

Market Risk:

FROM VALUE-AT-RISK TO
STRESS TESTING

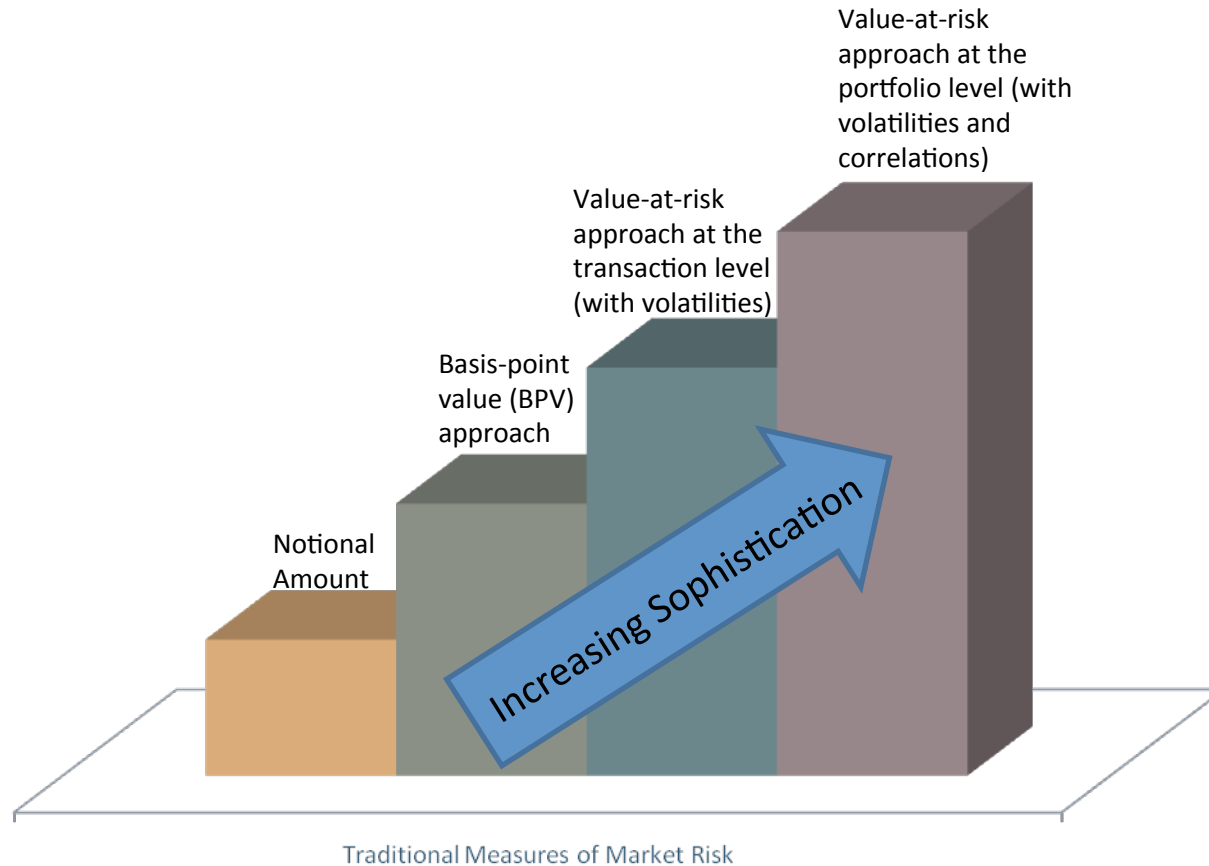
Agenda

- The Notional Amount Approach
- Price Sensitivity Measure for Derivatives
 - Weakness of the Greek Measure
- Define Value-at-Risk
 - 1-Day to VaR to 10-Day VaR
- How is VaR used to limit risk in practice?
- How do we generate distributions for calculating VaR?
 - Selection of the Risk Factors
 - Choice of a Methodology for Modeling Changes in Market Risk Factor
 - Variance-Covariance Approach
 - Historical Simulation Approach
 - Monte Carlo Approach

Agenda (Cont.)

- Stress Testing and Scenario Analysis
 - Risk-Factor Stress Testing
 - Stress-Testing Envelopes
 - Advantages of Stress Testing and Scenario Analysis
 - Limitations of Stress Testing and Scenario Analysis
- Summary of Key Risk – VaR and Stress Testing

Traditional Measures of Market Risk



The Notional Amount Approach

- Notional Amount is the nominal or face amount that is used to calculate payments made on the instruments

Price Sensitivity Measure for Derivatives

- Price sensitivities : The Greeks

Delta (Price) Gamma (Convexity)

Vega (Volatility) Theta (Time Decay)

Rho (Discounted rate risk)

Delta	Gamma	Theta	Vega	Rho
Measure impact of a change in price of underlying asset	Measure impact of a change in Delta	Measure impact of a change in time remaining	Measure impact of a change in volatility	Measure impact of a change in interest rate

Weakness of the Greek Measures

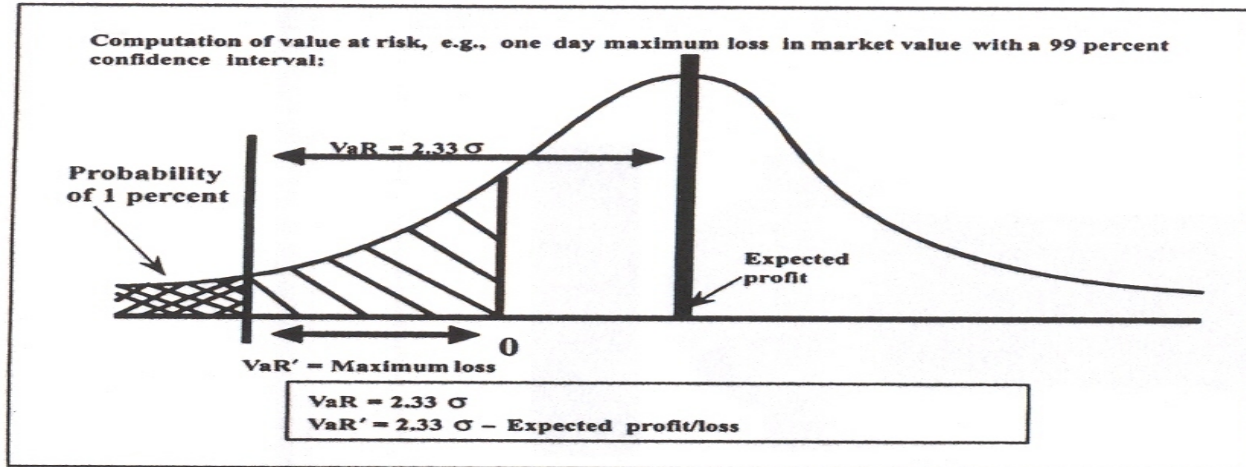
- Cannot be added up across risk types
- Cannot be added up across markets
- Cannot be used directly to measure amount of capital which banks is putting at risk
- Not facilitate financial risk control – not represent maximum dollar loss acceptable for the position

Defining Value at Risk

- Value at Risk (VaR) is worst-loss that might be expected from holding portfolio or securities over a given period of time and given the specified level of probability (confidence level)
- Example : If the portfolio has a daily VaR of 10 million with a 99 percent confidence level

Defining Value at Risk

Defining Value at Risk



- $VaR = \text{expected profit/loss} - \text{worst case loss at given confidence level and a period of time}$
- VaR' (Absolute VaR) is the maximum value of the portfolio that firm can stand with given probability of the loss

Defining Value at Risk

- Example : Given probability confidence level of 99 percent. Average daily revenue C\$0.451, only 1 percent of revenue that might less than C\$0.451 is the revenue at 25.919

Defining Value at Risk

- Step in calculation VaR
 - Derive the distribution
 - Select the percentile of the distribution in order to read the number of loss

From 1-Day VaR to 10-Day VaR



- Example : If we need 10-Day VaR, we can get it from

Strength and Wide Ranges of Uses

- VaR provides a common, consistent, and integrated measure of risk across risk factor, instrument, and asset classes
- VaR can provides an aggregate measure of risk and risk-adjusted performance
- Business-line risk limits can be set in term of VaR

Strength and Wide Ranges of Uses

- VaR provides senior management, the board of directors, and regulators with a risk measure that they can understand
- A VaR system allows a firm to assess the benefits from portfolio diversification within a line of activity and across businesses
- VaR has become an industry-standard internal and external reporting tool

How is VaR used to limit risk in practice?

- VaR is
 - Aggregate measure of risk across of all risk factor
 - Can be calculated at each level of activity in the business hierarchy of a firm
 - Good way of representing “risk appetite” of firm

Generating distributions for VaR

- Two processes of generate distributions for calculation VaR
 - Selection of the Risk Factors
 - Choice of a Methodology for Modeling Changes in Market Risk Factor

Selection of the Risk factors

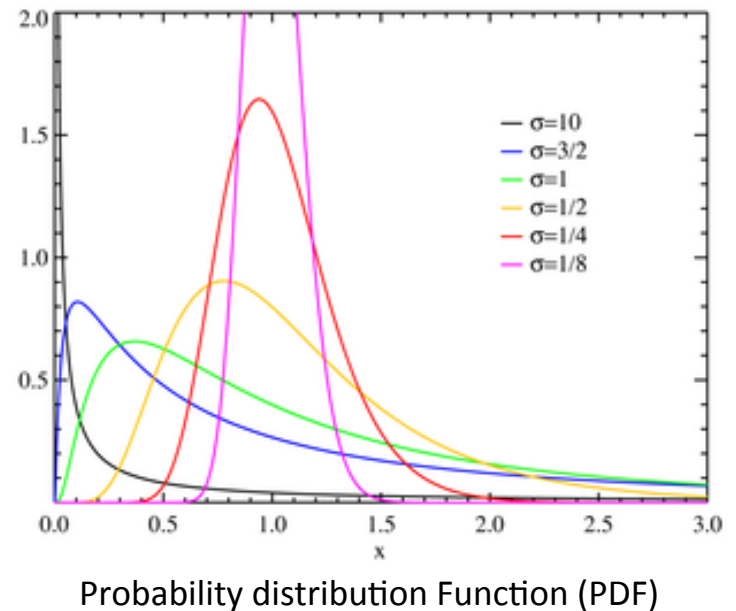
- Changes in value of portfolio is driven by changes in the influenced market factors
- Risk factor of simple security is straightforward, while more complex securities require judgment
- Example : US\$/Euro forward value is affected only by US \$/Euro forward rate, while US\$/Euro call option value depends on US\$/Euro exchange rate, US\$ interest rates, Euro interest rates and US\$/Euro volatility
- Stock Portfolio risk factor are price of individual stocks. Bond portfolio risk factor depends on degree of granularity.

Variance-Covariance Approach

- Fastest method , quick estimates of VaR though relies heavily on assumptions.
- **Assumptions**
 1. Delta normal and thus Log-normally distributed Risk factors and portfolio value
 2. Multivariate distribution of the underlying market factors
 3. Expected change in the portfolio's market value is assumed to be zero.

