A&D Ch.3

Behavioral: Prospect Theory

Prospect Theory

- Prospect Theory is a core theory in the field of behavioral finance. This is an alternative to the Expected Utility Theory.
- Prospect Theory is developed by Daniel Kahneman and Amos Tversky, in their study in 1979 titled ““Prospect Theory: An Analysis of Decision under Risk”, published in Econometrica No. 47 (2).
- The theory begins with the contention that standard Expected Utility Theory cannot full account for observed decision-making under risk. This contention is based on empirical observations.
- We will proceed by looking at the related empirical observations, and see related components of the Prospect Theory.

Prospect

- Recall that we use prospect to represent a risky situation under consideration.
- For simplicity, assume that there are only *two* states of the world: *low* wealth and *high* wealth.
- A **prospect** is a series of wealth outcomes, each of which is associated with a probability.
- Let $P(p_1, w_1, p_2, w_2)$ represent a prospect that outcome w_i occurs with probability p_i, $i = 1,2$.
- Let $P(w)$ represent a certain outcome w (with probability 1).

Problem 1

Imagine that you face the following pair of concurrent decisions. First examine both decisions, and then indicate the options you prefer.

- Decision 1: Choose between $P1(\$240)$ and $P2(0.25, \$1000, 0.75, \$0)$.
- Decision 2: Choose between $P3(-\$750)$ and $P4(0.75, -\$1000, 0.25, \$0)$.

Note on Problem 1

- From an experiment,
  - For decision 1, 84% of the respondents chose $P1$ ➔ Risk aversion
  - For decision 2, 87% of the respondents chose $P4$ ➔ Risk seeking

- **Key aspect 1**: People sometimes exhibit risk aversion and sometimes exhibit risk seeking, depending on the nature of the prospect.

Problem 2

- Decision 1: Assume yourself richer by $\$300$ than you are today. Then choose between $P5(\$100)$ and $P6(0.5, \$200, 0.5, \$0)$.

- Decision 2: Assume yourself richer by $\$500$ than you are today. Then choose between $P7(-\$100)$ and $P8(0.5, -\$200, 0.5, \$0)$.

Note on Problem 2

- From an experiment,
  - For decision 1, 72% chose $P5$ ➔ Risk aversion
  - For decision 2, 64% chose $P8$ ➔ Risk seeking

- **Key aspect 2**: People's valuations of prospects depend on gains and losses relative to a reference point. This reference point is usually the status quo.

Problem 3

What value of x would make you indifferent between $P9(\$0)$ and $P10(0.5, \$x, 0.5, -\$25)$.

