Tidapa Luewirojn
(6204640459)

Assignment #1

Instructions:

- For all questions, answer up to 4 decimal places.
- This assignment is due on **Tuesday, Mar 9, 2021 before class (11.00).**
- Write your answer in either digital or ordinary paper. For digital paper, export pages into a single PDF file. For ordinary paper, take photos of your writing and convert them into a single PDF file as well.
- There is no need to rewrite the question. Assign number item, ie. 1 a., clearly before your answer is sufficient.
- Submit your assignment into Moodle.
- Name your file as StudentID_Nickname (in Thai) such as 123456789_โบ้

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n} (X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_i Y_i = 7{,}524$	$\sum_{i=1}^{n} \left(X_i - \overline{X}\right)^2 = 1098.8$	$\sum_{i=1}^{n} (Y_i - \overline{Y})^2 = 882.97$	
$\sum_{i=1}^{n} (X_i - \overline{X})(Y_i - \overline{Y}) = -174.20$		$\sum_{i=1}^{n} \hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0, \sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

Tidapa Luewirojn
(6204640459)

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \quad (411.8)$$

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11,

$\bar{X} = 7.45,$

$\hat{\sigma}^2 = 212{,}877,$

$\sum(X_i - \bar{X})^2 = 78.73,$

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

Tidapa Luewirojn (6204640459)

1. Given this information

$n = 30$	$\sum_{i=1}^{n} X_i = 366$	$\sum_{i=1}^{n} Y_i = 631$	$\overline{X} = 12.20$	$\overline{Y} = 21.03$
$\sum_{i=1}^{n}(X_i)^2 = 5{,}564$	$\sum_{i=1}^{n} X_iY_i = 7{,}524$	$\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = 1098.8$		$\sum_{i=1}^{n}(Y_i - \overline{Y})^2 = 882.97$
$\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y}) = -174.20$		$\sum_{i=1}^{n}\hat{u}_i^2 = 873.14$		

Answer the following questions. Show your work.

a) From regression model: $Y_i = \beta_1 + \beta_2 X_i + u_i$, $u_i \sim NIID\left(0,\sigma^2\right)$ (normally, identically and independently distributed), find the estimators of β_1 and β_2 with OLS method and explain the meaning of the model.

b) Find r^2 and explain its meaning.

c) If $X_i = 5$, estimate the value of $\hat{Y}_i$ and explain its meaning.

d) Find the estimators of $\text{var}(u_i)$, $\text{var}\left(\hat{\beta}_1\right)$ and $\text{var}\left(\hat{\beta}_2\right)$

e) Test the hypothesis whether coefficients are different from zero at 0.05 level of significance.

f) Test the hypothesis whether coefficients are less than zero at 0.01 level of significance.

a) Estimator of β_2: $\hat{\beta}_2 = \frac{\Sigma(X_i-\bar{X})(Y_i-\bar{Y})}{\Sigma(X_i-\bar{X})^2} = \frac{-174.20}{1098.8} = -0.1585$ *

Estimator of β_1: $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X} = 21.03 - (-0.1585)(12.2) = 22.9637$ *

∴ The regression model: $\hat{Y}_i = 22.9637 - 0.1585\hat{X}_i$ *

Slope explanation: If X increases by 1 unit, Y will decrease by 0.1585 unit holding other things constant. *

Intercept explanation: Assumed X = 0, the average or conditional mean of Y is 22.9637. *

b) $r^2 = \frac{ESS}{TSS} = 1 - \frac{RSS}{TSS} = 1 - \frac{\Sigma\hat{u}_i^2}{\Sigma(Y_i-\bar{Y})^2} = 1 - \frac{873.14}{882.97} = 0.0111$ * → X can explain Y for 0.0111 or 1.11%, which is the explained part. *

c) $E(\hat{Y}_i \mid X_i = 5)$: $\hat{Y}_i = 22.9637 - 0.1585(5)$
$= 22.9637 - 0.7925$
$= 22.1712$ *

Given X = 5, the average or conditional mean of Y is 22.1712. *

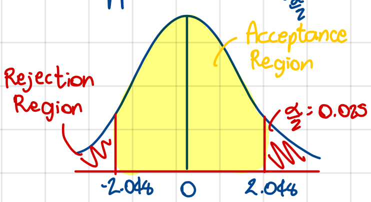
d) Estimator of $\text{var}(u_i)$: $\hat{\sigma}^2 = \frac{\Sigma\hat{u}_i^2}{n-k} = \frac{873.14}{30-2} = \frac{873.14}{28} = 31.1836$ * $\%^2$

Estimator of $\text{var}(\hat{\beta}_1)$: $\hat{\sigma}^2_{\hat{\beta}_1} = \frac{\Sigma X_i^2\,\hat{\sigma}^2}{n\Sigma x_i^2} = \frac{\Sigma X_i^2\,\hat{\sigma}^2}{n\Sigma(X_i-\bar{X})^2} = \frac{(5564)(31.1836)}{30(1098.8)} = 5.2635$ * $\%^2$

Estimator of $\text{var}(\hat{\beta}_2)$: $\hat{\sigma}^2_{\hat{\beta}_2} = \frac{\hat{\sigma}^2}{\Sigma x_i^2} = \frac{31.1836}{1098.8} = 0.0284$ * $\%^2$

e) level of significance = $\alpha = 0.05$ ∴ $\frac{\alpha}{2} = 0.025$

1. Hypothesis Testing
   $H_0: \beta_2 = 0$ → Null Hypothesis
   $H_a: \beta_2 \neq 0$ → Alternative Hypothesis
2. Calculate test statistics
   $t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{-0.1585-0}{\sqrt{0.0284}} = -0.9405$
3. State decision rule: d.f. = 30 − 2 = 28
   → $\alpha = 0.05$
   → Lower bound: $0 - t_{\frac{\alpha}{2}} = -2.044$
   → Upper bound: $0 + t_{\frac{\alpha}{2}} = 2.044$
   Critical Value

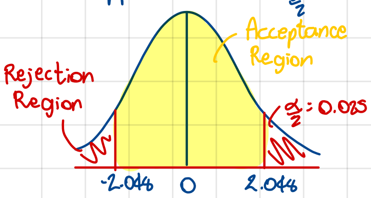


$t_{cal} = -0.9405$ is in the acceptance region.

4. We cannot reject the null hypothetic at the significant level of 5%.

∴ We cannot say for sure that β_2 is not 0, 95 out of 100 times.

1. Hypothesis Testing
   $H_0: \beta_1 = 0$ → Null Hypothesis
   $H_a: \beta_1 \neq 0$ → Alternative Hypothesis
2. Calculate test statistics
   $t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{22.9637-0}{\sqrt{5.2635}} = 10.0093$
3. State decision rule: d.f. = 30 − 2 = 28
   → $\alpha = 0.05$
   → Lower bound: $0 - t_{\frac{\alpha}{2}} = -2.044$
   → Upper bound: $0 + t_{\frac{\alpha}{2}} = 2.044$
   Critical Value



t cal = 10.0093 is in the rejection region.

4. We can reject the null hypothesis at the significant level of 0.05.

∴ We can say for sure that β_1 is not 0, 95 out of 100 times.

f) level of significance, α : 0.01

1. Hypothesis Testing

$H_0 : \beta_2 \leq 0 \rightarrow$ Null Hypothesis

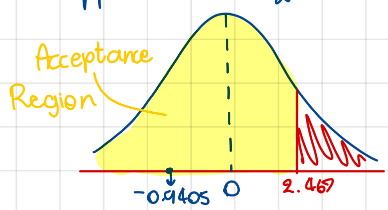
$H_a : \beta_2 > 0 \rightarrow$ Alternative Hypothesis

2. Test Statistics

$$t_{cal} = \frac{\hat{\beta}_2 - \beta_2}{\hat{\sigma}_{\hat{\beta}_2}} = \frac{-0.1585 - 0}{\sqrt{0.0284}} = -0.9405$$

3. Decision rule : d.f. = 30 - 2 = 28, α = 0.01 from t-table,

Upper bound : $0 + t_\alpha$ = 2.467 } Critical Value



$\therefore t_{cal} = -0.9405$ is in the acceptance region

4. We cannot reject the null hypothetic at the significant level of 0.01.

$\therefore$ We cannot say for sure that β_2 is not less than or equal to 0, 99 out of 100 times.

Tidapa Luewirojn (6204640459)

1. Hypothesis Testing

$H_0 : \beta_1 \leq 0 \rightarrow$ Null Hypothesis

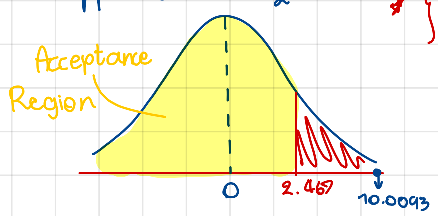
$H_a : \beta_1 > 0 \rightarrow$ Alternative Hypothesis

2. Calculate test statistics

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\hat{\sigma}_{\hat{\beta}_1}} = \frac{22.9639 - 0}{\sqrt{5.2635}} = 10.0093$$

3. State decision rule : d.f. = 30 - 2 = 28, α = 0.01 from t-table,

Upper bound : $0 + t_\alpha$ = 2.467 } Critical Value



$\therefore t_{cal} = 10.0093$ is in the rejection region.