Log-normally Distributed

- A random variable is lognormally distributed if the logarithm of the random variable is normally distributed.
- If x is a random variable with normal distribution then $\text{Exp}(x)$ has log-normally distributed.
- If $\text{Log}(y)$ is lognormally distributed. Then y has normal distribution.



Variance-Covariance Approach

- Some simple example
- Suppose that you invested \$100,000 in MARK today and the daily standard deviation of MARK is 2%. Then, the one-day at 99% confidence level VaR of your position in MARK is given by:
 - $\text{VaR} = \text{value of the position in MARK} * 2.33 * \sigma \text{ MARK}$
 - $\text{VaR} = \$100,000 * 2.33 * 2\% = \$ 4,660 ,$
- That is under normal conditions with 99% confidence level, you expect not to lose more than \$ 4,660 by holding MARK until tomorrow (daily horizon.)
- From this, it should be clear that the computation of the standard deviation of changes in portfolio value is to be focused

Risk Mapping

- Just a nice Jargon, don't be afraid.
- Meaning : taking the actual instruments and “mapping” them into a set of simpler, standardized position or instruments.

Variance-Covariance Approach

- **Step 1 :**

- Identify basic market factors, say, that generate returns in portfolio and,
- Map each single market factor with each standardized position.

Variance-Covariance Approach

● Step 2 :

- Assume % change in basic market factors to be multivariate normal distribution,
- then estimate parameters, i.e. standard deviation and correlation coefficient.
- This is the point at which variance-covariance procedure captures the variability and comovement of the market factors.

Variance-Covariance Approach

- **Step 3 :**
- Use such market factors' parameters to determine standard deviation and correlations of changes in the value of standardized position
- Note that we can apply the risk-factor analysis here to the present portfolio because we assume Multivariate normal distribution.

Variance-Covariance Approach

- **Step 4 :**
- Now we have standard deviations(σ) of and correlations (ρ) between changes in the values of the standardized positions, we can calculate portfolio variance and standard deviation using this formula :

Variance-Covariance Approach

● case of two assets :

4. One-day and 10-day VaR calculation for the stock portfolio

The return on the portfolio, as defined in (14), being a linear combination of normal distributions, is also normally distributed:

$$R_V \sim N(\mu_V, \sigma_V)$$

with

$$\begin{aligned}\mu_V &= \sum_{i=1}^2 \omega_i \mu_i \\ \sigma_V^2 &= \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2\omega_1 \omega_2 \text{cov}(R_1, R_2) \\ &= \omega_1^2 \sigma_1^2 + \omega_2^2 \sigma_2^2 + 2\omega_1 \omega_2 \rho \sigma_1 \sigma_2 \\ &= (\omega_1 \ \omega_2) \begin{pmatrix} \sigma_1^2 & \rho \sigma_1 \sigma_2 \\ \rho \sigma_1 \sigma_2 & \sigma_2^2 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \\ &= (\omega_1 \ \omega_2) \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{pmatrix} \begin{pmatrix} \omega_1 \\ \omega_2 \end{pmatrix} \\ &= w \Omega w^T = w \sigma C \sigma w^T\end{aligned}\tag{16}$$

where $w = (\omega_1 \ \omega_2)$ denotes the 1×2 weight vector, Ω is the variance-covariance matrix, σ is the 2×2 diagonal standard deviation matrix, C is the 2×2 correlation matrix and w^T denotes the trans-

Variance-Covariance Approach

- Back to the calculation, now we get the standard deviation (σ)
- Suppose that you invested \$100,000 in MARK today and the daily standard deviation of MARK is 2%. Then, the one-day at 99% confidence level VaR of your position in MARK is given by:
 - $\text{VaR} = \text{value of the position in MARK} * 2.33 * \sigma \text{ MARK}$
 - $\text{VaR} = \$100,000 * 2.33 * 2\% = \$ 4,660 ,$
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Variance-Covariance Approach

