The Logic of Compound Statements: II

TU251: Fundamental Mathematics

Saifon Chaturantabut

Department of Mathematics and Statistics
TU

2/2014

Outline

1. Valid and Invalid Arguments: Introduction & Definitions

2. Rules of Inference
   - Modus Ponens and Modus Tollens
   - Other Rules of Inference

3. Fallacies

Valid and Invalid Arguments: Introduction & Definitions

Definition

An **argument** is a sequence of statements, and an **argument form** is a sequence of statement forms.

- *All statements in an argument and all statement forms in an argument form, except for the final one*, are called **premises** (or **assumptions** or **hypotheses**).
- The final statement or statement form is called the **conclusion**.
- The symbol "$\therefore$" , which is called *"therefore"* is normally placed just *before the conclusion*.

Valid & Invalid Statements

- To say that an argument form is **valid** means that...

  whenever the premises are all true, then the conclusion is also true,

  no matter what particular statements are substituted for the statement variables in its premises.
- An argument is **invalid** means that there is an argument of that form whose premises are all true and whose conclusion is false.

When an argument is valid and its premises are true, the truth of the conclusion is said to be *inferred* or *deduced* from the truth of the premises.

Valid & Invalid Arguments

Testing an Argument Form for Validity

1. Identify the premises and conclusion of the argument form.
2. Construct a truth table showing the truth values of all the premises and the conclusion.
3. Find rows in which all premises are true. Each of these row is called a **critical row**.
    - If there is a critical row in which the conclusion is false, then *the argument form is invalid*.
    - If the conclusion in every critical row is true, then *the argument form is valid*.

Valid & Invalid Arguments

Example: Show that the statements:

"All lawyers have gone to law school. Mark is a lawyer.
Therefore Mark went to law school."

form an argument. Determine if this argument is valid or invalid.
Answer: Let

p: Mark is a lawyer.

q: All lawyers have gone to law school.

r: Mark went to law school.

Then

Valid & Invalid Arguments

Example: Show that the argument

$$p \to q$$
$$q \to p$$
$$\therefore \quad p \vee q$$

is invalid

Answer:

Rules of inference

Definition

- A rule of inference is a form of argument that is valid.
- An argument form consisting of two premises and a conclusion is called a **syllogism**. The first and second premises are called the *major premise* and *minor premise*, respectively.
  - Modus Ponens
  - Modus Tollens

Valid Arguments: Modus Ponens

Modus Ponens (the method of affirming)

$$\begin{array}{rl} & p \rightarrow q \\ & p \\ \therefore & q \end{array}$$

Truth table:

Example:

Valid Arguments: Modus Tollens

Modus Tollens (the method of denying)

$$\begin{array}{rl} & p \to q \\ & \sim q \\ \therefore & \sim p \end{array}$$

Truth table:

Example:

Modus Ponens & Modus Tollens: Example

Example: Use modus ponens or modus tollens to fill in the blanks of the following arguments so that they become valid inferences.

1. If there are more pigeons than there are pigeonholes, then at least two pigeons roost in the same hole. There are more pigeons than there are pigeonholes.

   $\therefore$ ....

2. If 870,232 is divisible by 6, then it is divisible by 3. 870,232 is not divisible by 3.

   $\therefore$ ....

Other Rules of Inference: Generalization

Generalization

(I)

$$\begin{array}{ll} & p \\ \therefore & p \vee q \end{array}$$

(II)

$$\begin{array}{ll} & q \\ \therefore & p \vee q \end{array}$$

Truth table:

Example:

Other Rules of Inference: Specialization

Specialization

(I)

$$\begin{array}{ll} & p \wedge q \\ \therefore & p \end{array}$$

(II)

$$\begin{array}{ll} & p \wedge q \\ \therefore & q \end{array}$$

Truth table:

Other Rules of Inference: Elimination

Elimination

(I)

$$\begin{array}{ll} & p \vee q \\ & \sim p \\ \therefore & q \end{array}$$

(II)

$$\begin{array}{ll} & p \vee q \\ & \sim q \\ \therefore & p \end{array}$$

Truth table:

Other Rules of Inference: Proof by Division into Cases

Proof by Division into Cases

$$
\begin{array}{rl}
 & p \vee q \\
 & p \rightarrow r \\
 & q \rightarrow r \\
\therefore & r
\end{array}
$$

Other Rules of Inference: Contradiction Rule

Contradiction Rule

- If you can show that the supposition that p is false leads logically to a contradiction, then you can conclude that p is true.
- If an assumption leads to a contradiction, then that assumption must be false.

Let c be a contradiction. Show that the following argument form is valid:

$$\begin{array}{ll} & \sim p \rightarrow c \\ \therefore & p \end{array}$$

Truth table:

Summary: Rules of Inferences

Valid Argument Forms

Modus Ponens	$p \to q$ p $\therefore q$	**Elimination**	**a.** $p \vee q$ $\sim q$ $\therefore p$ **b.** $p \vee q$ $\sim p$ $\therefore q$
Modus Tollens	$p \to q$ $\sim q$ $\therefore \sim p$	**Transitivity**	$p \to q$ $q \to r$ $\therefore p \to r$
Generalization	**a.** p $\therefore p \vee q$ **b.** q $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$ $p \to r$ $q \to r$ $\therefore r$
Specialization	**a.** $p \wedge q$ $\therefore p$ **b.** $p \wedge q$ $\therefore q$		
Conjunction	p q $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to c$ $\therefore p$

Fallacies

Definition (Fallacies)

A fallacy is an error in reasoning that results in an invalid argument.

Invalid argument: Converse Error

$$\begin{array}{ll} & p \to q \\ & q \\ \therefore & p \end{array}$$

Invalid argument: Inverse Error

$$\begin{array}{ll} & p \to q \\ & \sim p \\ \therefore & \sim q \end{array}$$

Example

Example (Converse Error):

Show that the following argument is invalid:

If John cheated on the exam, then John sits in the back row.
John sits in the back row.
$\therefore$ John cheated on the exam.

Answer: