EE 320 Introductory Mathematical Economics

Semester 2/2012

Problem Set 9 – Suggested Answers

Constrained Optimization[1]

1. a) $(x^*, y^*) = (40,24)$

   b) $(x^{**}, y^{**}) = (34,20)$.

   [Hint: Since *x* + *y* = *54*, substitute *y* = *54* – *x* in the profit function and then solve for the optimal values of *x* and *y*, respectively.]

2. a) Rewrite the problem as: max $U = (108 - 4y - 4z)yz$. Then, take find $\frac{\partial U}{\partial y}$ and $\frac{\partial U}{\partial z}$, and it follows that $(x^*, y^*, z^*) = (36, 12, 9)$.

   b) You should get the same answer as in part a).

3. a) $(x^*, y^*) = (230, 5)$

   b) $(x^{**}, y^{**}) = (195, 5)$.

4. $\mathcal{L} = -40Q_1 + Q_1^2 - 2Q_1Q_2 - 20Q_2 + Q_2^2 + \lambda[15 - Q_1 - Q_2]$.

   ➔ $(Q_1^*, Q_2^*) = (10,5)$.

$$|\bar{H}| = \begin{vmatrix} 0 & 1 & 1 \\ 1 & 2 & -2 \\ 1 & -2 & 2 \end{vmatrix} = -8 < 0$$. Thus, SOSC for a minimum are satisfied.

5. $(x^*, y^*) = (250, 125)$

$$|\bar{H}| = \begin{vmatrix} 0 & 2 & 4 \\ 2 & 0 & 100 \\ 4 & 100 & 0 \end{vmatrix} = 1600 > 0.$$ Thus, SOSC for a maximum are satisfied.

6. $m = \frac{awT_0}{(a+b)}$, and $l = \frac{bT_0}{(a+b)}$.

7.

a) $P(x, y) = (96 - 4x)x + (84 - 2y)y - (2x^2 + 2xy + y^2)$

$$= -6x^2 - 3y^2 - 2xy + 96x + 84y$$

[1] All questions in this problem set are from Sydsaeter and Hammond, 2008.

b) $P_x = -12x - 2y + 96$; $P_y = -6y - 2x + 84$

$(x^*, y^*) = (6, 12)$ and $P(x^*, y^*) = 792$.

c) $(x^{**}, y^{**}) = (4, 7)$; $P(x^*, y^*) = 673$.

8. a) $(x^*, y^*) = (4, 24)$ ➔ $U(x^*, y^*) = 26$ [Note: $\lambda = 0.25$]

b) $(x^{**}, y^{**}) = (4, 24.25)$ ➔ $U(x^{**}, y^{**}) = 26.25$.

Thus, $U(x^{**}, y^{**}) - U(x^*, y^*) = 0.25 = \lambda$.

c) $x^* = \frac{q^2}{4p^2}$; $y^* = \frac{m}{q} - \frac{q}{4p}$. Note: $y^* > 0 \Leftrightarrow m > q^2/4p$.

9. a) From FONC:

$$x^* = a + \frac{\alpha}{\lambda p};\ y^* = b + \frac{\beta}{\lambda q};\ \text{and}\ \frac{1}{\lambda} = m - (pa + qb).$$

Thus, it follows that

$$px^* = \alpha m + pa - \alpha(pa + qb);\ qy^* = \beta m + qb - \beta(pa + qb).$$

b) $\frac{\partial U^*}{\partial m} = \frac{\alpha}{m-(pa+qb)} + \frac{\beta}{m-(pa+qb)} = \frac{1}{m-(pa+qb)} = \lambda > 0$;

$\frac{\partial U^*}{\partial p} = \frac{-\alpha a}{m-(pa+qb)} - \frac{\alpha}{p} + \frac{-\beta a}{m-(pa+qb)} = \frac{-a}{m-(pa+qb)} - \frac{\alpha}{p} = -a\lambda - \frac{\alpha}{p}$.

Also, $-\frac{\partial U^*}{\partial m} x^* = -\lambda\left(a + \frac{\alpha}{\lambda p}\right) - a\lambda - \frac{\alpha}{p}$. Thus, $\frac{\partial U^*}{\partial p} = -\frac{\partial U^*}{\partial m} x^*$.

$\frac{\partial U^*}{\partial q} = -\frac{\partial U^*}{\partial m} y^*$ can be shown in the same manner.