FN 451

Self-Practice 1: Return, risks, and distributions

1. Contrast the use of the arithmetic mean return to the geometric mean return of an investment from the perspective of an investor concerned with the investment's terminal value.

Contrast the use of the arithmetic mean return to the geometric mean return of an investment from the perspective of an investor concerned with the investment's average one-year return.

The following table repeats the annual total returns on the MSCI Germany Index previously given and also gives the annual total returns on the JP Morgan Germany five- to seven-year government bond index (JPM 5–7 Year GBI, for short). During the period given in the table, the International Monetary Fund Germany Money Market Index (IMF Germany MMI, for short) had a mean annual total return of 4.33 percent. Use that information and the information in the table to answer Problems 2 and 3.

Year	MSCI Germany Index	JPM Germany 5–7 Year GBI
1993	46.21%	15.74%
1994	−6.18%	−3.40%
1995	8.04%	18.30%
1996	22.87%	8.35%
1997	45.90%	6.65%
1998	20.32%	12.45%
1999	41.20%	−2.19%
2000	−9.53%	7.44%
2001	−17.75%	5.55%
2002	−43.06%	10.27%

Source: Ibbotson EnCorr Analyzer.

2. Calculate the annual returns and the mean annual return on a portfolio 60 percent invested in the MSCI Germany Index and 40 percent invested in the JPM Germany GBI.

3. A. Using the IMF Germany MMI as a proxy for the risk-free return, calculate the Sharpe ratio for

i. the 60/40 equity/bond portfolio described in Problem 2.

ii. the MSCI Germany Index.

iii. the JPM Germany 5–7 Year GBI.

B. Contrast the risk-adjusted performance of the 60/40 equity/bond portfolio, the MSCI Germany Index, and the JPM Germany 5–7 Year GBI, as measured by the Sharpe ratio.

4. You are forecasting sales for a company in the fourth quarter of its fiscal year. Your low-end estimate of sales is € 14 million, and your high-end estimate is €15 million. You decide to treat all outcomes for sales between these two values as equally likely, using a continuous uniform distribution.

A. What is the expected value of sales for the fourth quarter?

B. What is the probability that fourth-quarter sales will be less than or equal to € 14, 125, 000?

5. State the approximate probability that a normal random variable will fall within the following intervals:

A. Mean plus or minus one standard deviation

B. Mean plus or minus two standard deviations

C. Mean plus or minus three standard deviations

6. You are evaluating a diversified equity portfolio. The portfolio's mean monthly return is 0.56 percent, and its standard deviation of monthly returns is 8.86 percent.

A. Calculate a one standard deviation confidence interval for the return on this portfolio.

Interpret this interval, with a normality assumption for returns.

B. Calculate an exact 95 percent confidence interval for portfolio return, assuming portfolio returns are described by a normal distribution.

C. Calculate an exact 99 percent confidence interval for portfolio return, assuming portfolio returns are described by a normal distribution.

7. Find the area under the normal curve up to $z = 0.36$; that is, find $P(Z \le 0.36)$. Interpret this value.

8. Economic globalization is defined as the integration of national economies into the international economy through trade, foreign direct investment, capital flows, migration, and the spread of technology. Although globalization is generally viewed favorably, it also increases the vulnerability of a country to economic conditions of the other country. An economist predicts a 60% chance that country A will perform poorly and a 25% chance that country B will perform poorly. There is also a 16% chance that both countries will perform poorly.

a. What is the probability that country A performs poorly given that country B performs poorly?

b. What is the probability that country B performs poorly given that country A performs poorly?

c. Interpret your findings.

9. A stockbroker knows from past experience that the probability that a client owns stocks is 0.60 and the probability that a client owns bonds is 0.50. The probability that the client owns bonds if he/she already owns stocks is 0.55.

a. What is the probability that the client owns both of these securities?

b. Given that the client owns bonds, what is the probability that the client owns stocks?

