0.13778	0.11168	0.09069	0.07379
24	0.94184	0.88719	0.83583	0.78757	0.69954	0.62172	0.55288	0.49193	0.39012	0.31007	0.24698	0.19715	0.15770	0.12640	0.10153	0.08170	0.06588
25	0.93949	0.88277	0.82961	0.77977	0.68921	0.60953	0.53939	0.47761	0.37512	0.29530	0.23300	0.18425	0.14602	0.11597	0.09230	0.07361	0.05882
30	0.92783	0.86103	0.79919	0.74192	0.63976	0.55207	0.47674	0.41199	0.30832	0.23138	0.17411	0.13137	0.09938	0.07537	0.05731	0.04368	0.03338
35	0.91632	0.83982	0.76988	0.70591	0.59387	0.50003	0.42137	0.35538	0.25342	0.18129	0.13011	0.09366	0.06763	0.04899	0.03558	0.02592	0.01894
40	0.90495	0.81914	0.74165	0.67165	0.55126	0.45289	0.37243	0.30656	0.20829	0.14205	0.09722	0.06678	0.04603	0.03184	0.02209	0.01538	0.01075
50	0.88263	0.77929	0.68825	0.60804	0.47500	0.37153	0.29094	0.22811	0.14071	0.08720	0.05429	0.03395	0.02132	0.01345	0.00852	0.00542	0.00346

PRESENT VALUE OF ORDINARY ANNUITY
(annuity in arrears -- end of period payments)

Periods	RATE PER PERIOD																
	0.25%	0.50%	0.75%	1.00%	1.50%	2.00%	2.50%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%	10.00%	11.00%	12.00%
1	0.99751	0.99502	0.99256	0.99010	0.98522	0.98039	0.97561	0.97087	0.96154	0.95238	0.94340	0.93458	0.92593	0.91743	0.90909	0.90090	0.89286
2	1.99252	1.98510	1.97772	1.97040	1.95588	1.94156	1.92742	1.91347	1.88609	1.85941	1.83339	1.80802	1.78326	1.75911	1.73554	1.71252	1.69005
3	2.98506	2.97025	2.95556	2.94099	2.91220	2.88388	2.85602	2.82861	2.77509	2.72325	2.67301	2.62432	2.57710	2.53129	2.48685	2.44371	2.40183
4	3.97512	3.95050	3.92611	3.90197	3.85438	3.80773	3.76197	3.71710	3.62990	3.54595	3.46511	3.38721	3.31213	3.23972	3.16987	3.10245	3.03735
5	4.96272	4.92587	4.88944	4.85343	4.78264	4.71346	4.64583	4.57971	4.45182	4.32948	4.21236	4.10020	3.99271	3.88965	3.79079	3.69590	3.60478
6	5.94785	5.89638	5.84560	5.79548	5.69719	5.60143	5.50813	5.41719	5.24214	5.07569	4.91732	4.76654	4.62288	4.48592	4.35526	4.23054	4.11141
7	6.93052	6.86207	6.79464	6.72819	6.59821	6.47199	6.34939	6.23028	6.00205	5.78637	5.58238	5.38929	5.20637	5.03295	4.86842	4.71220	4.56376
8	7.91074	7.82296	7.73661	7.65168	7.48593	7.32548	7.17014	7.01969	6.73274	6.46321	6.20979	5.97130	5.74664	5.53482	5.33493	5.14612	4.96764
9	8.88852	8.77906	8.67158	8.56602	8.36052	8.16224	7.97087	7.78611	7.43533	7.10782	6.80169	6.51523	6.24689	5.99525	5.75902	5.53705	5.32825
10	9.86386	9.73041	9.59958	9.47130	9.22218	8.98259	8.75206	8.53020	8.11090	7.72173	7.36009	7.02358	6.71008	6.41766	6.14457	5.88923	5.65022
11	10.83677	10.67703	10.52067	10.36763	10.07112	9.78685	9.51421	9.25262	8.76048	8.30641	7.88687	7.49867	7.13896	6.80519	6.49506	6.20652	5.93770
