B.E. International Program
Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics

Semester 1/2019

Homework 3

Question 1

Calculate $\frac{\partial f}{\partial x}$, $\frac{\partial f}{\partial y}$ and $\frac{\partial f}{\partial z}$ (if possible) of the following functions.

a. $f(x,y) = \frac{5xy^2}{x^2+y^2}$

b. $f(x,y) = \ln(x^2y + xy^2) - x^2 - y^2$

c. $f(x,y,z) = \mathrm{xz}^2 \ln(y) - \frac{y}{z^2+x-y}$

d. $f(x,y,z) = \mathrm{e}^{x+\ln(z)} - \ln(x^2)y^2z^3$

Question 2

The optimal profit function of a firm can be given by,

$$\pi^*(p, w_1, w_2, A) = A * p^{\frac{1}{1-\beta}}({w_1}^{\gamma} + {w_2}^{\gamma})^{\frac{\beta}{\gamma(\beta-1)}}$$

where $0 < \beta < 1$ and $\gamma < 1$, A is the level of technology, P is price of output, $w_1$is the factor price of capital, and $w_2$is the factor price of labor.
Consider the following problem

a. Use the partial derivative to conclude about the relationship between price and the level of profit.
b. How does the technical progress affect the level of profit? Show your result by using the partial derivative.
c. How does the level of factor price of inputs affect the level of profit? Show your result for both types of input, using the partial derivative. Then, explain the intuition of your result in economics.
d. Show that the profit function is convex in factor price of inputs. That is, the second-order partial derivatives of profit with respect to factor price of both capital and labor are greater than zero.

<u>Question 3</u>

Consider a simple macroeconomic model given below,

$$Y = C + G$$
$$C = C_0 + c_1(Y - T)$$
$$T = T_0 + t_1 Y$$
$$G = G_0$$

where Y is national income, C is consumption, T is the amount of tax collected, G is the level of government expenditure.

Answer the following questions:

a. State all the endogenous variables. What are the parameters and exogenous variables in the model?
b. Derive the equilibrium solution of all the endogenous variables?
c. Use the partial derivative to show the effect change in c_1 and G_0 on the equilibrium of endogenous variables?
d. How does the marginal propensity to tax (t_1) affect the equilibrium level of income (Y*) and consumption (C*)?

Question 4

Write the Hessian Matrix for each of the following functions:

a. $U(x, y) = 7x^2 + 8xy + 3y^2$
b. $z(x, y) = 5(13x - 5y)^2$
c. $f(x, y) = 4x^3 - 11xy - 7y^5$
d. $Q(K, L) = (2K + 1)(3L^2 + 2)$

Question 5

The demand for a product depends on the price p_1 of the product and on the price p_2 charged by a competing producer, and it is given by:

$$D(p_1, p_2) = 36 - \frac{8p_1}{\sqrt{p_2}}.$$

a. Find $\frac{\partial D}{\partial p_1}$ and $\frac{\partial D}{\partial p_2}$, and comment on the signs of the partial derivatives.
b. Calculate the own-price and cross-price elasticities of demand when $p_1 = 3$ and $p_2 = 4$.

Question 6

Suppose the production function Q depends on the number of workers L according to the formula:

$$Q = L * g(\frac{ln(L)}{L})$$

where g(.) is a differentiable function. Find expressions for $\frac{dQ}{dL}$ and $\frac{d^2Q}{dL^2}$.

Question 6: Given the production function $Q = f(K, L) = A[K^n + L^n]$ where A is the level of technology, K is capital and L is labor. Suppose that $n > 0$. Consider the following problem.

a) To ensure that the above production function exhibits a *decreasing return to scale technology*, what additional restrictions do one need to place on n?
b) Under the assumption used in (a), show that the production function satisfies the law of diminishing returns.
c) Calculate the marginal rate of technical substitution (MRTS) of labor (L) for capital (K).
d) Show that MRTS is a decreasing function in L. That is, as labor increases, the value of MRTS decreases.
e) Under the condition(s) assumed in (a), does the production function have the *global* concave property? (Optional)

Suppose that $K(t) = \frac{1}{2}t^2 + 2t + 3$ and $L(t) = e^t + 3$, where $t \geq 0$ is the number of periods from now. Consider the following problem

f) Show that Q is increasing over time.
g) Compute $\frac{\frac{dQ}{Q}}{dt}$ when t = 0, i.e. growth of output in the initial period.