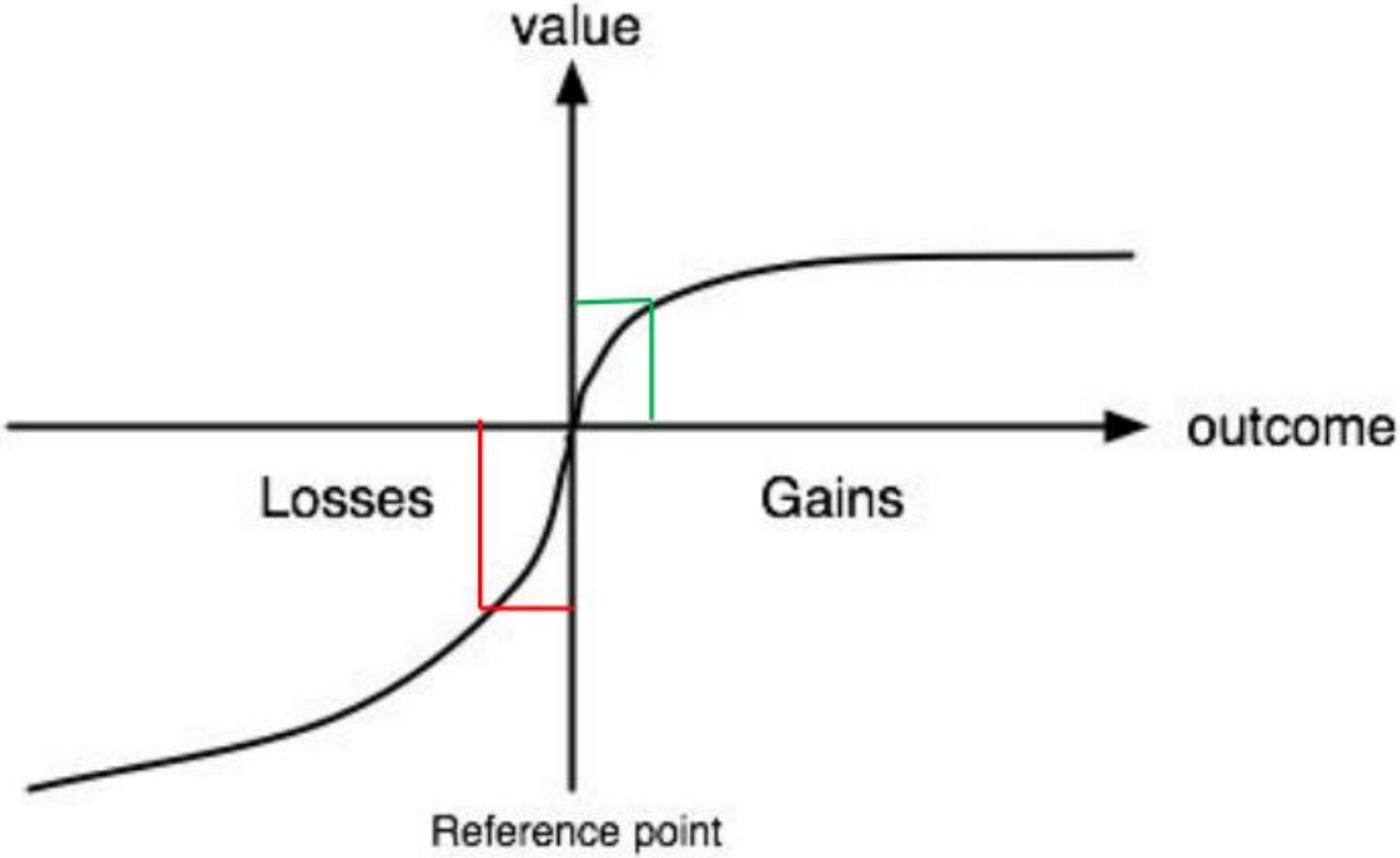
Note on Problem 3

- From an experiment, the average response was $x = \$61$.

- **Key aspect 3**: People are averse to losses because losses loom larger than gains.

Value Function

- The value function in Prospect Theory replaces the (Bernoulli) utility function in Expected Utility Theory.
- While utility is usually measured in terms of the level of wealth, value is defined by gains and losses relative to a reference point.
- The value function is concave in the positive domain and convex in the negative domain.
- People dislike losses, so the value function is steeper for losses than for gains.


value
outcome
Losses
Gains
Reference point

Value Function

- Let
  - $V(P)$ denote value of prospect P
  - $v(z_i)$ denote value of a wealth change equal to z_i
  - $\pi(p_i)$ denote decision weight associated with probability p_i determined by the weighting function (more details later).
- For prospect $P(p_1, z_1, p_2, z_2)$, value is

$$V(p_1, z_1, p_2, z_2) = V(P) = \pi(p_1) \cdot v(z_1) + \pi(p_2) \cdot v(z_2)$$

Hypothetical Value Function

- Kahneman and Tversky proposed a functional form of value function as

$$v(z) = \begin{cases} z^{\alpha} & , \text{if } z \geq 0 \\ -\lambda(-z)^{\beta}, & \text{if } z < 0 \end{cases}$$

