H.W. (1)

$y = 10 + \sqrt{x}$ Find $\frac{dy}{dx}$ and approximate Δy when $x = 2$, $\Delta x = 0.1$ and $\Delta x = -0.2$

Compare the actual Δy to find the errors

x	y	$\frac{dy}{dx}$
0	10	0
1	11	0.5
2	11.41	0.35
3	11.73	0.29

Solution

$$\frac{dy}{dx} = \frac{d(10+\sqrt{x})}{dx}$$

$$= \frac{d(10+x^{\frac{1}{2}})}{dx}$$

$$= \frac{1}{2}x^{-\frac{1}{2}}$$

when $x = 0$;

$\frac{dy}{dx} = \frac{1}{2}(0)^{-\frac{1}{2}} = 0$.

when $x = 1$;

$\frac{dy}{dx} = \frac{1}{2}(1)^{-\frac{1}{2}} = \frac{1}{2} = 0.5$

when $x = 2$;

$\frac{dy}{dx} = \frac{1}{2}(2)^{-\frac{1}{2}} = 0.35$

when $x = 3$;

$\frac{dy}{dx} = \frac{1}{2}(3)^{-\frac{1}{2}} = 0.29$.

Given $x = 2$, approximate Δy when $\Delta x = 0.1$

from $\frac{\Delta y}{\Delta x}$, slope.

$\Delta y = 0.35(0.1)$

$\Delta y = 0.035$

Compare to actual Δy,

$y = 10 + \sqrt{x}$ ——— ①

$y = 10 + \sqrt{x+\Delta x}$ ——— ②

② − ①; $\Delta y = 10 + \sqrt{2+0.1} - (10 + \sqrt{2}) = \sqrt{2.1} - \sqrt{2}$

$= 1.45 - 1.41$

$\Delta y = 0.04$

∴ the error is equal to $0.04 - 0.035 = 0.005$.

H.W. (2)

Find 2nd order derivative of $y = 10 + \sqrt{x}$ and plot the graph of y and $\frac{dy}{dx}$

Is the slope of slope is a constant?

$y = 10 + \sqrt{x}$

x	y	$\frac{dy}{dx}$	$\frac{d^2y}{dx^2}$
0	10	0	-
1	11	0.5	-0.25
2	11.41	0.35	-0.09
3	11.73	0.29	-0.05

Solution

$y = 10 + \sqrt{x}$

$$\frac{dy}{dx} = \frac{d(10+\sqrt{x})}{dx}$$

$$= \frac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} = \frac{d(10+\sqrt{x})}{dx}$$

$$= -\frac{1}{4}x^{-\frac{3}{2}}$$

when $x = 0$;

$\frac{d^2y}{dx^2} = -\frac{1}{4}(0)^{-\frac{3}{2}}$ = undefined.

when $x = 1$;

$\frac{d^2y}{dx^2} = -\frac{1}{4}(1)^{-\frac{3}{2}} = -\frac{1}{4} = -0.25$

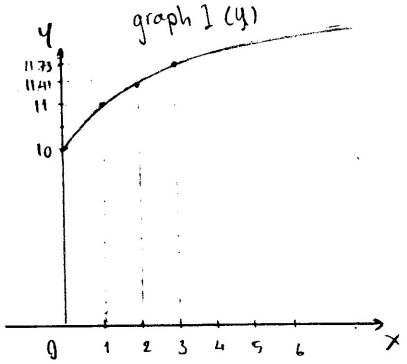
when $x = 2$

$\frac{d^2y}{dx^2} = -\frac{1}{4}(2)^{-\frac{3}{2}} = -\frac{1}{4}\left(\frac{1}{2\sqrt{2}}\right) = -0.088$

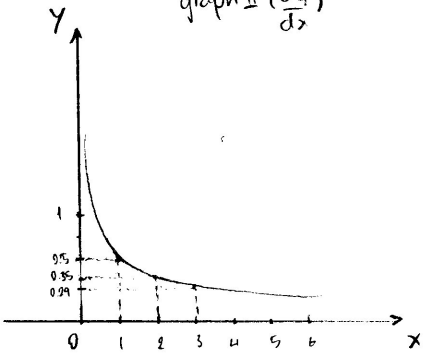
when $x = 3$

$\frac{d^2y}{dx^2} = -\frac{1}{4}(3)^{-\frac{3}{2}} = -\frac{1}{4}\left(\frac{1}{3\sqrt{3}}\right) = -0.048$

graph I (y)



graph II ($\frac{dy}{dx}$)



∴ the slope of slope is not a constant.