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1.1) $E[a_t]$

$$E[a_t] = E[E[a_t | F_{t-1}]]$$

$$= E[E[\sigma_t \varepsilon_t | F_{t-1}]] \Rightarrow \text{by law of iterated expectation}$$

$$= E[\sigma_t E[\overset{0}{\cancel{\varepsilon_t}} | F_{t-1}]] \Rightarrow \text{as } \sigma_t^2 \text{ is ARCH (1)}$$

$$E[a_t] = 0$$

$$E[r_t] = E[\mu_t] + E[a_t]$$

$$= \mu_t + 0$$

$$= \mu_t \text{ or constant.}$$

$$= 0.002 \quad \#$$

1.2) $$Var(a_t) = E[(a_t - \overset{0}{\cancel{E(a_t)}})^2]$$

$$= E[a_t^2]$$

$$= E[E[a_t^2 | F_{t-1}]]$$

$$= E[\sigma_t^2] = E[\alpha_0 + \alpha_1 + a_{t-1}^2] = \sigma_t^2$$

$$\text{var} \Rightarrow E[a_t^2] \cdot \frac{\alpha_0}{1-\alpha_1} \quad \text{by property of weak stationarity}$$

$$= \frac{0.01}{1-0.1} = 0.0111 \quad \#$$

$$Var(a_t | F_{t-1}) = E[(a_t - E[a_t | F_{t-1}])^2 | F_{t-1}]$$

$$= E[(a_t)^2 | F_{t-1}]$$

$$= E[\sigma_t^2 \varepsilon_t^2 | F_{t-1}]$$

$$= \sigma_t^2 E[\overset{1}{\cancel{\varepsilon_t^2}} | F_{t-1}]$$

$$= \sigma_t^2 = 0.01 + 0.1a_{t-1}^2$$

13) $h = 100$, $a_h = 0.015$ $\sigma_h = 0.2$

$r_h = \mu_h + a_t$

$r_{h+1} = \mu_{h+1} + a_{h+1}$

$E(r_{h+1}) : r_h(1) = \mu_{101} + E[a_{101}]$

$= \mu_{101} + E(\sigma_{101} \cdot \varepsilon_{101})$

assume
$\mu_{101} = \mu_{100} = 0.002$

$= \mu_{101} + \sigma_{101} E(\varepsilon_{101})$

$= 0.002 + 0$

$\boxed{= 0.002}$

$\sigma_h^2 = \alpha_0 + \alpha_1 a_{h-1}^2$

$\sigma_h^2 = 0.01 + 0.1 a_{h-1}^2$

1-step ahead

$\hookrightarrow : E[\sigma_{101}^2 | F_{100}] = 0.01 + 0.1 E[a_{100}^2 | F_h]$

$\sigma_h^2(1) = 0.01 + 0.1\, a_{100}^2$

$= 0.01 + 0.1 (0.015)^2$

$\sigma_h^2(1) = 0.0100225$ #

$\sigma_h(1) = \sqrt{\sigma_h^2(1)} = \boxed{0.1001}$

1.4) as $\lim_{l \to \infty} r_h(l)$ will goes to mean or unconditional mean which is 0 #

(converge.)

as $\lim_{l \to \infty} \sigma_h^2(l)$ will goes to unconditional variance which is $E[a_t^2] = \dfrac{\alpha_0}{1-\alpha_1}$

$= 0.0111$ #