10. (This one is more challenging and involves thinking about Bayes' theorem). Yaowaluck is a security analyst for a telecommunications firm called iTalk. Although she is optimistic about the firm's future, she is concerned that its stock price will be considerably affected by the condition of credit flow in the economy. She believes that the probability is 0.20 that credit flow will improve significantly, 0.50 that it will improve only marginally, and 0.30 that it will not improve at all. She also estimates that the probability that the stock price of iTalk will go up is 0.90 with significant improvement in credit flow in the economy, 0.40 with marginal improvement in credit flow in the economy, and 0.10 with no improvement in credit flow in the economy.

a. Based on Yaowaluck estimates, what is the probability that the stock price of iTalk goes up?

b. If we know that the stock price of iTalk has gone up, what is the probability that credit flow in the economy has improved significantly?

11. Following the United Kingdom (UK) referendum in which people voted to leave the European Union (EU), economists have made these predictions.

Table 1

Pre-Brexit	Probability
EU GDP growth is negative	0.4
UK GDP growth is negative	0.5
Both UK and EU GDP growth is negative	0.1

Table 2

Post-Brexit	Probability
EU GDP growth is negative	0.35
UK GDP growth is negative	0.6
Both UK and EU GDP growth is negative	0.1

Compute the probability that UK GDP growth is negative pre-Brexit and post-Brexit, given EU GDP growth is negative pre-Brexit and post-Brexit, respectively. Analyze and discuss results.

Solutions

1. The geometric mean return is more meaningful than the arithmetic mean return for an investor concerned with the terminal value of an investment when they have a buy-and-hold strategy. The geometric mean return is the compound rate of growth, so it directly relates to the terminal value of an investment. By contrast, a higher arithmetic mean return does not necessarily imply a higher terminal value for an investment.

The arithmetic mean return is more meaningful than the geometric mean return for an investor concerned with the average one-period performance of an investment (rebalancing investment). The arithmetic mean return is a direct representation of the average one-period return. In contrast, the geometric mean return, as a compound rate of growth, aims to summarize what a return series means for the growth rate of an investment over many periods.

Note that GM and AM are equal when each period return is constant. There is little difference in GM and AM when period returns have small variations. This is why when inter-period return is small such as daily or intra-daily return is considered, AM and GM are close approximates.

2.

The following table shows the calculation of the portfolio's annual returns, and the mean annual return.

Year	Weighted Mean Calculation	Portfolio Return
1993	$0.60(46.21) + 0.40(15.74) =$	34.02%
1994	$0.60(-6.18) + 0.40(-3.40) =$	−5.07%
1995	$0.60(8.04) + 0.40(18.30) =$	12.14%
1996	$0.60(22.87) + 0.40(8.35) =$	17.06%
1997	$0.60(45.90) + 0.40(6.65) =$	30.20%
1998	$0.60(20.32) + 0.40(12.45) =$	17.17%
1999	$0.60(41.20) + 0.40(-2.19) =$	23.84%
2000	$0.60(-9.53) + 0.40(7.44) =$	−2.74%
2001	$0.60(-17.75) + 0.40(5.55) =$	−8.43%
2002	$0.60(-43.06) + 0.40(10.27) =$	−21.73%
	Sum =	96.46%
	Mean Annual Return =	9.65%

Note: The sum of the portfolio returns carried without rounding is 96.48.

3.

A. i. For the 60/40 equity/bond portfolio, we earlier computed a mean return and standard deviation of return of 9.65 percent and 18.31, respectively. The statement of the problem gave the mean annual return on the proxy for the risk-free rate, the IMF Germany MMI, as 4.33 percent. We compute the Sharpe ratio as

$$S_h = \frac{\overline{R}_p - \overline{R}_F}{s_p} = \frac{9.65 - 4.33}{18.31} = 0.29$$

ii. For the MSCI Germany Index,

$$S_h = \frac{\overline{R}_p - \overline{R}_F}{s_p} = \frac{10.80 - 4.33}{29.95} = 0.22$$

iii. For the JPM Germany 5–7 Year GBI,

$$S_h = \frac{\overline{R}_p - \overline{R}_F}{s_p} = \frac{7.92 - 4.33}{6.94} = 0.52$$

The Sharpe ratio measures excess return per unit of risk as measured by standard deviation. Because we are comparing positive Sharpe ratios, a larger Sharpe ratio reflects better risk-adjusted performance. During the period, the JPM Germany GBI had the best risk-adjusted performance and the MSCI Germany Index had the worst risk-adjusted performance, as measured by the Sharpe ratio. The 60/40 quity/bond portfolio was intermediate in risk-adjusted performance.

