B.E. International Program

Faculty of Economics, Thammasat University

Final Examination: 1/2012

Subject: MA 217 Calculus for Social Sciences 2

Date: Saturday 15 December 2011 Time: 09.00 – 12.00 hrs.

Seat No…………………..

ID.No……………………

INSTRUCTIONS

DO NOT TURN OVER UNTIL TOLD THAT YOU MAY DO SO.

*This paper has __**11**__ questions. Students are to answer __**ALL**__ questions. Show all steps in answering the questions. Simplify your answers where possible. There are __**23 pages**__ in this paper. Necessary formulas are on Page No. 23. You may detach Page No. 23 for your convenience __**BUT**__ do not detach other pages. The mark for each question is given next to the problem – __**use your time wisely**__. Full mark of __**60**__ can be obtained without completing all. (TOTAL: **66** marks)*

Students:

1. Non-graphic scientific calculators are allowed. Textbooks, lecture notes or any reading materials are __**NOT**__ allowed in the examination room. If you are caught doing so you will automatically receive an **"F"** for the course and be suspended for one academic year.
2. All communication equipment (mobile phones, pagers, etc.) __**must be switched off**__.
3. Write in __**black or blue ink only**__. Any writing in pencil will __**NOT**__ be marked except for curve sketching.
4. **All of the Thammasat University examination rules are applied.**

Question	1	2	3	4	5	6	7	8	9	10	11
Marks											

Total ____________

1. Use the **Lagrange Multiplier method** to determine the critical point of a three-variable function

$$f(x, y, z) = x^2 + y^2 + z^2$$

subjected to $\quad x + y = 2$

and $\quad x + z = 2$

(a) Solve the system of linear questions using **algebra of real numbers**. [Not using matrix]

(b) Solve the system of linear questions using **matrix**.

(10 marks)

2. (a) If $\underline{\mathbf{A}} = \begin{bmatrix} 2 & 1 & 1 \\ -1 & 0 & 2 \\ 3 & 1 & 3 \end{bmatrix}$, determine $\underline{\mathbf{A}}^{-1}$ and **show that your answer is correct**.

(b) For a system of linear questions $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 4 \\ 2 \\ -2 \end{bmatrix}$, determine the solution $\underline{\mathbf{x}}$.

(8 marks)

3. If $\underline{\mathbf{A}} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, use **row operations** to determine $\underline{\mathbf{A}}^{-1}$ and **show that your answer is correct**.

(5 marks)

4. (a) Find the missing entries *

$$\begin{bmatrix} 1 & 8 & -9 & 7 & 5 \\ 0 & 1 & 0 & 4 & 4 \\ 0 & 0 & 1 & 2 & 5 \\ 0 & 0 & 0 & 1 & -5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -8 & 9 & * & * \\ 0 & 1 & 0 & -4 & * \\ 0 & 0 & 1 & -2 & -15 \\ 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Find the inverse of $\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 8 & 1 & 0 & 0 & 0 \\ -9 & 0 & 1 & 0 & 0 \\ 7 & 4 & 2 & 1 & 0 \\ 5 & 4 & 5 & -5 & 1 \end{bmatrix}$; using information from (a) if possible.

(5 marks)

5. (a) If $\underline{\mathbf{A}} = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}$, for which three numbers c is the matrix **<u>not invertable</u>**, and **<u>why not</u>**?

(b) If $c = 0$ and $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$, find the solution of this system of linear equations. If there are infinite solutions, give your answer in **<u>vector form</u>**.

(5 marks)

6. If $\underline{\mathbf{A}}$ has **row 1 + row 2 = row 3**,

(a) Show that $\underline{\mathbf{A}}$ is singular.

(b) Explain why $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ cannot have a solution.

(c) Which right side $\underline{\mathbf{b}} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ might allow a solution of $\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$? Explain what type of solution it can be.

(5 marks)

7. (a) Determine the determinant of $\underline{\mathbf{A}} = \begin{bmatrix} 2 & 1 & 5 & 1 & 3 \\ 2 & 1 & 5 & 1 & 2 \\ 4 & 3 & 2 & 1 & 1 \\ 4 & 3 & 2 & 0 & 1 \\ 2 & 1 & 6 & \pi & 7 \end{bmatrix}$.

<u>Remark</u>: The entry π is not a misprint!

(a) If $\underline{\mathbf{B}}$ is a square matrix of the 6×6 size and $\det(\underline{\mathbf{B}}) = 3$, find $\det(\underline{\mathbf{A}}^2 \underline{\mathbf{B}}^{-1})$.

(5 marks)

8. (a) Determine the determinant of $\underline{\mathbf{A}} = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -a & 0 \\ -1 & 0 & 1 & 1 \\ -t & 0 & 0 & 1 \end{bmatrix}$.

(b) For a system of linear questions $\underline{\mathbf{A}}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} i+g \\ b \\ 0 \\ T \end{bmatrix}$, determine x_2. Give the answer in term of a, b, i, g, t, T.

(6 marks)

9. Determine this integral $I = \int_{-2}^{4} \int_{0}^{3x} e^{x^2}\, dy\, dx$. Give you answer to **2 decimal places**.

(5 marks)

10. Determine this integral $I = \int_{-1}^{1} \int_{0}^{x} \int_{0}^{xy} (6z + 3y)\, dz\, dy\, dx$.

(5 marks)

11. A company sells red and yellow T-shirts and has determined that the profit function for selling x red T-shirts and y yellow T-shirts is given by

$$P(x, y) = 10{,}000 + 2{,}100x - 3x^2 + 3(y - 400)^2$$

Find **the average profit** if the company sells between 200 and 400 red T-shirts and 300 and 400 yellow T-shirts.

(7 marks)

Formulas

Differentiation

We assume that u is a differentiable function of x.

$\frac{d}{dx}(c) = 0$	$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$
$\frac{d}{dx}\left(x^n\right) = n\, x^{n-1}$	$\frac{d}{dx}\left(u^n\right) = n\, u^{n-1} \frac{du}{dx}$
$\frac{d}{dx}\left[c\ f(x)\right] = c\ f'(x)$	$\frac{d}{dx}(\ln u) = \frac{1}{u} \frac{du}{dx}$
$\frac{d}{dx}\left[f(x) \pm g(x)\right] = f'(x) \pm g'(x)$	$\frac{d}{dx}\left(e^u\right) = e^u \frac{du}{dx}$
$\frac{d}{dx}\left[f(x)g(x)\right] = f(x)g'(x) + f'(x)g(x)$	$\frac{d}{dx}(\log_b u) = \frac{1}{(\ln b)u} \cdot \frac{du}{dx}$
$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{\left[g(x)\right]^2}$	$\frac{d}{dx}\left(a^u\right) = a^u (\ln a) \frac{du}{dx}$

Integration

$\int k dx = kx + C$	$\int \left[f(x) \pm g(x)\right] dx = \int f(x) dx \pm \int g(x) dx$
$\int x^n dx = \frac{x^{n+1}}{n+1} + C,\ n \neq -1$	$\int [u(x)]^n du = \frac{u^{n+1}}{n+1} + C$
$\int e^x dx = e^x + C$	$\int e^u du = e^u + C$
$\int kf(x) dx = k \int f(x) dx$	$\int \frac{1}{u} du = \ln\lvert u\rvert + C,\ u \neq 0$