Topic 4 Part 1

Risk and Information (Chapter 15)

Tools for Describing Risky Outcomes

Definition: A **lottery** is any event with an uncertain outcome.

Examples: Investment, Roulette, Football Game.

Definition: A **probability** of an outcome *(of a lottery)* is the likelihood that this outcome occurs.

Example: The probability often is estimated by the historical frequency of the outcome.

Tools for Describing Risky Outcomes

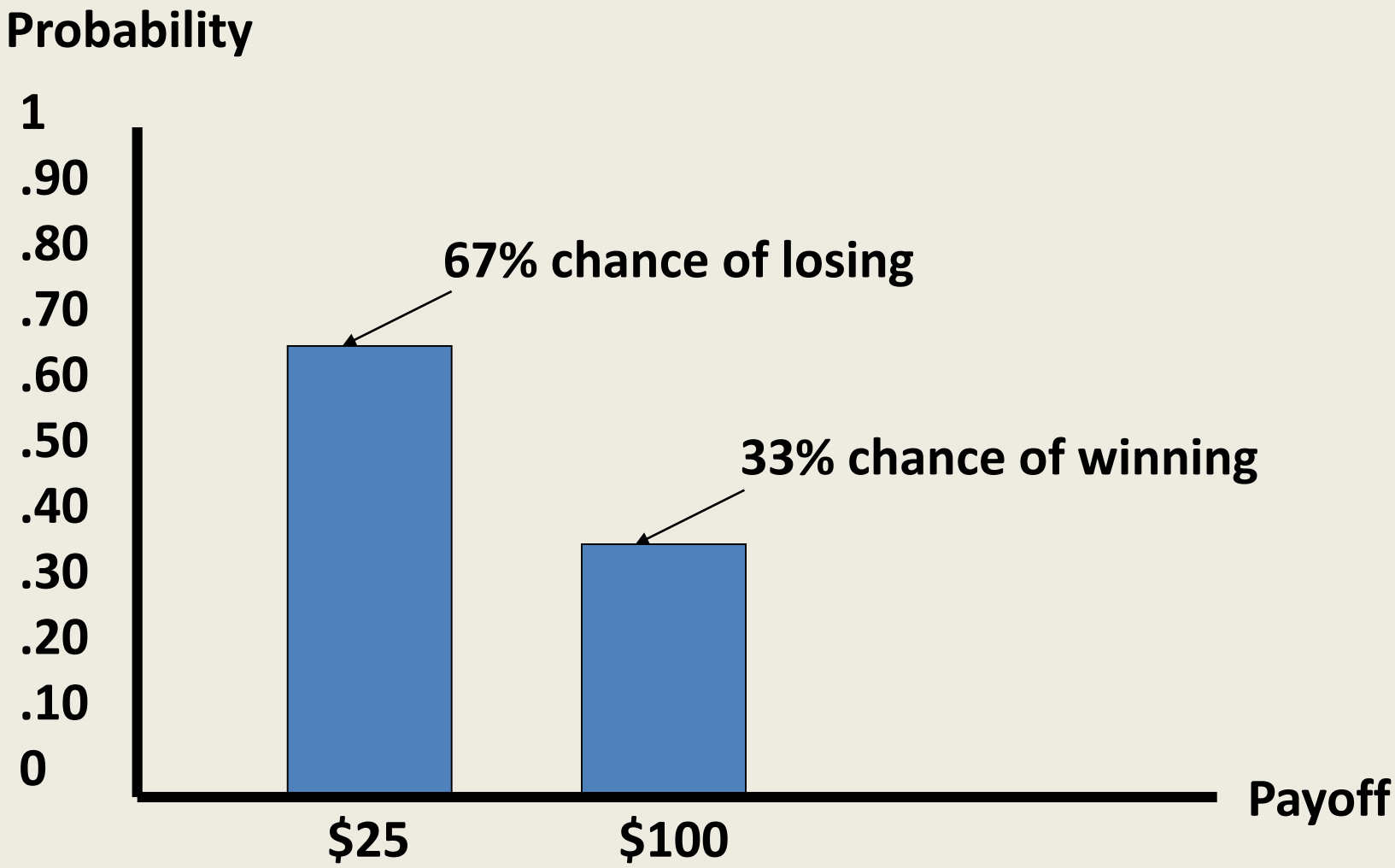
Definition: The **probability distribution** of the lottery depicts all possible payoffs in the lottery and their associated probabilities.

Property:

- The probability of any particular outcome is between 0 and 1.
- The sum of the probabilities of all possible outcomes equals 1.

Definition: Probabilities that reflect subjective beliefs about risky events are called **subjective probabilities**.

