Quiz 2

1. Let $A = \{0, 1, 2\}$ be the domain of variable x. Let the set of real number $\mathbb{R}$ be the domain of variable y. Find the truth set of the predicate

$$P(y): \quad \exists x \in A, \ (x < y) \wedge (x + y > 10).$$

   (a) Write the negation of $P(y)$ (without using negation symbol "∼" in the final answer).

   (b) Determine the **truth set** of the above statement. Explain your answer.

<u>**Answer:**</u>

(a) Write the negation of $P(y)$ (without using negation symbol "∼" in the final answer).

$$\sim P(y): \quad \forall x \in A, \ (x \geq y) \vee (x + y \leq 10).$$

(b) Determine the **truth set** of the above predicate $P(y)$. Explain your answer.

Consider each value of $x \in A = \{0, 1, 2\}$.

(i) For $x = 0$, we have $(0 < y) \wedge (y > 10)$,
which is true for $y > 0$ and $y > 10$. That is, $\boxed{y \in (10, \infty)}$.

(ii) For $x = 1$, we have $(1 < y) \wedge (1 + y > 10)$,
which is true for $y > 1$ and $y > 9$. That is, $\boxed{y \in (9, \infty)}$.

(iii) For $x = 2$, we have $(2 < y) \wedge (2 + y > 10)$,
which is true for $y > 2$ and $y > 8$. That is, $\boxed{y \in (8, \infty)}$.

The given statement has the existential quantifier $\exists$, so we need this to be true at least one of the values $x \in A = \{0, 1, 2\}$ and the value of y can be anything in the union of all cases from (i)-(iii), i.e.

$$y \in (10, \infty) \cup (9, \infty) \cup (8, \infty) = (8, \infty).$$

That is, the truth set for $P(y)$ is the interval $\boxed{(8, \infty)}$.