12	11.80725	11.61893	11.43491	11.25508	10.90751	10.57534	10.25776	9.95400	9.38507	8.86325	8.38384	7.94269	7.53608	7.16073	6.81369	6.49236	6.19437
13	12.77532	12.55615	12.34235	12.13374	11.73153	11.34837	10.98318	10.63496	9.98565	9.39357	8.85268	8.35765	7.90378	7.48690	7.10336	6.74987	6.42355
14	13.74096	13.48871	13.24302	13.00370	12.54338	12.10625	11.69091	11.29607	10.56312	9.89864	9.29498	8.74547	8.24424	7.78615	7.36669	6.98187	6.62817
15	14.70420	14.41662	14.13699	13.86505	13.34323	12.84926	12.38138	11.93794	11.11839	10.37966	9.71225	9.10791	8.55948	8.06069	7.60608	7.19087	6.81086
16	15.66504	15.33993	15.02431	14.71787	14.13126	13.57771	13.05500	12.56110	11.65230	10.83777	10.10590	9.44665	8.85137	8.31256	7.82371	7.37916	6.97399
17	16.62348	16.25863	15.90502	15.56225	14.90765	14.29187	13.71220	13.16612	12.16567	11.27407	10.47726	9.76322	9.12164	8.54363	8.02155	7.54879	7.11963
18	17.57953	17.17277	16.77918	16.39827	15.67256	14.99203	14.35336	13.75351	12.65930	11.68959	10.82760	10.05909	9.37189	8.75563	8.20141	7.70162	7.24967
19	18.53320	18.08236	17.64683	17.22601	16.42617	15.67846	14.97889	14.32380	13.13394	12.08532	11.15812	10.33560	9.60360	8.95011	8.36492	7.83929	7.36578
20	19.48449	18.98742	18.50802	18.04555	17.16864	16.35143	15.58916	14.87747	13.59033	12.46221	11.46992	10.59401	9.81815	9.12855	8.51356	7.96333	7.46944
21	20.43340	19.88798	19.36280	18.85698	17.90014	17.01121	16.18455	15.41502	14.02916	12.82115	11.76408	10.83553	10.01680	9.29224	8.64869	8.07507	7.56200
22	21.37995	20.78406	20.21121	19.66038	18.62082	17.65805	16.76541	15.93692	14.45112	13.16300	12.04158	11.06124	10.20074	9.44243	8.77154	8.17574	7.64465
23	22.32414	21.67568	21.05331	20.45582	19.33086	18.29220	17.33211	16.44361	14.85684	13.48857	12.30338	11.27219	10.37106	9.58021	8.88322	8.26643	7.71843
24	23.26598	22.56287	21.88915	21.24339	20.03041	18.91393	17.88499	16.93554	15.24696	13.79864	12.55036	11.46933	10.52876	9.70661	8.98474	8.34814	7.78432
25	24.20547	23.44564	22.71876	22.02316	20.71961	19.52346	18.42438	17.41315	15.62208	14.09394	12.78336	11.65358	10.67478	9.82258	9.07704	8.42174	7.84314
30	28.86787	27.79405	26.77508	25.80771	24.01584	22.39646	20.93029	19.60044	17.29203	15.37245	13.76483	12.40904	11.25778	10.27365	9.42691	8.69379	8.05518
35	33.47243	32.03537	30.68266	29.40858	27.07559	24.99862	23.14516	21.48722	18.66461	16.37419	14.49825	12.94767	11.65457	10.56682	9.64416	8.85524	8.17550
40	38.01986	36.17223	34.44694	32.83469	29.91585	27.35548	25.10278	23.11477	19.79277	17.15909	15.04630	13.33171	11.92461	10.75736	9.77905	8.95105	8.24378
50	46.94617	44.14279	41.56645	39.19612	34.99969	31.42361	28.36231	25.72976	21.48218	18.25593	15.76186	13.80075	12.23348	10.96168	9.91481	9.04165	8.30450