Extra Practice problem sets

Question 1

A firm's production function is given by

$$Q = [0.64K^{0.5} + 0.36L^{0.5}]^2$$

1.1. Determine the marginal rate of technical substitution (MRTS) of *L* for *K*.

1.2. Determine the values of the marginal products of *K* and *L*, when K = 256 and L = 324.

1.3. Suppose now that the capital input (*K*) and the labor input (*L*) change with time (*t*) according to the functions:

$$K(t) = 232 + 10t - t^2;$$

$$L(t) = 292 + 12t - t^2.$$

Use total derivatives to find the rate of change of the output (*Q*) with respect to time (*t*) at the time *t = 4*.

Question 2

A business conglomerate produces three different types of products, namely A, B and C. Each product has the market demand equation given by:

Product A: $P_A = 36 - Q_A^2$;

Product B: $P_B = 35 - 2Q_B^2 + Q_C$;

Product C: $P_C = 20 + Q_B - Q_C$;

Suppose that the cost function for producing the three types of products is given by:

$$TC(Q_A, Q_B, Q_C) = 8 + 2Q_A^3 - 2Q_B^3 + 3Q_BQ_c + 30Q_B + 12Q_C + \frac{1}{2}Q_B^2,$$

and the conglomerate is the sole supplier in each of the three markets.

Consider the following problems

2.1. Write down the profit function of this conglomerate firm.

2.2. Using the first-derivative test, solve for the profit-maximizing level of output for each of the three products.

2.3. Confirm your answer in (2.2) with the second-order derivative test.

2.4. Determine the level of maximized profit.

Question 3

Joe's utility function depends on consumption and leisure, and it is given by:

$$U(c, l) = 8\sqrt{c} + l,$$

where c is consumption l is leisure. Each day Joe works n hours and spends the rest of the time on leisure; that is, $l + n = 24$. Suppose further that Joe's total income is the sum of a fixed non-wage income (Y_0) and a wage income (Y_n). The wage income (Y_n) is equal to wn, where w is the hourly wage rate. If Joe spends all of his income on the consumption, his budget constraint becomes:

$$p_c c = Y_0 + wn = Y_0 + w(24 - l),$$

which can also be written as:

$$p_c c + wl = Y_0 + 24w.$$

3.1. Use Lagrange method to derive the optimal values of c^* and l^* in terms of the parameters p_c, w, and Y_0. State the condition under which the consumer demands some leisure (i.e., $l^* > 0$).

3.2. Based on your answer in (3.1), derive the impact of an increase in the hourly wage (w) on the demand for leisure (l^*). Interpret its economic meaning.

3.3. Suppose now that the hourly wage rate (w) is \$24, the price of consumption (p_c) is \$12, and the fixed non-wage income (Y_0) is 384. Use Lagrange method to determine the levels of c^* and l^* that maximizes Joe's utility subject to the budget constraint, and determine the level of constrained maximum utility.

3.4. Verify your answers in (3.3) by using the second-order sufficient condition.

Question 4

A monopoly is facing a non-linear inverse market demand given by $P = \frac{100}{\sqrt{Q}}$ where P is the price per unit of output, and Q is the quantity of output. Assume further that the cost function of this monopolist is given by $C = 20Q + 10$.

4.1. Calculate the level of profit maximizing output and price under non-discriminatory pricing regime, and compute the consumer surplus and producer surplus.

4.2. Calculate the deadweight loss from the monopoly. [Hint: *You need to compute the optimal quantity in the perfectly competitive market*.]

4.3. Suppose a $10 subsidy is given to the monopolist. Calculate the change in social welfare, and discuss your result.