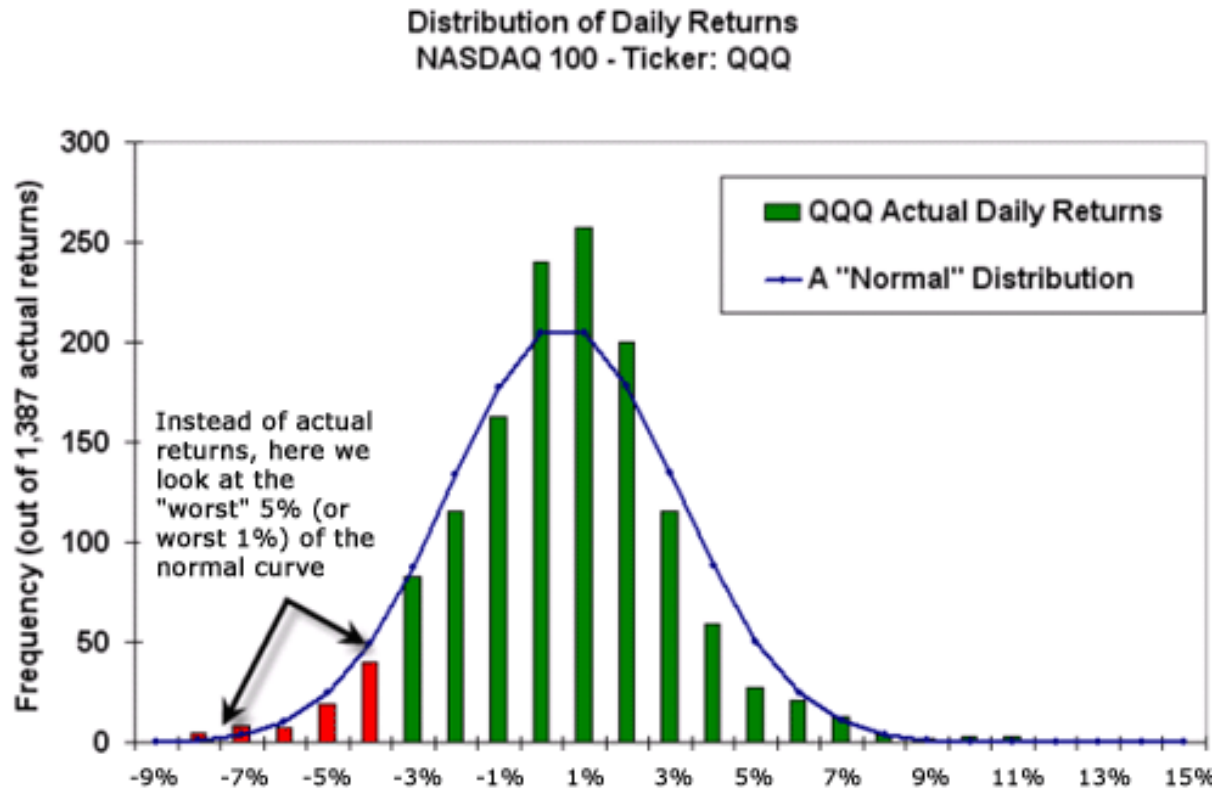
- **Look at some further details**
- Correlation Risk factor is important
- Perfectly correlated : VaR will be the sum of VaRs of each individual asset
- Mostly they are not strongly correlated; see example here

Variance-Covariance Approach

- An example : Microsoft(1) and Exxon(2) stocks
- One-day VaRs at 99% confidence level are:

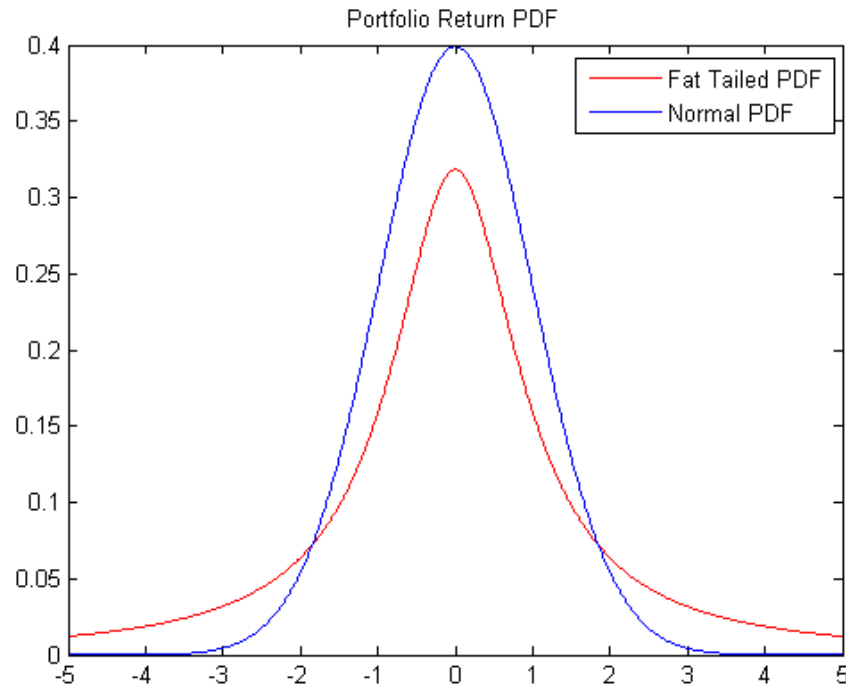


Variance-Covariance Approach



Variance-Covariance Approach

- Is it ok to assume that returns are Normally Distributed?



Variance-Covariance Approach

- Not particularly appropriate for poorly diversified portfolios or individual securities at the daily horizon due to 'fat tails';
- Fat-tailed : individual return distribution
- Normal : diversified portfolio distribution which implies:
 - well diversified
 - risk-factors returns are sufficiently independent from one another
 - Central limit theorem

Variance-Covariance Approach

- **Central Limit Theorem**

- if the sum of the variables has a finite variance, then it will be approximately normally distributed.

Pro VS Con

● PRO

- No pricing model is required. Only the Greeks are essential.
- Easy to handle the incremental

• CON

- Estimation of volatilities of risk factors and correlations of their returns required.
- May not sufficient to capture option risk.
- CANNOT BE USED to conduct sensitivity analysis .
- CANNOT BE USED to derive the confidence interval for VaR.

Historical Simulation Approach

- Simple approach and not oblige to any analytical assumption
- To produce meaningful result, need 2-3 years historical data
- Three step of Historical Simulation Approach
 - Select a sample of actual daily risk factor change over a given period of time
 - Apply daily changes to the current value of risk factor
 - Construct the histogram of portfolio values

Historical Simulation Approach

- Example :- Current portfolio composed of 3-month US\$/DM call option
- Market risk factor include
 - US\$/DM exchange rate
 - US\$ 3-month interest rate
 - DM 3-month interest rate
 - 3-month implied volatility of the US\$/DM exchange rate

Historical Simulation Approach

- First STEP : report historical data

Historical Market Values for the Risk Factors Over the Last 100 Days

Day (t)	\$US/DM (FX _t)	FX Volatility (σ _t)
-100	1.3970	0.149
-99	1.3960	0.149
-98	1.3973	0.151
...
-2	1.4015	0.163
-1	1.4024	0.164