4. A. The expected value of fourth-quarter sales is € 14, 500, 000, calculated as (€ 14,000,000 + € 15,000,000)/2. With a continuous uniform random variable, the mean or expected value is the midpoint between the smallest and largest values. (See Example 5-7.)

B. The probability that fourth-quarter sales will be less than € 14, 125, 000 is 0.125 or 12.5 percent, calculated as (€ 14, 125, 000 –€ 14, 000, 000)/(€ 15, 000, 000 – € 14, 000, 000).

5.

A. Approximately 68 percent of all outcomes of a normal random variable fall within plus or minus one standard deviation of the mean.

B. Approximately 95 percent of all outcomes of a normal random variable fall within plus or minus two standard deviations of the mean.

C. Approximately 99 percent of all outcomes of a normal random variable fall within plus or minus three standard deviations of the mean.

6. A. The lower limit of a one standard deviation confidence interval is the sample mean return (0.56 percent) minus the sample standard deviation (8.86 percent):

0.56% − 8.86% = −8.30%. The upper limit is the sample mean return (0.56 percent) plus the sample standard deviation (8.86 percent): 0.56% + 8.86% = 9.42%. Summarizing, the one standard deviation confidence interval runs from −8.30 percent to 9.42 percent, written as [−8.30%, 9.42%]. If the portfolio return is normally distributed, approximately 68 percent (precisely 68.27 percent) of monthly returns will fall in this interval.

B. The lower limit of a 95 percent confidence interval is the sample mean return minus 1.96 standard deviations: 0.56% − 1.96 × 8.86% = −16.81%. The upper limit is the sample mean return plus 1.96 standard deviations: 0.56% + 1.96 × 8.86% = 17.93%. Summarizing, an exact 95 percent confidence interval runs from −16.81 percent to 17.93 percent, written as [−16.81%, 17.93%]. Only 5 percent of a large number of returns should fall outside of this interval, under the normality assumption. In that sense, we have 95 percent confidence in this interval.

C. The lower limit of a 99 percent confidence interval is the sample mean return minus 2.58 standard deviations: 0.56% − 2.58 × 8.86% = −22.30%. The upper limit is the sample mean return plus 2.58 standard deviations: 0.56% + 2.58 × 8.86% = 23.42%. Summarizing, an exact 99 percent confidence interval runs from −22.30 percent to 23.42 percent, written as [−22.30%, 23.42%]. Only 1 percent of a large number of returns should fall outside of this interval, under the normality assumption.

In that sense, we have 99 percent confidence in this interval.

7. The area under the normal curve for z = 0.36 is 0.6406 or 64.06 percent. The table below presents an excerpt from the tables of the standard normal cumulative distribution. To locate z = 0.36, find 0.30 in the fourth row of numbers, then look at the column for 0.06 (the second decimal place of 0.36). The entry is 0.6406.

$P(Z \leq x) = N(x)$ for $x \geq 0$ or $P(Z \leq z) = N(z)$ for $z \geq 0$

x or z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.00	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.10	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.20	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.30	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	**0.6406**	0.6443	0.6480	0.6517
0.40	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.50	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224

The interpretation of 64.06 percent for $z = 0.36$ is that 64.06 percent of observations on a standard normal random variable are smaller than or equal to the value 0.36. (So 100% − 64.06% = 35.94% of the values are greater than 0.36.)

8.

