Ex 13.3 $\quad x^2y + 2y = 6 \quad \Rightarrow f(x) = y = \frac{6}{x^2+2}$

1. Domain : $y = \frac{6}{x^2+2}$

$\therefore$ The domain is $\mathbb{R}$

Range : $x^2 = \frac{6-2y}{y}$

Since $x^2 \geq 0$ and $x^2 = \frac{6-2y}{y}$, hence

$\frac{6-2y}{y} \geq 0$

		0		3	
$\frac{6-2y}{y}$	$-$		$+$		$-$

$\therefore$ The range is $(0, 3]$.

2. Intercepts :

x-int : Set $y=0$, solve for x

$$0 = \frac{6}{x^2+2}$$

$0 = 6$ which is impossible.

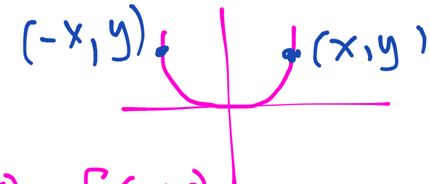
$\therefore$ There is no x-intercept.

y-int: Set $x=0$, solve for y

$$y = \frac{6}{0^2+2} = 3$$

$\therefore$ y-int is $(0,3)$.

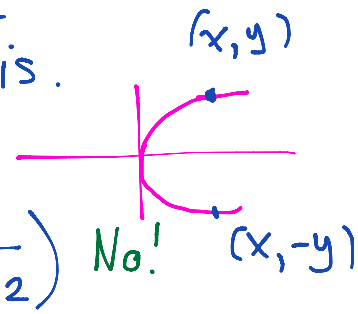
3. Symmetry



about y-axis: $f(x) = f(-x)$

$$\frac{6}{x^2+2} = \frac{6}{(-x)^2+2}$$ Yes!

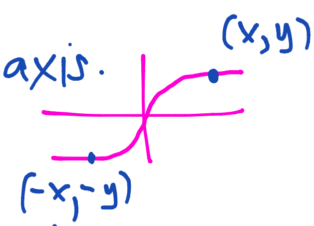
$\therefore$ It is symmetric about the y-axis.



about x-axis: $f(x) = -f(x)$

$$\frac{6}{x^2+2} \neq -\left(\frac{6}{x^2+2}\right)$$ No!

$\therefore$ It is not symmetric about the x-axis.



about the origin: $f(-x) = -f(x)$

$$\frac{6}{(-x)^2+2} \neq -\left(\frac{6}{x^2+2}\right)$$ No!

$\therefore$ It is not symmetric about the origin.

4. Asymptotes:

Horizontal Asymptote:

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{6}{x^2+2} = 0$$

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{6}{x^2+2} = 0$$

$\therefore$ $y = 0$ is a HA.

Vertical Asymptote:

There is no VA b/c there is no value of x such that $y = \frac{6}{x^2+2}$ is undefined.

5. Interval of Increase/Decrease

6. Local max/min

step(1) Find $f'(x)$

$$f(x) = \frac{6}{x^2+2}$$

$$f'(x) = \frac{-12x}{(x^2+2)^2}$$

step(2) Find critical numbers

To find critical numbers:

(i) Find x where $f'(x)$ DNE

None!

(ii) Find x where $f'(x) = 0$.

$$\frac{-12x}{(x^2+2)^2} = 0$$

$$x = 0$$

$\therefore$ critical number is $x=0$.

Step(3) Find the sign of $f'(x)$

		max	
		0	
$f'(x)$	$+$		$-$
$f(x)$	inc		dec

$\therefore$ f is increasing on $(-\infty, 0)$

and decreasing on $(0, \infty)$

f has a local maximum at $x=0$.

(The local max value is $f(0)=3$)

7. Concavity and Inflection Point:

Step(1) Find $f''(x)$

$$f'(x) = \frac{-12x}{(x^2+2)^2}$$

$$f''(x) = \frac{12(3x^2-2)}{(x^2+2)^3}$$

Step(2) Find the critical numbers

To find critical #:

(i) Find x where $f''(x)$ DNE

None!

(ii) Find x where $f''(x) = 0$

$$\frac{12(3x^2-2)}{(x^2+2)^3} = 0$$

$$3x^2 - 2 = 0$$

$$x = \pm\sqrt{\frac{2}{3}} \approx \pm 0.82$$

Step(3) Find the sign of $f''(x)$

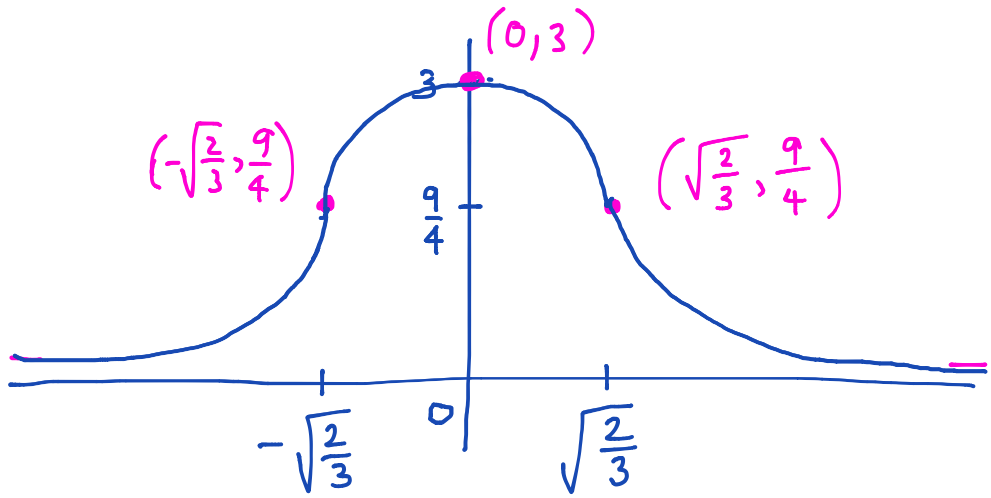
		IP		IP	
		$-\sqrt{\frac{2}{3}}$		$\sqrt{\frac{2}{3}}$	
$f''(x)$	$+$		$-$		$+$
$f(x)$	CU		CD		CU

$\therefore$ f is concave upward on $\left(-\infty, -\sqrt{\frac{2}{3}}\right) \cup \left(\sqrt{\frac{2}{3}}, \infty\right)$

f is concave downward on $\left(-\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}\right)$.

f has inflection points at $x = \pm\sqrt{\frac{2}{3}}$

and the inflection points are $\left(\pm\sqrt{\frac{2}{3}}, \frac{9}{4}\right)$.



—— inc ——|—— dec ——

—— CU ——|—— CD ——|—— CU ——