- Second STEP : repricing of the position using historical distribution of the risk factor

Simulating Portfolio Values Using Historical Data
(Current Value of the Portfolio: \$1.80)

	Change from Current Value (\$1.80)
Alternate price 100 = $C(FX_{100}; \sigma_{100}) = \1.75	-\$0.05
Alternate price 99 = $C(FX_{99}; \sigma_{99}) = \1.73	-\$0.07
Alternate price 98 = $C(FX_{98}; \sigma_{98}) = \1.69	-\$0.11
.....	
Alternate price 2 = $C(FX_2; \sigma_2) = \$1.87$	+\$0.07
Alternate price 1 = $C(FX_1; \sigma_1) = \$1.88$	+\$0.08

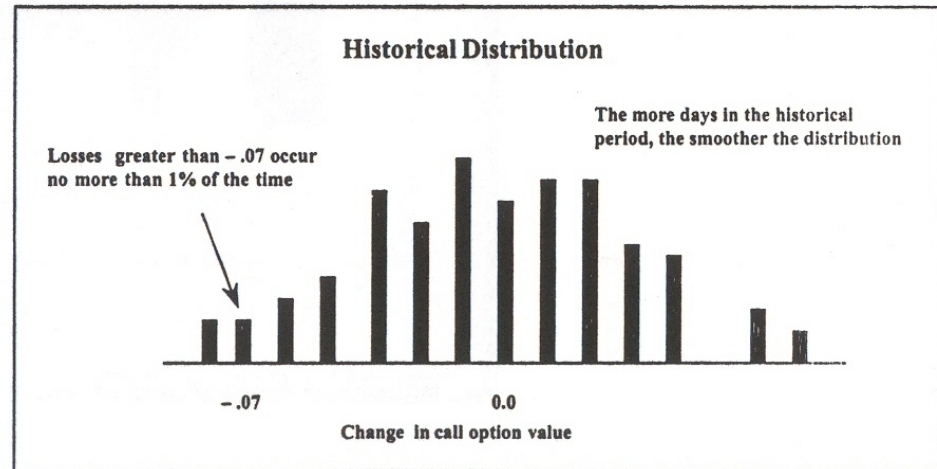
Historical Simulation Approach

- Last STEP : construct the histogram of the portfolio

Identifying the First Percentile of the Historical Distribution of the Portfolio Return

Rank	Change from Current Value
100	-\$0.05
99	-\$0.07
98	-\$0.11
...	...
2	+\$0.07
1	+\$0.08

VaR from Historical Simulations



Historical Simulation Approach

- Major attraction
 - No need to make any assumption about the distribution of the risk factors
 - No need to estimate volatilities and correlations
 - Extreme events are contained in the data set
 - Aggregation across market is straightforward
 - Allows the calculation of confidence intervals for VaR

Historical Simulation Approach

- Drawback

- Complete depends on historical data
- Cannot accommodate change in the market structure
- Short data set may lead to biased and imprecise estimation of VaR
- Cannot be used to conduct sensitivity analysis
- Not always computationally efficient when the portfolio contains complex securities

History of Monte Carlo Method



Monte Carlo Method

- A trivial example that can introduce you about The Monte Carlo Method

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$
$$S_t = S_{t-1} \exp\left[\left(\mu - \frac{1}{2} \sigma^2\right)\Delta t + \sigma\sqrt{\Delta t}Z\right]$$
$$S_t = S_0 \exp\left[\left(\mu - \frac{1}{2} \sigma^2\right)t + \sigma W_t\right]$$

Monte Carlo Method

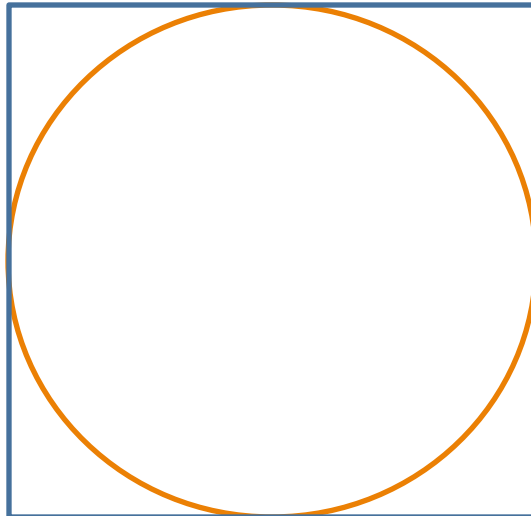
- **Step 1**

- Draw a square on a piece of paper the length of whose sides are the same as the diameter of the circle



Monte Carlo Method

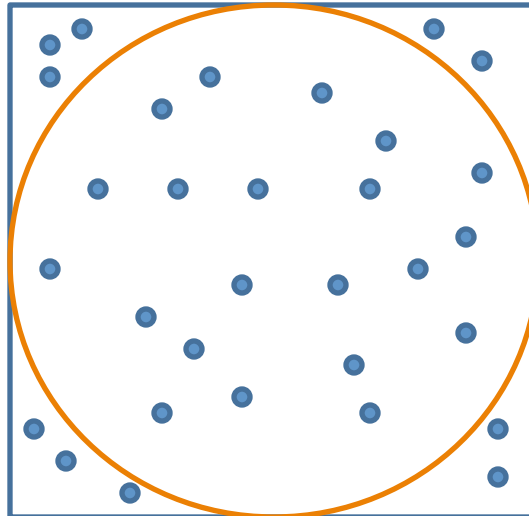
- **Step 2**
- Draw a circle in the square such that the centre of the circle and the square are the same