SOLUTION: We first write down the available information in probability terms. Defining A as "country A performing poorly" and B as "country B performing poorly," we have the following information: $P(A) = 0.60$, $P(B) = 0.25$, and $P(A \cap B) = 0.16$.

a. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.16}{0.25} = 0.64.$

b. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.16}{0.60} = 0.27.$

c. It appears that globalization has definitely made these countries vulnerable to the economic woes of the other country. The probability that country A performs poorly increases from 60% to 64% when country B has performed poorly. Similarly, the probability that country B performs poorly increases from 25% to 27% when conditioned on country A performing poorly.

9.

a. Let A correspond to the event that a client owns stocks and B correspond to the event that a client owns bonds. Thus, the unconditional probabilities that the client owns stocks and that the client owns bonds are $P(A) = 0.60$ and $P(B) = 0.50$, respectively. The conditional probability that the client owns bonds given that he/she owns stocks is $P(B|A) = 0.55$. We calculate the probability that the client owns both of these securities as $P(A \cap B) = P(B|A)P(A) = 0.55 \times 0.60 = 0.33$.

b. We need to calculate the conditional probability that the client owns stocks given that he/she owns bonds, or $P(A|B)$. Using the formula for conditional probability and the answer from part a, we find $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.33}{0.50} = 0.66$.

10. As always, we first define the relevant events and their associated probabilities. Let *S* , *M* , and *N* denote significant, marginal, and no improvement in credit flow, respectively. Then $P(S) = 0.20$, $P(M) = 0.50$, and $P(N) = 0.30$. In addition, if we allow *G* to denote an increase in stock price, we formulate $P(G \mid S) = 0.90$, $P(G \mid M) = 0.40$, and $P(G \mid N) = 0.10$. We need to calculate $P(G)$ in part a and $P(S \mid G)$ in part b.

Prior Probability	Conditional Probability	Joint Probability	Posterior Probability
$P(S) = 0.20$	$P(G\|S) = 0.90$	$P(G \cap S) = 0.18$	$P(S\|G) = 0.4390$
$P(M) = 0.50$	$P(G\|M) = 0.40$	$P(G \cap M) = 0.20$	$P(M\|G) = 0.4878$
$P(N) = 0.30$	$P(G\|N) = 0.10$	$P(G \cap N) = 0.03$	$P(N\|G) = 0.0732$
$P(S) + P(M) + P(N) = 1$		$P(G) = 0.41$	$P(S\|G) + P(M\|G) + P(N\|G) = 1$

a. In order to calculate $P(G)$, we use the total probability rule, $P(G) = P(G \cap S) + P(G \cap M) + P(G \cap N)$. The joint probabilities are calculated as a product of conditional probabilities with their corresponding prior probabilities.

$$P(G \cap S) = P(G|S)P(S) = 0.90 \times 0.20 = 0.18.$$

Therefore, the probability that the stock price of iTalk goes up equals $P(G) = 0.18 + 0.20 + 0.03 = 0.41$.

b. According to Bayes' theorem, $P(S|G) = \frac{P(G \cap S)}{P(G)} = \frac{P(G \cap S)}{P(G \cap S) + P(G \cap M) + P(G \cap N)}$.
We use the total probability rule in the denominator to find $P(G) = 0.18 + 0.20 + 0.03 = 0.41$. Therefore, $P(S|G) = \frac{P(G \cap S)}{P(G)} = \frac{0.18}{0.41} = 0.4390$. Note that the prior probability of a significant improvement in credit flow is revised upward from 0.20 to a posterior probability of 0.4390.

11. Pre-Brexit: P(UK-|EU-) = P(UK-,EU-)/P(EU-) = 0.1/0.4 = 0.25

Post-Brexit : P(UK-|EU-) = 0.1/0.3 = 0.33

It appears post-Brexit, the likelihood of UK experiencing negative GDP has grown from 0.5 to 0.6 whereas EU's likelihood of negative GDP has dropped slightly. Perhaps UK is experiencing some economic backlash from the withdrawal from EU from less favorable trade terms to loss of financial sector jobs. Post-Brexit, notice that P(EU-|UK-) has gone down from 0.1/0.5 to 0.1/0.6 suggesting lower UK's influence on EU's negative GDP. Of course, if numbers are available we should cross-check if this analysis holds true in event of positive GDP as well.