Probability Distribution



Expected Value and Expected Utility

Definition: The **expected value** of a lottery is a measure of the average payoff that the lottery will generate.

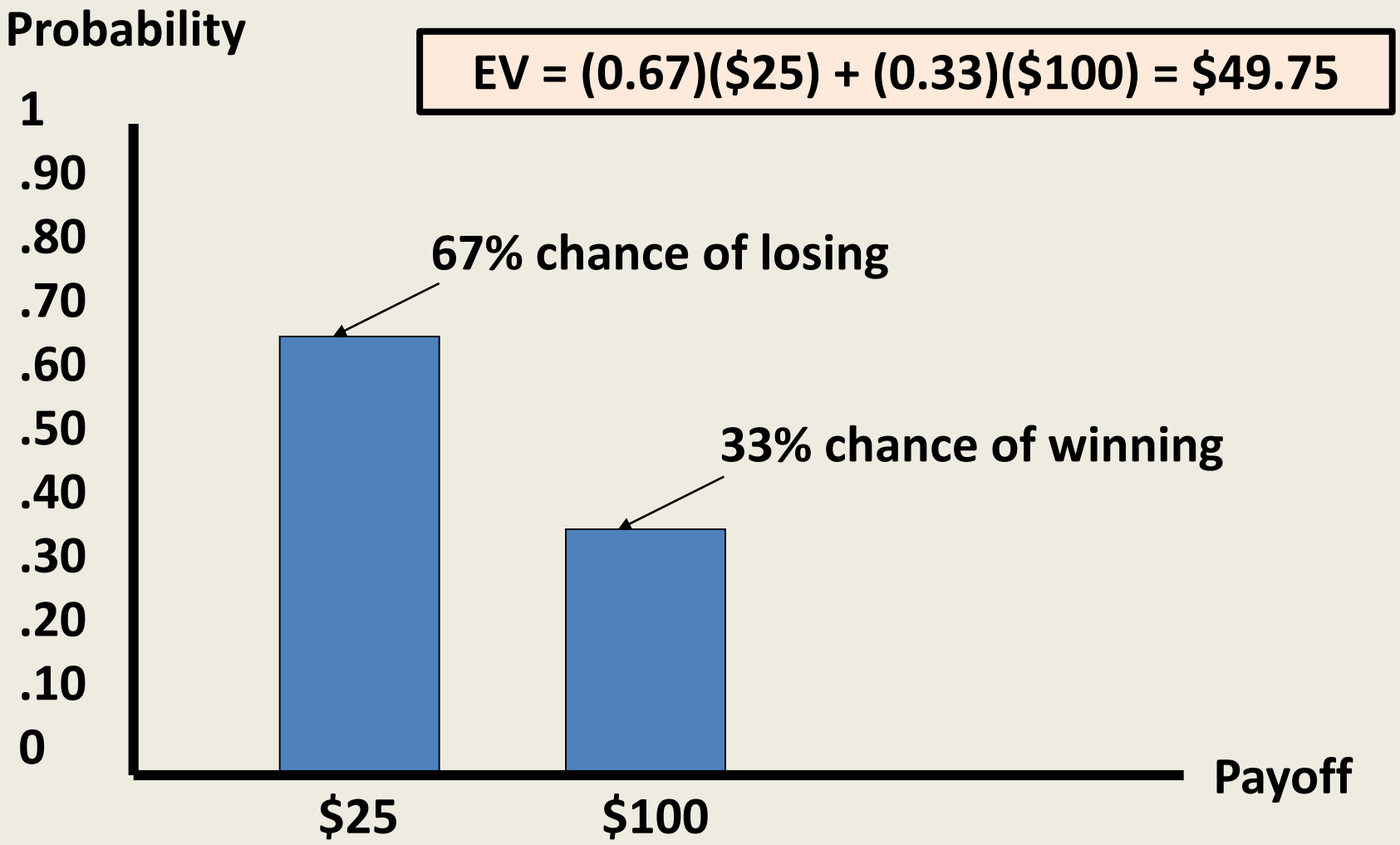
Let X be a lottery with three possible outcomes A, B, and C.

$$\mathbf{E[X] = Pr(A)\cdot A + Pr(B)\cdot B + Pr(C)\cdot C}$$

Definition: The **expected utility** of a lottery is the average value of the utility levels that the lottery will generate.

$$\mathbf{E[U(X)] = Pr(A)\cdot U(A) + Pr(B)\cdot U(B) + Pr(C)\cdot U(C)}$$

Expected Value



Variance & Standard Deviation

Definition: The **variance** of a lottery is a measure of the lottery's riskiness.

$$\text{Var}[X] = E[X^2] - (E[X])^2$$

$$\text{Var}[X] = (x - E[X])^2 \cdot \Pr(X = x)$$

Definition: The **standard deviation** of a lottery is the square root of the variance. It is an alternative measure of risk.