Monte Carlo Method

- **Step 3**

- Randomly cover the surface of the square with dots, so it looks like this



Monte Carlo Method

- Step 4
- Count all the dots, then count the ones which fall inside the circle, the area of the circle is estimated thus

$$\text{Area of Circle (est)} : : \text{Area of Square} \times \frac{\text{dots inside circle}}{\text{all dots}}$$

Monte Carlo Method

- Conclusion
- The larger the number of dots, the greater the accuracy of the estimate
- But it is also the more time is taken to complete the process

Monte Carlo Approach in the world of Finance

- Consists of repeatedly simulating the random processes that govern market prices and rates at the target horizon e.g. 10 days
- If we generate enough of these scenarios, we will get the simulated distribution that will converge toward the true.
- Thus the VaR can be easily inferred from the distribution

Monte Carlo Simulation

- Involves three steps

1. Specify all the relevant risk factors.
2. Construct price paths
3. Value the portfolio for each path(scenario)

Monte Carlo Simulation

1. Specify all relevant risk factors and specify the dynamic of these factors and estimate the parameters such as expected values, volatilities, and correlations
- For example a commonly used model for stock price is the geometric Brownian motion which is described by the stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Monte Carlo Simulation

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Stock price

The Drift

Volatility

The noise

Monte Carlo Simulation

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Deterministic

Noise
(assumed to be
uncorrelated overtime
Which means it does
not depend on the past
information)

Monte Carlo Simulation

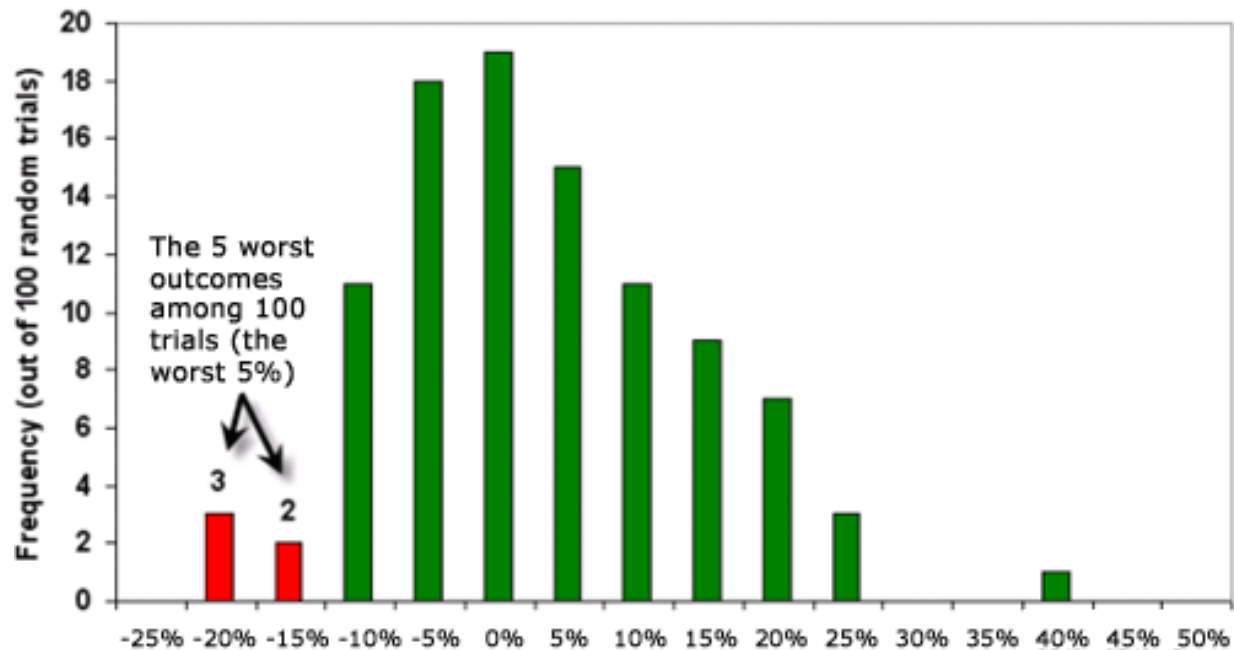
2. Construct price paths using a random number generator
 - When several risk factor are involved we need to simulate multivariate distributions. Only in the case that the distribution has no correlation, then the randomization can be formed independently for each variable.

Monte Carlo Simulation

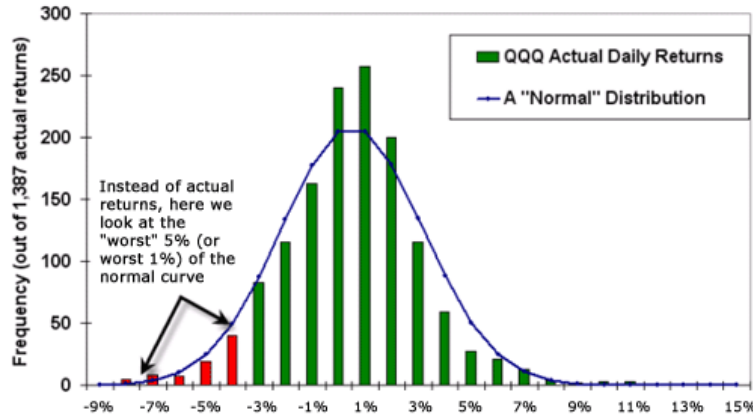
3. Value the portfolio for each scenario. Each path generates a set of values for the risk factors that are used as inputs into the pricing models, for each security composing the portfolio. This process is repeated a large number of times, to generate a distribution of portfolio returns at the risk horizon.

