

Theorem: For all statements p , q , and r , the following statements are tautologies.

1. Law of Identity: $p \leftrightarrow p$
2. Law of Double Negation: $p \leftrightarrow \sim(\sim p)$
3. Law of Excluded Middle: $p \vee \sim p$
4. Law of Contradiction: $\sim(p \wedge \sim p)$
5. Idempotent Laws: $(p \wedge p) \leftrightarrow p$, $(p \vee p) \leftrightarrow p$
6. Law of Addition: $p \rightarrow (p \vee q)$
7. Law of Equivalence: $[p \leftrightarrow q] \leftrightarrow [(p \rightarrow q) \wedge (q \rightarrow p)]$
8. Law of Contraposition: $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$
9. Law of (Hypothetical) Syllogism: $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow [p \rightarrow r]$
10. Commutative Laws: $(p \wedge q) \leftrightarrow (q \wedge p)$
 $(p \vee q) \leftrightarrow (q \vee p)$
11. Associative Laws: $[p \wedge (q \wedge r)] \leftrightarrow [(p \wedge q) \wedge r] \leftrightarrow [p \wedge q \wedge r]$
 $[p \vee (q \vee r)] \leftrightarrow [(p \vee q) \vee r] \leftrightarrow [p \vee q \vee r]$
12. Distributive Laws: $[p \wedge (q \vee r)] \leftrightarrow [(p \wedge q) \vee (p \wedge r)]$
 $[p \vee (q \wedge r)] \leftrightarrow [(p \vee q) \wedge (p \vee r)]$
13. Absorption Laws: $[p \wedge (p \vee q)] \leftrightarrow p$
 $[p \vee (p \wedge q)] \leftrightarrow p$
14. De Morgan's Laws: $\sim(p \wedge q) \leftrightarrow (\sim p \vee \sim q)$
 $\sim(p \vee q) \leftrightarrow (\sim p \wedge \sim q)$
15. Law of Implication: $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
 $(p \vee q) \leftrightarrow (\sim p \rightarrow q)$
16. Law of Negation for Implication: $\sim(p \rightarrow q) \leftrightarrow (p \wedge \sim q)$

Rules for Proving:

1. Rule of Modus Ponens or Rule of Implication

If $p \rightarrow q$ and p , then q

2. Rule of Substitution

2.1 Substitute the statement that is tautology (or contradiction) with the other statement

2.2 Substitute the statement with its equivalent statement.

3. Definition of Conjunction

3.1 If $p \wedge q$, then p and q .

3.2 If p and q , then $p \wedge q$

4. Rule of Modus Tollens

If $p \rightarrow q$ and $\sim q$, then $\sim p$

5. Disjunctive Syllogism

5.1 If $p \vee q$ and $\sim p$, then q .

5.2 If $p \vee q$ and $\sim q$, then p .

6. Constructive Dilemma

If $(p \rightarrow q) \wedge (r \rightarrow s)$ and $p \vee r$, then $q \vee s$