2 One-step Profit Maximization

- Nicholson, Ch. 11, pp. 383-393

- One-step procedure: maximize profits

- Perfect competition. Price p is given
  - Firms are small relative to market
  - Firms do not affect market price p_M

  - Will firm produce at $p > p_M$?
  - Will firm produce at $p < p_M$?
  - $\Longrightarrow p = p_M$

- Revenue: $py = pf\left(L, K\right)$

- Cost: $wL + rK$

- Profit $pf\left(L, K\right) - wL - rK$

- Agent optimization:

$$\max_{L,K} pf(L,K) - wL - rK$$

- First order conditions:

$$pf'_L(L,K) - w = 0$$

and

$$pf'_K(L,K) - r = 0$$

- Second order conditions? $pf''_{L,L}(L,K) < 0$ and

$$\begin{aligned} |H| &= \begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix} = \\ &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0 \end{aligned}$$

- Need $f''_{L,K}$ not too large for maximum

- Comparative statics with respect to to p, w, and r.

- What happens if w increases?

$$\frac{\partial L^*}{\partial w} = -\frac{\begin{vmatrix} -1 & pf''_{L,K}(L,K) \\ 0 & pf''_{K,K}(L,K) \end{vmatrix}}{\begin{vmatrix} pf''_{L,L}(L,K) & pf''_{L,K}(L,K) \\ pf''_{L,K}(L,K) & pf''_{K,K}(L,K) \end{vmatrix}} < 0$$

and

$$\frac{\partial L^*}{\partial r} =$$

- Sign of $\partial L^*/\partial r$ depends on $f''_{L,K}$.

3 Second Order Conditions in P-Max: Cobb-Douglas

- How do the second order conditions relate for:
  - Cost Minimization
  - Profit Maximization?

- Check for Cobb-Douglas production function
$$y = AK^{\alpha}L^{\beta}$$

- **Cost Minimization.** S.o.c.:
$$c_y''(y^*, w, r) > 0$$

- As we showed, for CD prod. ftn.,
$$c_y''(y^*, w, r) = -\frac{1}{\alpha+\beta}\frac{1-(\alpha+\beta)}{\alpha+\beta}\frac{B}{A^2}\left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$
which is > 0 as long as $\alpha + \beta < 1$ (DRS)

- **Profit Maximization.** S.o.c.: $pf''_{L,L}(L,K) < 0$ and

$$|H| = p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0$$

- As long as $\beta < 1$,

$$pf''_{L,L} = p\beta(\beta - 1) AK^{\alpha} L^{\beta-2} < 0$$

- Then,

$$\begin{aligned} |H| &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] = \\ &= p^2 \left[\begin{array}{c} \beta(\beta-1) AK^{\alpha} L^{\beta-2} * \\ \alpha(\alpha-1) AK^{\alpha-2} L^{\beta} - \\ \left(\alpha\beta AK^{\alpha-1} L^{\beta-1} \right)^2 \end{array} \right] = \\ &= p^2 A^2 K^{2\alpha-2} L^{2\beta-2} \alpha\beta [1 - \alpha - \beta] \end{aligned}$$

- Therefore, $|H| > 0$ iff $\alpha + \beta < 1$ (DRS)

- The two conditions coincide

4 Introduction to Market Equilibrium

- Nicholson, Ch. 12, pp. 409–419

- Two ways to analyze firm behavior:
  - Two-Step Cost Minimization
  - One-Step Profit Maximization

- What did we learn?
  - Optimal demand for inputs L^*, K^* (see above)
  - Optimal quantity produced y^*

- **Supply function**. $y = y^*(p, w, r)$

  – From profit maximization:

  $$y = f\left(L^*(p, w, r), K^*(p, w, r)\right)$$

  – From cost minimization:

  $$MC \text{ curve above } AC$$

  – Supply function is increasing in p

- Market Equilibrium. Equate demand and supply.

- Aggregation?

- Industry supply function!

5 Aggregation

5.1 Producers aggregation

- J companies, $j = 1, ..., J$, producing good i

- Company j has supply function

$$y_i^j = y_i^{j*}(p_i, w, r)$$

- Industry supply function:

$$Y_i(p_i, w, r) = \sum_{j=1}^{J} y_i^{j*}(p_i, w, r)$$

- Graphically,

5.2 Consumer aggregation

- *One-consumer economy*

- Utility function $u(x_1, ..., x_n)$

- prices $p_1, ..., p_n$

- Maximization $\Longrightarrow$

$$\begin{aligned} x_1^* &= x_1^*(p_1, ..., p_n, M), \\ &\vdots \\ x_n^* &= x_n^*(p_1, ..., p_n, M). \end{aligned}$$

- Focus on good i. Fix prices $p_1, ..., p_{i-1}, p_{i+1}, ..., p_n$ and M

- **Single-consumer demand function**:

$$x_i^* = x_i^* \left(p_i | p_1, ..., p_{i-1}, p_{i+1}, ..., p_n, M \right)$$

- What is sign of $\partial x_i^* / \partial p_i$?

- Negative if good i is normal

- Negative or positive if good i is inferior

- *Aggregation:* J consumers, $j = 1, ..., J$

- Demand for good i by consumer j :

$$x_i^{j*} = x_i^{j*}\left(p_1, ..., p_n, M^j\right)$$

- Market demand X_i:

$$\begin{aligned} & X_i\left(p_1, ..., p_n, M^1, ..., M^J\right) \\ = & \sum_{j=1}^{J} x_i^{j*}\left(p_1, ..., p_n, M^j\right) \end{aligned}$$

- Graphically,

- Notice: market demand function depends on distribution of income M^J

- Market demand function X_i:
  - Consumption of good i as function of prices $\mathbf{p}$
  - Consumption of good i as function of income distribution M^j

6 Market Equilibrium in the Short-Run

- What is equilibrium price p_i?

- Magic of the Market...

- Equilibrium: No excess supply, No excess demand

- Prices $\mathbf{p}^*$ equates demand and supply of good i:

  $Y^* = Y_i^S(p_i^*, w, r) = X_i^D\left(p_1^*, ..., p_n^*, M^1, ..., M^J\right)$

- Graphically,

- Notice: in short-run firms can make positive profits

- Comparative statics exercises with endogenous price p_i :
  - increase in wage w or interest rate r:

  - change in income distribution