Monte Carlo Simulation

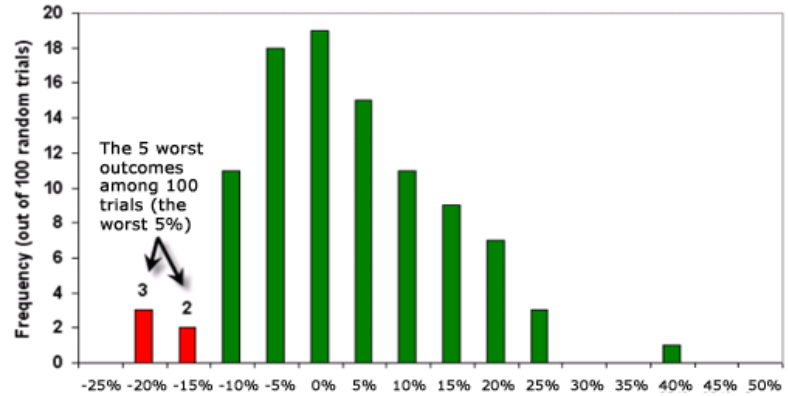
Monte Carlo Simulation
100 Random Trials
(using historical volatility of QQQ)



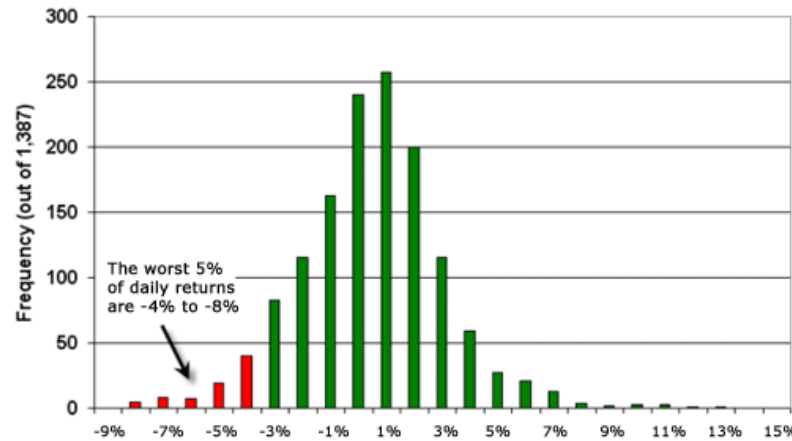
Distribution of Daily Returns
NASDAQ 100 - Ticker: QQQ



Monte Carlo Simulation
100 Random Trials
(using historical volatility of QQQ)



Distribution of Daily Returns
NASDAQ 100 - Ticker: QQQ



Why do we need to understand the stress testing?

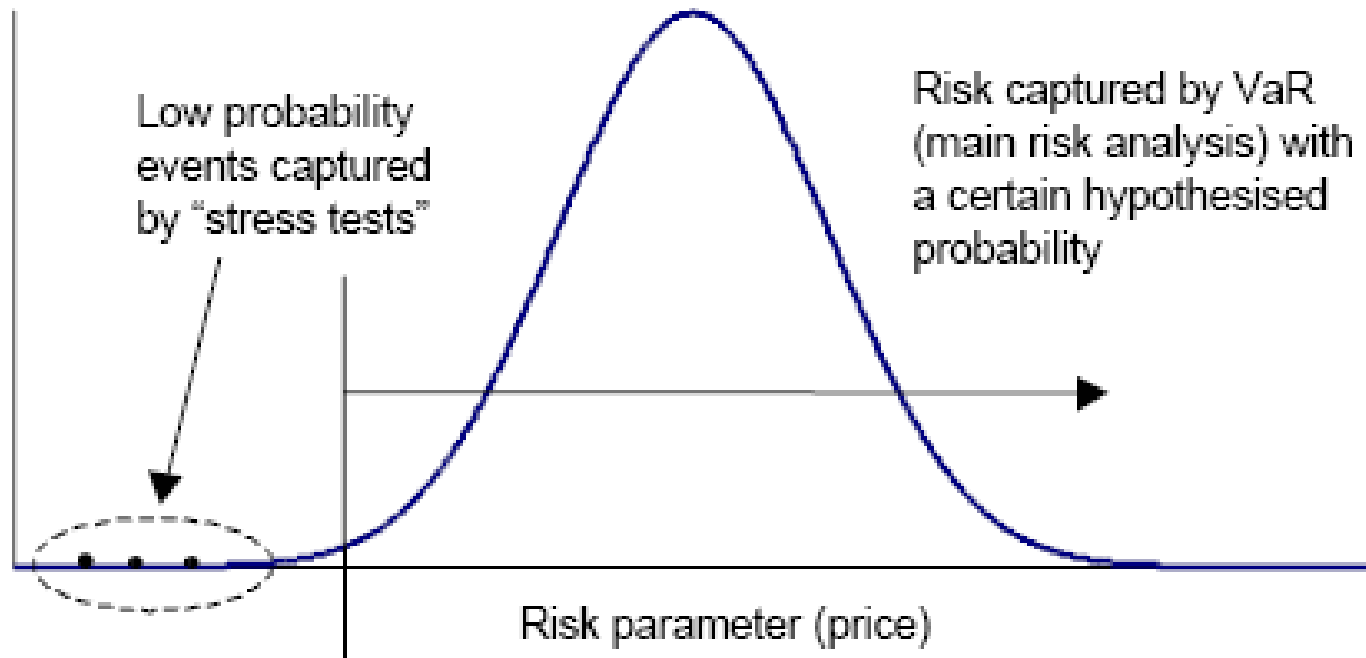
- We don't yet know how to construct a VaR model that would combine a periods of normal market condition with period of market crisis
- VaR is usually calculated within a static framework and is therefore appropriate only for only a short time horizon.