4. We can reject the null hypothesis at the significant level of 1%.

$\therefore$ We can say for sure that β_1 is not less than or equal to 0, 99 out of 100 times.

2. Given that Y is market price of a car (USD) while X is how long a car aged (years), results of the regression are as follows.

$$\hat{Y}_i = 7{,}836 - 502.4X_i$$
$$(52) \qquad (411.8)$$

price; age; SD

Given that u_i is normally, identically and independently distributed with zero mean and σ^2 variance, total number of observations is 11, → n

$\bar{X} = 7.45$,

$\hat{\sigma}^2 = 212{,}877$,

$\sum(X_i - \bar{X})^2 = 78.73$,

Answer the following questions. Show your work.

a) Does the sign of $\hat{\beta}_2$ make economic sense? Provide your explanation.

b) If you are a car expert and someone asks you to estimate how much his car will be **averagely** priced at when his car is 5 years old, how much is the market price range that you would estimate that you can make sure that for 95% of the time, market price will be within the specific range?

c) If you multiply all the X with 10, report the new SRF with the standard error resulted from the multiplication.

d) Calculate the elasticity of market price when a car is 10 years old.

a) From $\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i$, $\hat{\beta}_2$ is -502.4. Meaning that when a car is one year older, the market price of the car decreases by 502.4 USD, holding other things constant. This makes sense because when capital is older, its value depreciates. So, the price should be lower. $\therefore$ Yes, it makes sense

b) $E(Y|X=5) = 7836 - 502.4(5) = 5324$ \$

$\therefore$ The averaged price of the car when it is 5 years old is 5,324 \$.

Confidence Interval

Given $\alpha = 0.05$, $\frac{\alpha}{2} = 0.025$, d.f. = n - k = 11 - 2 = 9

① Find $var(\hat{Y}_0) = \hat{\sigma}^2 \left[\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X_i - \bar{X})^2}\right]$

$= 212{,}877\left(\frac{1}{11} + \frac{(5-7.45)^2}{78.73}\right)$

$= 35{,}582.5345$

② $\hat{\sigma}_{\hat{Y}_0} = \sqrt{35{,}582.5345} = 188.6333$

③ Find the 95% of confidence interval from $E(Y|X_0=5)$

Lower bound : $\hat{Y}_0 - (t_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{Y}_0}) = 5{,}324 - (2.262 \times 188.6333)$

$= 4{,}897.3115$ USD

Upper bound : $\hat{Y}_0 + (t_{\frac{\alpha}{2}} \cdot \hat{\sigma}_{\hat{Y}_0}) = 5{,}324 + (2.262 \times 188.6333)$

$= 5{,}750.6885$ USD

$\therefore$ The market price range where we can be 95% sure that $E(Y_0|X_0=5)$ is within is 4,897.3115 USD to 5,750.6885 USD

c) Data Scaling : even though the unit changes, the interpretation should be the same

Let w_1 be the unit of β_1 and w_2 be the unit of β_2.

From $\hat{Y}_i = 7836 - 502.4X_i$
$(52) \quad (411.8)$

We get $\hat{\beta}_1^* = w_1 \beta_1 = 1(7836) = 7836$

$\hat{\beta}_2^* = \left(\frac{w_1}{w_2}\right)\beta_2 = \frac{1}{0.1}(502.4) = 5024$

$\hat{Y}_i = 7836 - 502.4X_i \rightarrow$ The car is 1 year older, the market price is 502.4 USD less.

$\rightarrow$ se = 411.8

$\hat{Y}_i = 7836 - 5024X_i \rightarrow$ The car is 10 years older (1 decade), the market price is 5024 USD less.

$\rightarrow$ se = 411.8(10) = 4118

d) $\hat{Y}_i = 7836 - 502.4(10) = 2812$ USD

Market price elasticity $= \frac{dY}{dX} \cdot \frac{X}{Y} = \text{slope} \cdot \frac{X}{Y}$

$= \beta_2 \cdot \frac{X}{Y} = -502.4 \times \frac{10}{2812}$

$= -1.7866 \rightarrow$ elasticity