Methodology of Econometrics

1. Statement of theory or hypothesis
2. Specification of mathematical model of the theory
3. Specification of econometric model of theory
4. Obtaining the data
5. Estimation of the parameters of the econometric model
6. Hypothesis testing
7. Forecasting or prediction
8. Using model for control or policy purposes

Ordinary Least Squares Method

Model $\underset{n\times 1}{y} = \underset{n\times k}{X}\,\underset{k\times 1}{\beta} + \underset{n\times 1}{u}$

$$\begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} 1 & x_{21} & x_{31} & \cdots & x_{k1} \\ 1 & x_{22} & x_{32} & \cdots & x_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_{2n} & x_{3n} & \cdots & x_{kn} \end{bmatrix} \cdot \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_k \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}$$

OLS $\underset{k\times 1}{\hat{\beta}} = (\underset{k\times k}{X'X})^{-1}\,\underset{k\times n}{X'}\,\underset{n\times 1}{y}$

Assume Normal Distribution: $u \sim N(0, \sigma^2 I)$

Assumption of OLS

1. $E(u) = 0$

   where u and 0 are $nx1$ column vectors

2. $E(uu') = \sigma^2 I$

   where I is an nxn identity matrix

3. $E(X'u) = 0$

4. nxk matrix X is nonstochastic

5. Rank of X is $r(X)=k$, k is # columns X & $< n$

6. $u \sim N(0, \sigma^2 I)$

Properties of Least-Squares Estimators

1. Linear
2. Unbiased
3. Efficient estimator

Gauss-Markov Theorem:

Given assumptions of CLRM, the least-squares estimators, in the class of unbiased linear estimators, have minimum variance, that is, they are Best Linear Unbiased Estimators (BLUE).

Evaluation of Estimated Results

1. Sign & meaning of the estimated coefficient
2. Overall Test – ANOVA F-test
3. Coefficient of Determination or R^2
4. Individual Test – t-test
5. Durbin-Watson Test / Heteroscedasticity Test

Variance-Covariance Matrix

Under OLS Assumptions

$$\sum_{n\times n} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \cdots & \sigma_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \cdots & \sigma_{\text{n}}^2 \end{bmatrix} = \begin{bmatrix} \sigma^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma^2 \end{bmatrix} = \sigma^2 I$$

Under Heteroskedasticity Problem

$$\sum_{n\times n} = \begin{bmatrix} \sigma_1^2 & 0 & 0 & \cdots & 0 \\ 0 & \sigma_2^2 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \sigma_n^2 \end{bmatrix}$$

Variance-Covariance Matrix

Under Autocorrelation Problem

Variance-Covariance Matrix $\underset{n\times n}{\sum} = \sigma^2 \underset{n\times n}{\Omega}$

where

$$\underset{\text{n}\times\text{n}}{\Omega} = \begin{bmatrix} 1 & \rho & \rho^2 & \rho^3 & \cdots & \rho^{n-1} \\ \rho & 1 & \rho & \rho^2 & \cdots & \rho^{n-2} \\ \rho^2 & \rho & 1 & \rho & \cdots & \rho^{n-3} \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho^{n-1} & \rho^{n-2} & \rho^{n-3} & \cdots & \rho & 1 \end{bmatrix}$$

Matrix

$$\underset{k\times 1}{\hat{\beta}} = (\underset{k\times n}{X'} \underset{\substack{n\times n \\ k\times k}}{\hat{\Omega}^{-1}} \underset{n\times k}{X})^{-1} \underset{k\times n}{X'} \underset{\substack{n\times n \\ k\times 1}}{\hat{\Omega}^{-1}} \underset{n\times 1}{Y}$$

OLS vs GLS vs FGLS

OLS Estimation

$$\underset{n\times n}{\Sigma} = \sigma^2 \underset{n\times n}{I}$$

$$\underset{k\times 1}{\hat{\beta}} = (\underset{k\times k}{X'X})^{-1} \underset{k\times n}{X'} \underset{n\times 1}{y}$$

WLS Estimation

$$\underset{n\times n}{\hat{\Sigma}} = \hat{\sigma}_i^2 \underset{n\times n}{I}$$

$$\underset{k\times 1}{\hat{\beta}} = (\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}^{-1}} \underset{n\times k}{X}}_{k\times k})^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}^{-1}} \underset{n\times 1}{y}}_{k\times 1}$$

Cochrane-Orcutt

$$\underset{n\times n}{\hat{\Sigma}} = \sigma^2 \underset{n\times n}{\hat{\Omega}}$$

$$\underset{k\times 1}{\hat{\beta}} = (\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Omega}^{-1}} \underset{n\times k}{X}}_{k\times k})^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Omega}^{-1}} \underset{n\times 1}{y}}_{k\times 1}$$

GLS Estimation

$$\underset{n\times n}{\Sigma} \text{ is known}$$

$$\underset{k\times 1}{\hat{\beta}} = (\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\Sigma^{-1}} \underset{n\times k}{X}}_{k\times k})^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\Sigma^{-1}} \underset{n\times 1}{y}}_{k\times 1}$$

FGLS Estimation

$$\underset{n\times n}{\Sigma} \text{ is notknown}$$

$$\underset{k\times 1}{\hat{\beta}} = (\underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}^{-1}} \underset{n\times k}{X}}_{k\times k})^{-1} \underbrace{\underset{k\times n}{X'} \underset{n\times n}{\hat{\Sigma}^{-1}} \underset{n\times 1}{y}}_{k\times 1}$$

Estimation Methods

<u>Nonparametric Estimation Methods</u>

- No assumption of distribution
- i.e. Linear Programming

<u>Parametric Estimation Methods</u>

- Assume distribution

Linear Estimation Method

- Least Squares Estimation Methods (LS)

Nonlinear Estimation Methods

- Maximum Likelihood Estimation Methods (MLE)
- General Method of Moment (GMM)

Least Squares Estimation Methods

Generalized Least Squares (GLS)

Feasible Generalized Least Squares (FGLS)

- Weighted Least Squares (WLS)
  Heteroscedasticity
- Cochrane-Orcutt Technique
  Autocorrelation

Ordinary Least Squares (OLS)

Other Least Squares Methods

- Nonlinear Least Squares (NLS)
- System Equation Estimation Methods

Least Squares Estimation Methods

System Equation Models

- Seemingly Unrelated Model
- Simultaneous Equation Model
  - Limited Information Estimation Methods (Single Equation Estimation Methods)
    - ILS, 2SLS, & (LIML)
  - Full Information Estimation Methods (System Equation Estimation Methods)
    - 3SLS, I3SLS, & (FIML)
- Vector Autoregressive Model (VARs)