Stress Testing

- Stress testing helps analyzing the possible effects of extreme event that lie outside normal market condition
- The calculation often begins with a set of hypothetical extreme scenario; either by creating from stylized extreme scenarios or come from actual extreme events.
- The purpose of stress testing and scenario analysis is to determine the size of potential losses related to specific scenario

Stress Testing

Stress tests capturing exceptional but plausible events



Source: <http://www.bis.org/publ/cgfs24.pdf>

Risk-Factor Stress Testing

- Help giving us a flavor of the range of stresses bank use to test out their derivative exposure.
- The followings are some of the risk factors that are recommended by the Derivative Policy Group in 1995.
 - Parallel yield curve shift of plus or minus 100 bp
 - Yield-Curve twist of plus or minus 25 bp
 - Equity index values change of plus or minus 10 %
 - Currency change of plus or minus 6%
 - Volatility change of plus or minus 20%

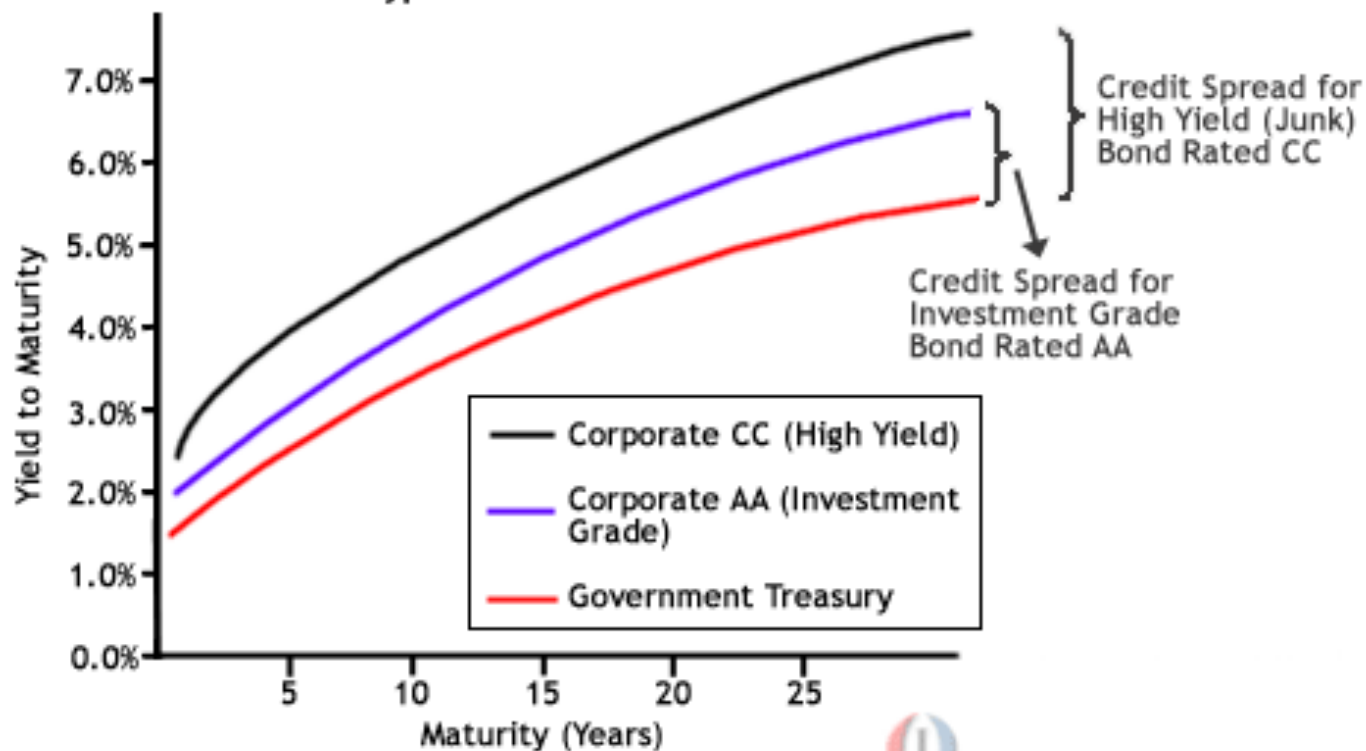
Stress-Testing Envelopes

- Stress-envelope combines stress categories with the worst possible “stress shocks” across all possible markets for every business.
- It is basically the “boundary” for calculated stress testing.

Advantage of the Stress Testing and Scenario Analysis

- Stress testing and scenario analyses are very useful in highlighting the unique vulnerabilities for senior management
- The major benefit is that they show how vulnerable a portfolio might be to a variety of extreme events
- For example, a high yield bond portfolio is vulnerable to a widening of credit spreads

Hypothetical Yield Curves



Limitation of Stress Testing and Scenario Analysis

- Scenarios are based on an arbitrary combination of stress shocks
- The potential number of combinations of basic stress shocks is overwhelming
- Market crises unfold over a period of time, during which liquidity may dry up

Summary of Key Risks-VaR and Stress Testing

- The stress-testing and scenarios methodologies presented in the previous section can be combined with the VaR approach to produce a summary of significant risks
- For example, a high-yield portfolio might well be most exposed to a widening of credit spreads, so the relevant scenario is based on stress-envelope values for a widening of credit spreads