Assignment 5

DUE DATE: Thursday 29^{th}, April 2021.

I pledge to the Honor Code and to obey all rules for taking and performing homework assignments as specified by the course instructor.

Full name Naravitch P. **Student ID.** 6104641136

All data are downloadable from BE moodle

There are four questions.

Question1.

Consider the daily log returns of Caterpillar stock (CAT) from January 3, 2006 to April 13, 2017.You may download the data using quantmod. Let r_t be the log returns, which can be obtained via

```
rt <- diff(log(as.numeric(CAT[,6])))
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(a) Are there any serial correlations in the log return series r_t ? Why?

(b) Are there any ARCH effects in the log return series r_t (the linear depedence of squared returns)? Why?

(c) Fit a Gaussian ARMA(1,0)-GARCH(1,1) model to the r_t series. Perform model chekcing, including showing the normal QQ-plot of the standardized residuals. Is the model adequate? Write down the fitted model.

mean eq : $\hat{r}_t = \underset{(3.075e^{-4})}{4.83e^{-4}}(1-\phi_1) + \underset{(0.030)}{0.0169}\, r_{t-1}$

vol. eq : $\sigma_t^2 = \underset{(1.278e^{-6})}{4.478e^{-6}} + \underset{(0.0082)}{0.04972}\, a_{t-1}^2 + \underset{(0.010)}{0.9387}\, \sigma_{t-1}^2$

(d) Build a GARCH(1,1) model with standardized Student-t innovations for the r_t series. Perform model checking, including the QQ-plot. Is the model adequate? Why?

(e) Write down the fitted model.

mean eq : $\hat{r}_t = \underset{(0.0003)}{5.978e^{-4}}$

vol. eq : $\sigma_t^2 = \underset{(1.571e^{-6})}{4.203e^{-6}} + \underset{(0.014)}{0.072}\, a_{t-1}^2 + \underset{(0.015)}{0.926}\, \sigma_{t-1}^2$

(f) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted GARCH(1,1) model with standardized Student-t innovations.

$$\hat{r}_t \pm 1.96(\hat{\sigma}_t) \begin{cases} (-0.02911, 0.0303) \\ (-0.0293, 0.0305) \\ (-0.0294, 0.03063) \\ (-0.0296, 0.0308) \\ (-0.0298, 0.03095) \end{cases}$$

(g) Compute the 95 % 1-step to 5-step interval predictions of the log return series using standardized student-t innovations.

Question2.

Consider the monthly returns of Coke (KO) stock from January 1951 to December 2016. The data are available from CRSP and in the file m-kovw-5116.txt. Obtain the log return series of KO stock.

(a) Is the expected value of KO log return zero? Why? Is there any serial correlation in the log returns? Why? Is there any ARCH effect in the log returns? Why?

(b) Build a AR(1)-GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

mean eq: $\hat{r}_t = \underset{(0.002)}{0.011}(1-\hat{\phi}_1) - \underset{(0.039)}{0.086}\, r_{t-1}$ vol eq: $\hat{\sigma}_t^2 = \underset{(5.856e^{-5})}{0.0002} + \underset{(0.019)}{0.095}\, a_{t-1}^2 + \underset{(0.028)}{0.849}\, \sigma_{t-1}^2$

(c) Fit a AR(1)-GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

mean eq: $\hat{r}_t = \underset{(0.002)}{0.011}(1-\hat{\phi}_1) - \underset{(0.037)}{0.099}\, r_{t-1}$ vol. eq: $\hat{\sigma}_t^2 = \underset{(6.596e^{-5})}{0.0002} + \underset{(0.023)}{0.964}\, a_{t-1}^2 + \underset{(0.033)}{0.850}\, \sigma_{t-1}^2$

(d) Build a GARCH(1,1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

mean eq. $\hat{r}_t = \underset{(0.002)}{0.011}$ vol eq: $\hat{\sigma}_t^2 = \underset{(5.89e^{-5})}{0.0002} + \underset{(0.019)}{0.095}\, a_{t-1}^2 + \underset{(0.0279)}{0.848}\, \sigma_{t-1}^2$

(e) Fit a GARCH(1,1) model with standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

mean eq. $\hat{r}_t = \underset{(0.002)}{0.011}$ vol eq. $\hat{\sigma}_t^2 = \underset{(6.627e^{-5})}{0.0002} + \underset{(0.023)}{0.096}\, a_{t-1}^2 + \underset{(0.033)}{0.850}\, \sigma_{t-1}^2$

(f) Compare the model (b)-(e) which model you select.

Question3.

Consider the daily returns of the stock $S\&P500$ from January 2, 2005 to March 31, 2021. Let r_t be the percentage log returns.

(a) Is the expected value of r_t zero? Why? Are there any serial correlations in r_t? Why?

(b) Fit a Gaussian ARMA-GARCH model to the r_t series. Obtain the normal QQ- plot of the standardized residuals, and write down the fitted model. Is the model adequate? Why?

mean eq: $\hat{r}_t = \underset{(1.164e^{-4})}{6.747e^{-4}}$ vol eq: $\hat{\sigma}_t^2 = \underset{(3.54e^{-7})}{2.763e^{-6}} + \underset{(0.012)}{0.102}\, a_{t-1}^2 + \underset{(0.012)}{0.887}\, \sigma_{t-1}^2$

(c) Build an ARMA-GARCH model with Student-t innovations for the log return series. Perform model checking and write down the fitted model.

mean eq; $\hat{r}_t = 8.434e^{-4}$ $(1.047e^{-4})$

vol eq; $\hat{\sigma}_t^2 = 1.750e^{-6} + 0.149 a_{t-1}^2 + 0.854 \sigma_{t-1}^2$

$(3.696e^{-7})$ (0.015) (0.013)

(d) Obtain 1-step to 5-step ahead mean and volatility forecasts using the fitted ARMA-GARCH model with Student-t innovations.

Question 4.

Consider the monthly returns log returns of the CRSP decile 9 portfolio from January 1951 to December 2010. The simple returns are in the file m-deciles.txt under the name CAP9RET.

(a) Is the expected value of the CRSP decile 9 portfolio log return zero? Why? Is there any serial correlation in the log returns? Why? If necessary, find an ARMA model to remove the serial correlations.

(b) Is there any ARCH effect in the log returns? Why?

(c) Build a AR(1)-ARCH(1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

mean eq. $\hat{r}_t = 0.011$ (0.002)

vol eq; $\hat{\sigma}_t^2 = 0.147 + 0.002 a_{t-1}^2 + 0.182 \sigma_{t-1}^2$

(0.042) (0.0001) (0.065)

(d) Fit a AR(1)-ARCH(1) model with Standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

mean eq; $\hat{r}_t = 0.012(1-\hat{\phi}_1) + 0.108 r_{t-1}$

(0.007) (0.040)

vol eq; $\hat{\sigma}_t^2 = 0.002 + 0.190 a_{t-1}^2$

(0.0001) (0.041)

(e) Build a ARCH(1) model with Gaussian innovations for the log return series. Perform model checking and write down the fitted model.

mean eq: $\hat{r}_t = 0.012$ (0.002)   vol eq ; $\hat{\sigma}_t^2 = 0.002 + 0.194 a_{t-1}^2$ (0.0001) (0.067)

(f) Fit a ARCH(1) model with Standardized Student-t innovations to the log return series. Perform model checking and write down the fitted model.

mean eq ; $\hat{r}_t = 0.013$ (0.008)   vol eq ; $\hat{\sigma}_t^2 = 0.002 + 0.204 a_{t-1}^2$ (0.0002) (0.072)

(g) Compare the model (c)-(f) which model you select.