Example

X = x	X = 0	X = 1	X = 2	X = 3
Pr(X = x)	0.1	0.2	0.4	0.3

E[X] = 0(0.1) + 1(0.2) + 2(0.4) + 3(0.3) = 1.9

Var[X] = (x – E[X])² · Pr(X = x)

$= (0.1)(0–1.9)^2 + (0.2)(1–1.9)^2 + (0.4)(2–1.9)^2 + (0.3)(3–1.9)$

$= 0.89$

Var[X] = E[X²] – (E[X])²

$E[X^2] = (0^2)(0.1) + (1^2)(0.2) + (2^2)(0.4) + (3^2)(0.3) = 4.5$

$Var[X] = 4.5 – 1.9^2 = 0.89$

Risk Preferences

Notes:
- Utility as a function of income only

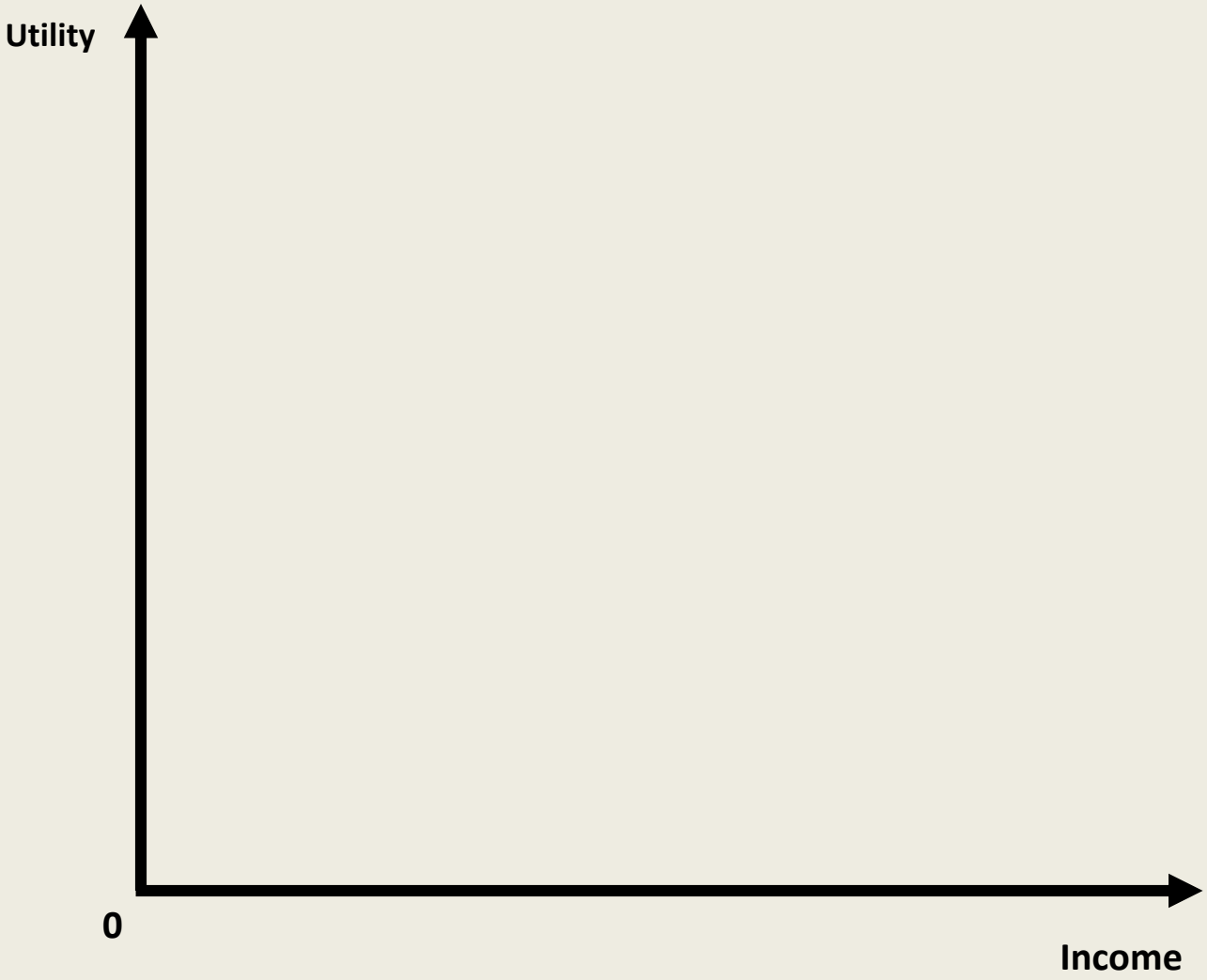
Definition: The risk preferences can be classified as follows:

