Past exam questions –Matrix and Integration

1. Determine this integral $I = \int_{-2}^{4} \int_{0}^{3x} e^{x^2}\, dy\, dx$. Give you answer to **2 decimal places**.

2. Determine this integral $I = \int_{-1}^{1} \int_{0}^{x} \int_{0}^{xy} (6z + 3y)\, dz\, dy\, dx$.

3. A company sells red and yellow T-shirts and has determined that the profit function for selling x red T-shirts and y yellow T-shirts is given by

$$P(x, y) = 10{,}000 + 2{,}100x - 3x^2 + 3(y - 400)^2$$

Find **the average profit** if the company sells between 200 and 400 red T-shirts and 300 and 400 yellow T-shirts.

Q7:

Determine this integral $I = \int_{1}^{4} \int_{1}^{2} \left(\frac{x}{y} + \frac{y}{x} \right) dydx$.

Q9:

Determine this integral $I = \int_{0}^{1} \int_{0}^{1} \int_{\sqrt{x^2+y^2}}^{2} xyz\, dzdydx$.

Q10:

Determine this integral $I = \int_{0}^{1} \int_{0}^{z} \int_{0}^{y} z e^{-y^2}\, dxdydz$.

a) Use **<u>cofactor expansion method together with row operations</u>** to determine $|\mathbf{A}|$ and $|\mathbf{A}^{-1}|$

b) Let $\mathbf{B}=\begin{bmatrix} 0 & a & 2 \\ 0 & 0 & 9 \\ a & 1 & 2 \end{bmatrix}$; $a>0$, if $\det \mathbf{B} = 81$, find the value of a

c) Determine $\det \mathbf{C}$ if $(\det \mathbf{B})^{\frac{1}{2}} \det(\mathbf{A}^2 \mathbf{A}^T) = \dfrac{\det(\mathbf{C}^3)}{\det(\mathbf{A}^{-1})}$

d) Using **Cramer's rule** to solve for $\underline{\mathbf{Bx}} = \begin{bmatrix} -3 \\ 0 \\ 2 \end{bmatrix}$

3. For a system of linear equation below where C and D are real constants

$$\begin{aligned} x_1 + 2x_2 + x_3 + 5x_5 + 2x_6 &= 1 + x_4 \\ 2x_1 + 5x_2 + 2x_3 + x_6 &= 2 - 3x_4 - 6x_5 \\ x_1 + 3x_2 + Cx_3 + 4x_4 + x_5 &= 5 - Dx_6 \end{aligned}$$

a) Write down the matrix equation ($\underline{\mathbf{Ax}} = \underline{\mathbf{b}}$) and the augmented matrix that represents the above system of linear equations.

b) What are values of C and D that make this system <u>inconsistent</u>?

c) If $C = 1$, what are all possible values of D that make this system consistent?

d) If $D = 0$, choose a value of C that makes this system consistent. Also solve the given linear system with your specified value of C. **Write down the solution in vector form.**

e) If $C = 2$ and $D = 0$, is it possible for the linear system to have a unique solution? Give the reason to your answer.

3. A health spa customizes the diet and vitamin supplements of each of its clients. The spa offers three different vitamin supplements (X,Y and Z) each containing different percentages of the recommended daily allowance (RDA) of vitamins A, C and D. The percentages of RDA of vitamins A, C and D provided by each tablet of supplements X, Y and Z are summarized in the table below

		Supplement		
		X	Y	Z
Vitamins	A	10%	20%	30%
	C	10%	30%	50%
	D	10%	50%	60%

The spa staff determines that one client should take 150% of RDA of vitamin A, 180% of the RDA of vitamin C and 210% of the RDA of vitamin D each day.

a) Write down the matrix equation ($\underline{\mathbf{A}}\underline{\mathbf{x}} = \underline{\mathbf{b}}$) representing the problem. Suppose this client should take x tablets of supplement X , y tablets of supplement Y and z tablets of supplement Z

b) Use **Gauss-Jordan** method to obtain the inverse of the matrix $\underline{\mathbf{A}}$ in part a).

c) Determine how many tablets of each supplement a client should take each day in order to obtain the recommended percentages of each vitamin. (hints: solve for *x, y* and *z*)

Ans. $A^{-1} = \begin{bmatrix} 7/30 & -0.1 & -1/30 \\ 1/30 & -0.1 & 2/30 \\ -2/30 & 0.1 & -1/30 \end{bmatrix}$, x = 10, y = 1, z = 1

4. Given

$$\underline{\mathbf{A}} = \begin{bmatrix} x^2 & y^2 & z^2 & 1 \\ 3 & 9 & 8 & 1 \\ 3 & 4 & 5 & 1 \\ 3 & 7 & 5 & 1 \end{bmatrix}, \ \underline{\mathbf{B}} = \begin{bmatrix} 0 & 0 & 1 & -13 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 4 & 0 & 0 & 3 \end{bmatrix} \text{ and } \underline{\mathbf{C}} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{2} & 0 & 0 \\ \frac{1}{6} & 0 & -4 & 0 \\ -17 & \frac{1}{8} & -7 & -\frac{1}{4} \end{bmatrix}$$

a) If $y = 4$ and $z = 13$, determine all possible values of x such that $\det \underline{\mathbf{A}} = \det(\underline{\mathbf{B}}\underline{\mathbf{C}}^2)$.

b) If $x = 3$, $y = 1$ and $z = 2$, determine $\det \underline{\mathbf{A}}$, $\det(-2\underline{\mathbf{A}}^T)$ and $\det \underline{\mathbf{D}}$, provided that $\underline{\mathbf{B}} = 9\underline{\mathbf{A}}\underline{\mathbf{D}}^3\underline{\mathbf{C}}^{-1}$.

c) If $x = 3$, $y = 1$ and $z = 2$, use Cramer's rule to solve $\underline{\mathbf{A}}\underline{\mathbf{x}} = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 0 \end{bmatrix}$.