Exercise

Given rational expectations and the following aggregate supply function:

$$y = y^* + (\pi - \pi^e) + \varepsilon$$

where y is output, y^* is the natural level of output, π is inflation, π^e is expected inflation and ε is a supply disturbance, which is independently distributed around its mean of zero

The government aims to minimize the following loss function by selecting π as it sees fit at any time:

$$L = \pi^2 + (y - 2y^*)^2$$

where the targets for inflation and output are 0 and $2y^*$ respectively

1. Under such discretionary policy, prove that there is an inflationary bias while output stays at y^* on average

2. If the government intends to eliminate inflationary bias by forming and announcing a rule of the form

$$\pi = \alpha + \beta\varepsilon$$

and targets output at y^*, find the optimal α and β and prove that rule gives superior outcome to discretion.

3. Given that rule is superior to discretion, prove that a better result in terms of output can be achieved if the government announces the same rule as in (2) and the private sector believes in the rule announced, but the government cheats by practicing discretion and targeting output at $2y^*$ instead.