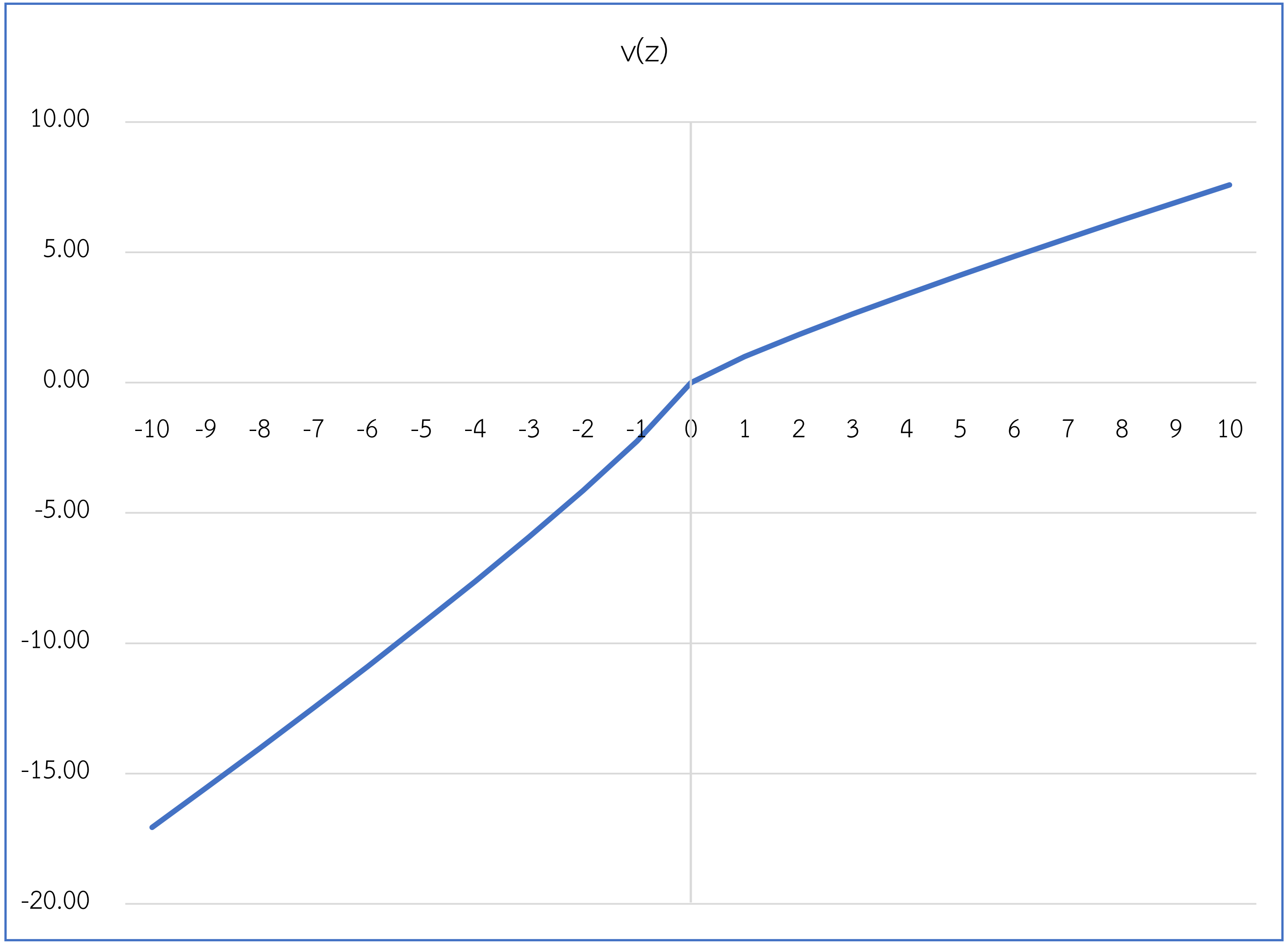
  where $0 < \alpha < 1, 0 < \beta < 1,$ and $\lambda > 1$.

- Kahneman and Tversky estimated α and β each to be approximately 0.88, and λ to be approximately 2.25.

- The way to view this function is that it is a *typical* decision-maker's value function. Some people will have lower/higher values for the relevant parameters.

The Typical Value Function

z	v(z)
-10	-17.07
-9	-15.56
-8	-14.02
-7	-12.47
-6	-10.89
-5	-9.27
-4	-7.62
-3	-5.92
-2	-4.14
-1	-2.25
0	0.00
1	1.00
2	1.84
3	2.63
4	3.39
5	4.12
6	4.84
7	5.54
8	6.23
9	6.91
10	7.59



Weighting Function

- We start this section by asking: Why do people who buy lottery tickets also purchase insurance?
- With Expected Utility Theory, this is puzzling because with a lottery a person is seeking risk. With insurance, the same person may pay to reduce risk, exhibiting risk aversion.
- Prospect Theory can account for the observation that some people buy lottery tickets and insurance at the same time. It does so by overweighting low-probability events via the use of weighting function.

