Topic 10 Extra

Mixed Strategy Nash Equilibrium

Game 5

Rooney

		L	R
Hart	*l*	$\underline{1}$,-1	-1,$\underline{1}$
	r	-1,$\underline{1}$	$\underline{1}$,-1

Game 5 is a zero sum game: in each strategy profile payoffs sum to zero.

It is a game of pure conflict, called matching pennies.

There does not exist a Nash equilibrium in *pure strategies*, since there is no cell in which both payoffs are underlined.

Suppose now that players can randomize between pure strategies.

Player 1 chooses l with prob p and r with prob $1-p$. Player 2 chooses L with prob q and R with prob $1-q$.

		q	$1-q$		
		L	R		
p	l	$1,-1$	$-1,1$	$\rightarrow$	$2q-1$
$1-p$	r	$-1,1$	$1,-1$	$\rightarrow$	$1-2q$
		$\downarrow$	$\downarrow$		
		$1-2p$	$2p-1$		

Player 2's expected payoff if he chooses L is

$$E(\pi_2(p,L)) = -1\cdot p + 1\cdot(1-p) = 1-2p.$$

The rest of the expected payoffs are as follows:

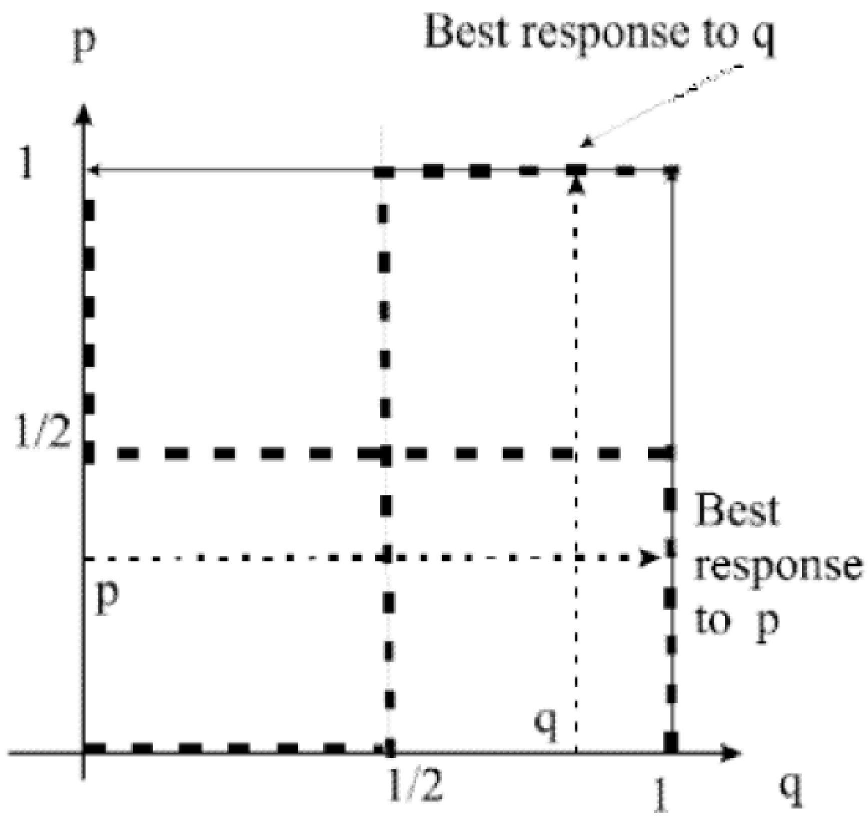
$$\begin{aligned}
E(\pi_2(p,R)) &= 1\cdot p + (-1)\cdot(1-p) = 2p-1 \\
E(\pi_1(l,q)) &= 1\cdot q + (-1)\cdot(1-q) = 2q-1 \\
E(\pi_1(r,q)) &= -1\cdot q + 1\cdot(1-q) = 1-2q.
\end{aligned}$$

Hence, each player best response to the other player's strategy is:

- Player 1's best response: $g_1(q) = \begin{cases} p = 1, & \forall q > 1/2; \\ p = 0, & \forall q < 1/2; \\ p \in [0,1], & \forall q = 1/2. \end{cases}$

- Player 2's best response: $g_2(p) = \begin{cases} q = 0, & \forall p > 1/2; \\ q = 1, & \forall p < 1/2; \\ q \in [0,1], & \forall p = 1/2. \end{cases}$

The graph below shows that there is a unique pair (p^*, q^*) such that $p^* = g_1(q^*)$ and $q^* = g_2(p^*)$; that is, there is a unique mixed strategy NE in Game 5, with $p^* = q^* = 1/2$ and expected payoff $E(\pi_i(p^*, q^*)) = 0$, $i = 1, 2$.


Best response to q
p
1
1/2
p
Best
response
to p
q
1/2
1
q

- If 2 choose $q^* = 0.5$, any $0 \leq p \leq 1$ would yield an expected payoff equal to 0 to player 1 and would thereby maximize 1's expected payoff; hence, $p^* = 0.5$ is a best response to $q^* = 0.5$.
- If 1 chooses $p^* = 0.5$, any $0 \leq q \leq 1$ would yield an expected payoff equal to 0 to player 2 and would thereby maximize 2's expected payoff; hence, $q^* = 0.5$ is a best response to $p^* = 0.5$.
- Therefore, $(p^*, q^*) = (0.5, 0.5)$ constitutes a mixed strategy NE.

More generally, it can be shown that every finite game has a Nash equilibrium, though not necessarily in pure strategies.