B.E. International Program
Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 2/2015

Homework 1

Due 11th February 2016 (during the lecture / before 11:15 am)

There are six questions in total. Each of them is worth equally.

1. Mr. Push produces shoes for sale, and his total cost function is given by $C(Q) = 105 + 23Q$, where Q is the output level.

   a. (2 points) Suppose that Mr. Push sells his shoes in a perfectly competitive shoe market at the price \$30 per pair. How many pairs of shoes should Mr. Push sell in order to break-even?

      Ans. $\pi_{BE} = 30Q - (105 + 23Q) = 0$ ➔ $Q_{BE} = 15$.

   b. (3 points) Suppose now that Mr. Push is the only producer in this market (because his shoes have special designs), and the demand function he faces is $Q_d = 153 - P$. If he is still able to sell his shoes at the break-even amount in the competitive market (i.e. the quantity found in part a.), what is his new profit?

      Ans. $Q = 153 - P$ ➔ $P = 153 - Q$

      $\pi = (153 - Q)Q - (105 + 23Q)$

      $\pi = 153Q - Q^2 - 105 - 23Q = -Q^2 + 130Q - 105$

      At $Q = 15,\ \pi = 1620$

2. Consider the following system of equations:

$$Q_{d1} = 33 - 4P_1 - 4P_2 + P_3 \qquad\qquad Q_{s1} = 2 + P_1$$

$Q_{d2} = 18 - 2P_1 - 3P_2 + 4P_3$ $\qquad\qquad Q_{s2} = 3 + 2P_2$

$Q_{d3} = 15 + P_1 + 5P_2 - 4P_3$ $\qquad\qquad Q_{s3} = 7 + 5P_3$

a. (2 points) What are the relationships among the three goods?

Ans. Good 1 and good 2 are complements to one another, but they are substitutes for good 3.

b. (3 points) Find the equilibrium price and quantity for the three goods.

Ans.

$$(Q_1^*, P_1^*) = (5,\ 3)$$
$$(Q_2^*, P_2^*) = (13,\ 5)$$
$$(Q_3^*, P_3^*) = (27,\ 4)$$

3. Consider the following IS-LM model:

Commodity market:

$$Y = C + I + G_0$$
$$C = C_0 + bY, \qquad (C_0 > 0,\ 0 < b < 1)$$
$$I = I_0 - ar, \qquad (I_0 > 0,\ a > 0)$$

Money market:

$$M_S = M_0$$
$$M_D = mY - hr, \qquad (m > 0,\ h > 0)$$

a. (2 points) Write out the explicit IS-LM system of equations, and determine the equilibrium national income and equilibrium interest rate.

Ans. IS: $Y = \frac{C_0 + I_0 + G_0 - ar}{(1-b)}$

LM: $Y = \frac{M_0 + hr}{m}$

$Y^* = \frac{h(C_0 + I_0 + G_0) + aM_0}{am + h(1-b)}$; $\quad r^* = \frac{m(C_0 + I_0 + G_0) - (1-b)M_0}{am + h(1-b)}$

b. (1 points) Find the impact of an exogenous increase in money supply (M_0) on the equilibrium interest rate found in part (a). Assume everything else remains constant.

Ans. $\frac{\Delta r^*}{\Delta M_0} = \frac{-(1-b)}{am + h(1-b)}$

c. (1 points) Suppose that $C_0 = 250$, $I_0 = 150$, $G_0 = 50$, $b = 0.8$, $a = 1000$, $h = 1500$, $M_0 = 200$, and $m = 0.4$. Find the equilibrium national income and interest rate.

Ans. $Y^* = 1250;\ \ r^* = 0.2$

d. (1 points) Based on the information in part (c), if the government expenditure (G_0) increases to 120, all else constant, what is the *change* in the equilibrium national income?

Ans. $\frac{\Delta Y^*}{\Delta G_0} = \frac{h}{am+h(1-b)} = \frac{1500}{700} = \frac{15}{7} \approx 2.14$

If ΔG_0 =70, then $\Delta Y^* = 150$.