Problem 4:

- Choose between $P11(0.001, \$5000, 0.999, \$0)$ and $P12(\$5)$.

Problem 5:

- Choose between $P13(0.001, -\$5000, 0.999, \$0)$ and $P14(-\$5)$.

Note on Problem 4 and Problem 5

- In an experiment, most of the respondents prefer $P11$ to $P12$ in problem 4, while choose $P14$ over $P13$ in problem 5
  - Choosing $P11$ in problem 4 ➔ risk seeking in the domain of gains
  - Choosing $P14$ in problem 5 ➔ risk aversion in the domain of losses.
- In sum, while we normally have risk aversion in the domain of gains, when there is a quite low probability of a payoff, this generally shifts to risk seeking.
- **Fourfold pattern of risk attitudes** ➔ People exhibit:
  - Risk aversion in the domain of gains and risk seeking in the domain of losses, when the outcome probability is high
  - Risk seeking in the domain of gains and risk aversion in the domain of losses, when the outcome probability is low

Weighting Function

- Tversky, A. and D. Kahneman (1992). Advances in prospect theory: cumulative representation of uncertainty. ***Journal of Risk and Uncertainty*** 5, pp. 297-323.

Problem 6:

- Decision 1: Choose between $P15(0.8, \$4000, 0.2, \$0)$ and $P16(\$3000)$
- Decision 2: Choose between $P17(0.2, \$4000, 0.8, \$0)$ and $P18(0.25, \$3000, 0.75, \$0)$

Weighting Function

- Tversky and Kahneman (1992) found that 80% of respondents chose $P16$ and 65% chose $P17$.
- Common Ratio Effect: Multiplying a common ratio to the probabilities of a prospect may switch the decision of people.
- Tversky and Kahneman argue that the reason for this is that people put a lot higher value to what is certain in comparison with what is merely probable. This implies that the slope of the weighting function in the neighborhood of certainty is relatively steep (i.e. has a slope greater than one).

Problem 7

- Decision 1: Choose between $P19(0.45, \$6000, 0.55, \$0)$ and $P20(0.9, \$3000, 0.1, \$0)$.
- Decision 2: Choose between $P21(0.001, \$6000, 0.999, \$0)$ and $P22(0.002, \$3000, 0.998, \$0)$.

Weighting Function

- Tversky and Kahneman (1992) reported that 86% of respondents chose $P20$ (risk aversion), while 73% chose $P21$ (risk seeking).

- While earlier we saw that people overweight low probabilities, it must be that the overweighting is greatest at the lowest probabilities, which implies that the weighting function is relatively steep in the neighborhood of zero.
- Let $\pi(p_i)$ represent the weighting function associated with a change in wealth z_i.
- The weighting function must be steep (slope is more than one) in the neighborhood of $p_i = 0$ and $p_i = 1$.
- Setting $\pi(0) = 0$ and $\pi(1) = 1$, it must be that for intermediate probabilities the slope of the weighting function is relatively flat (less than one).

Hypothetical Weighting Function

- Kahneman and Tversky propose a functional form of the weighting function as

$$\pi(p_1, p_2) = \begin{cases} \dfrac{{p_1}^{\gamma}}{({p_1}^{\gamma} + {p_2}^{\gamma})^{1/\gamma}}, & if\ z \geq 0 \\ \dfrac{{p_1}^{\chi}}{({p_1}^{\chi} + {p_2}^{\chi})^{1/\chi}}, & if\ z < 0 \end{cases}$$

  where $\gamma = 0.61$ and $\chi = 0.69$.

- However, for convenience let's assume that $\gamma = \chi = 0.65$.

The Weighting Function

p1	π(p)
0.00	0.00
0.10	0.18
0.20	0.26
0.30	0.32
0.40	0.38
0.50	0.44
0.60	0.50
0.70	0.56
0.80	0.64
0.90	0.75
1.00	1.00