An individual who prefers a sure thing to a lottery with the same expected value is **risk-averse.**

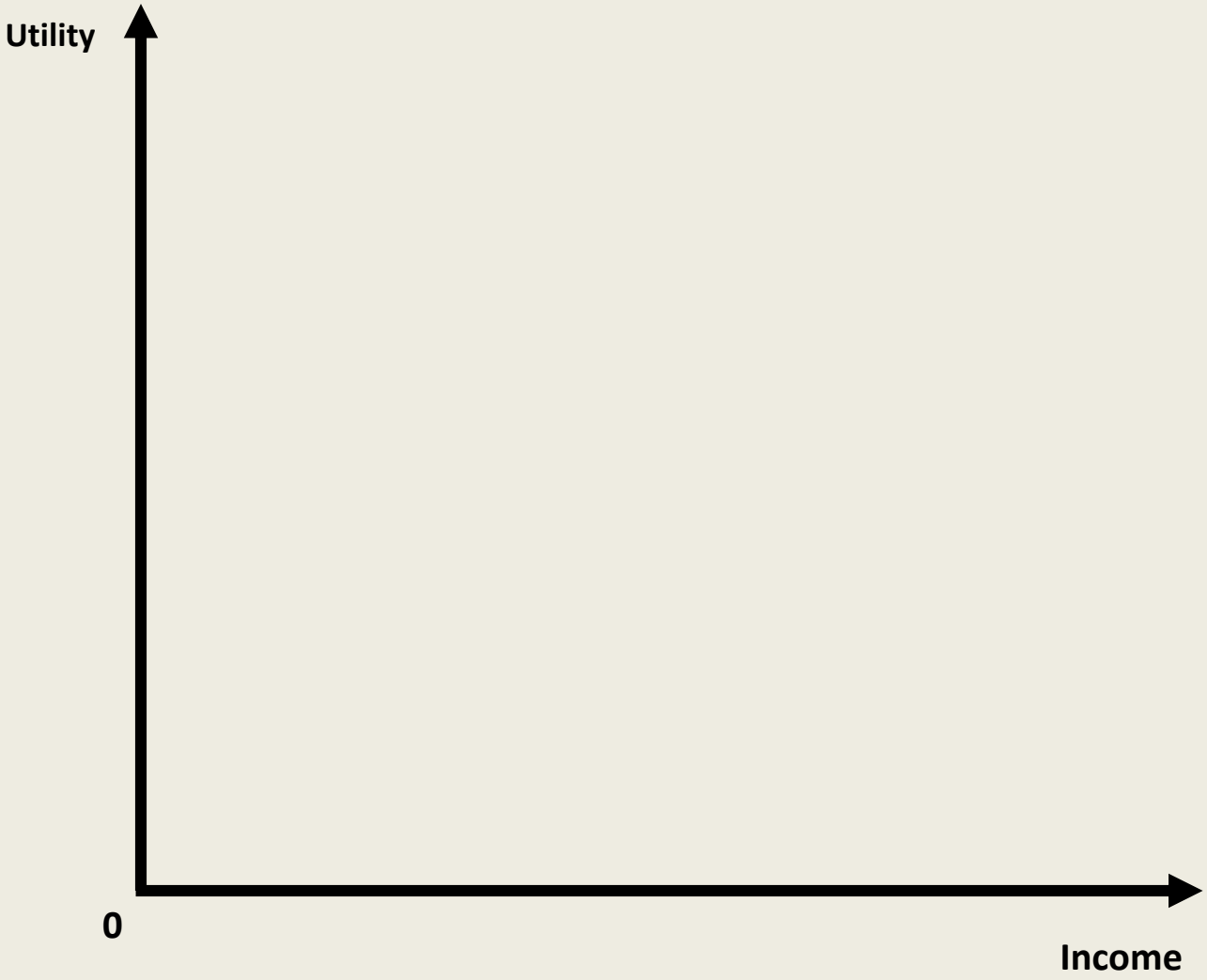
An individual who is indifferent about a sure thing or a lottery with the same expected value is **risk-neutral.**

An individual who prefers a lottery to a sure thing that equals the expected value of the lottery is **risk-loving.**

Risk-Averse Individuals



Risk-Neutral Individuals



Risk-Loving Individuals