4. Consider a café that serves both pizza and cookies. Suppose the demand curve and supply curve for pizza in a café are given by

Demand: Q = 300 – 20P – 20Pc

Supply: Q = 10P – 10

where P is the price of a pizza, and Pc is price of a cookie.

Answer the following questions:

a. (3 points) Suppose the price of a cookie is $P^c = 5$. Find the equilibrium price and quantity of pizza.

Given Pc = 5, demand equation would be Q = 300 - 20P - 20*5 = 200 - 20P

Setting demand equal to supply, we obtain that

10P - 10 = 200 - 20P ⇒ P* = 7

Q* = 10(7) - 10 = 60 units

b. (4 points) Due to a change in the price of cookies, the *new* equilibrium price of pizza changes to P*’ = 5. What must be the price of a cookie now?

Solve for the solution of equilibrium price/quantity in term of Pc.

300 – 20P – 20Pc = 10P – 10 $\Rightarrow P^* = \frac{310-20P_c}{30} \Rightarrow Q^* = 10 * (\frac{310-20P_c}{30}) - 10$.

Set $P^* = 5$ and solve for P_c. The answer is that $P_c = 8$.

c. (3 points) Are pizza and cookies substitutes or complements in this example? Use your economic intuition to justify your answer.

They are complementary product. Under the new equilibrium, quantity is equal to 40 units. As quantity decreases when price of "c" increases, this implies that both products are complementing to each other.

5. AT&T, T-mobile and Verizon all want to purchase some iPhone6 phones from Apple and hold these phones in stock so that they can promote their new contract with the iphone6 incorporated as an important element. Their respective demand curves for iPhone6 phones are as follows where Qa is the quantity of iPhone6 phones demanded by AT & T, Qt is the quantity of iPhone6 phones demanded by T-mobile, Qv is the quantity of iPhone6 phones demanded by Verizon, and P is the price per iPhone6 phone:

AT&T: $Qa = 30 - P$

T-mobile: $Qt = 20 - 0.5P$

Verizon: $Qv = 40 - 2P$

5.1) Justify the domain set of prices that allow all the three providers staying active in the market.

AT&T: max-price = \$30

T-mobile: max-price = \$40

Verizon: max-price = \$20

Ranking types: T-mobile > AT&T > Verizon

All three remain active only when P is lower than \$20.

5.2) Derive the equation for market demand

$$Q = \begin{cases} 0 & ; \ P \geq 40 \quad \text{(no one)} \\ 20 - 0.5P & ; \ 30 \leq P < 40 \quad \text{(T-mobile only)} \\ 50 - 1.5P & ; \ 20 \leq P < 30 \quad \text{(T-mobile + AT\&T)} \\ 90 - 3.5P & ; \ 0 \leq P < 20 \quad \text{(T-mobile + AT\&T + Verizon)} \end{cases}$$

Now continue with a new piece of information given. On the supply side in this market suppose Apple, the manufacturer or provider of these phones, outsources their production of the iPhone6 phones to two firms, Firm A and Firm B. These firms' supply curves are given below where Qa is the quantity of iPhone6 phones supplied by Firm A, Qb is the quantity of iPhone6 phones supplied by Firm B, and P is the price per iPhone6 phone:

Firm A: $P = 15 + Qa$

Firm B: $P = 10 - 2Qb$

5.3) Derive the market supply equation.

Firm A: min-price = $15

Firm B: min-price = $10

In terms of cost competitiveness, B is better than A as it needs only $10 to stay in the market.

Rewrite the two equations in Q-equal form:

$Qa = P - 15$;

$Qb = 0.5P - 5$;

$$Q = \begin{cases} 0 & ; \ 0 \leq P \leq 10 \quad \text{(no firm)} \\ 0.5P - 5 & ; \ 10 < P \leq 15 \quad \text{(B only)} \\ 1.5P - 20 & ; \ P > 15 \quad \text{(B and then A)} \end{cases}$$

5.4) (Hard) Based on the market demand curve in question (5.1), what is equilibrium price and quantity in this market? Explain how you found your answer and how you decided which segments of the demand curve and the supply curve were the relevant segments to consider.

Case 1: Two firms occur only when $P > 15$. One consumer (when $30 \leq P < 40$)

- $1.5P - 20 = 20 - 0.5P \Rightarrow P = 20$. But, when $P = 20$, there are two consumers. Contradict!

Case 2: Two firms occur only when $P > 15$. Two consumers (when $20 \leq P < 30$)

- $1.5P - 20 = 50 - 1.5P \Rightarrow P = 70/3 = 23.33$. Check if Q is positive. For $P = 23.33 \Rightarrow Q = 15$ units. This is the equilibrium.

Case 3: Two firms occur only when P > 15. Three consumers (when $0 \le P < 20$)

- 1.5P - 20 = 90 - 3.5P ⇒ P = 110/5 = 22 ⇒ BUt at this price, we would have only 2 consumers. Contradict!

Case 4: One firms occur only when $10 < P \le 15$. One consumer (when $30 \le P < 40$)

- 0.5P - 5 = 20 - 0.5P ⇒ P = 15. But if this is the equilibrium price, there must be three consumers in the market. Contradict!

Case 5: One firms occur only when $10 < P \le 15$. Two consumers (when $20 \le P < 30$)

- 0.5P - 5 = 50 - 1.5P ⇒ P = 45/2 = 22.5, but if P = 22.5, there must be two firms. Contradict!

Case 6: One firms occur only when $10 < P \le 15$. Three consumers (when $0 \le P < 20$)

- 0.5P - 5 = 90 - 3.5P ⇒ P = 85/4 = 21.25. But if price is 21.25, there would be only 2 consumers in the market. Contradict!

Case 2 is the equilibrium where P = 70/3 and Q = 15. There are two consumers in the market; meanwhile, both firms stay active in the market. Can you figure out how much each firm produce, and how much does each consumer buy?

6. The demand and supply curves in the market for whiskey are given by the following equations:

Demand: $Q_d = 120 - 3P$

Supply: $P = 0.5Q_s$

where Q is the quantity of whiskey in bottles and P is the price per bottle of whiskey.

Answer the following questions:

a. (1 point) Find the pre-tax equilibrium price and quantity, i.e. (P*,Q*).

   Setting demand equal to supply, we obtain that P*=24 and Q*48.

b. (1 point) Calculate the price elasticities of demand and supply at the equilibrium.

   elasticity of demand = -3*(24/48) = -3/2 and elasticity of supply = 2*(24/48) = 1

c. (1 point) Based on the values of calculated elasticities, can you tell us anything about the change in the total revenue if the market price can be increased by 1%?

   Demand is elastic. So, it’s above the midpoint. If price increases by 1%, quantity would decrease by more than 1%,which implies that total revenue would drop accordingly.

Now suppose that the government in this economy has levied an excise tax of $5 on the *suppliers* of whiskey.

d. (2 points) Find the post-tax equilibrium price and quantity.

   Tax at the producer would imply that ps = pd - t = pd - 5.

   ps = pd - 5

   0.5Q = (⅓)*(120-Q) - 5

   Q = 42.

   Ps = 0.5*(42) = 21

   Pd = Ps + 5 = 26.

e. (2 point) How much total revenue can government collect?

   Tax revenue = t*Q = 5*42 = 210.

f. (3 point) In terms of economic incidence, what percentage of the tax is actually borne by suppliers?

   Initial price was $24. After imposing $5 unit tax, Ps = 21 and Pd = 26. Producers get $3 less than before. Consumers pay $2 more than before. These number of tax burdens that each would have to bear to a total of $5 unit tax. Consumers' burden is then 40% and producers' burden is 60%. This makes sense because consumers' demand is relatively more elastic than producers' supply.