Chapter 17 Long-Run Production

Long-Run Production: Production in the time frame long enough to change every input $\Longrightarrow$ No Fixed Cost

• In Long-Run, when output $Q = 0$, Total Cost $= 0$.

A firm in Long-Run faces two possible problems:

1. $\begin{cases} \max Q = f(L,K) \\ \text{St. } wL + rK = C_0 \end{cases}$ } similar to consumption problem

utility

max $U(x,y)$

St. $p_x x + p_y y = B$.

Budget line.

2. $\begin{cases} \min C = wL + rK \\ \text{St. } f(L,K) = Q_0 \end{cases}$

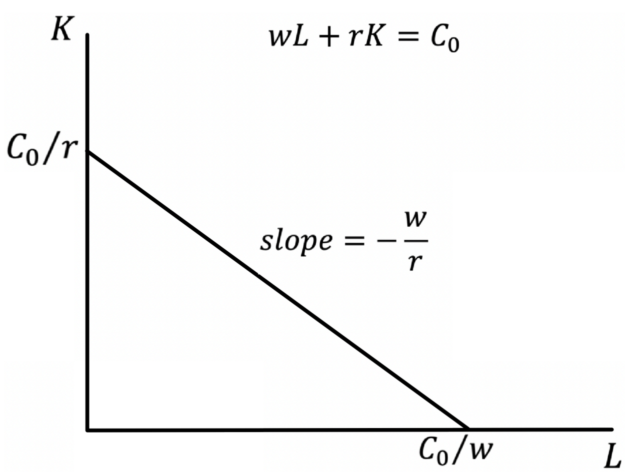
Isocost.

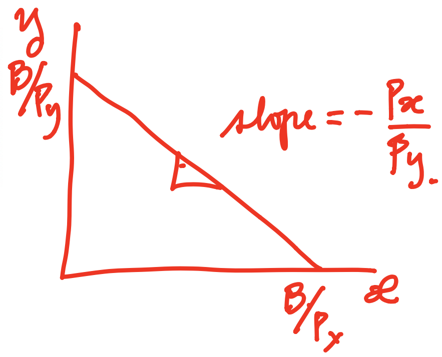
where $f(L,K)$ is the production function
w is wage, the price of labor L
r is interest, the price of capital K

Just like Budget Line and Indifference Curve in the consumption problem, we have Isocost and Isoquant in production problem.

Isocost Given cost level C_0, $wL + rK = C_0$ is a line called Isocost where every point on this line is the amount of L and K that cost the firm the same cost C_0.

• Isocost is similar to the Budget Line as given below:

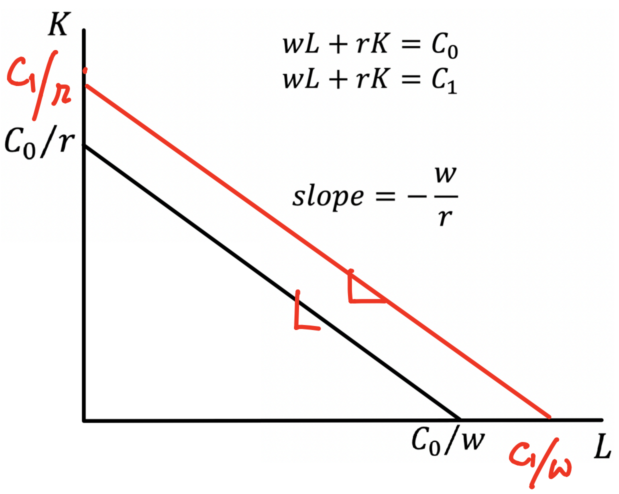




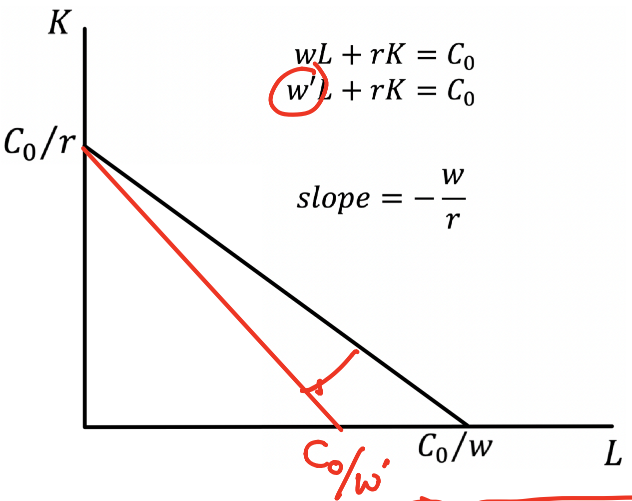
$$slope = -\frac{w}{r} = relative\ price\ of\ L\ in\ terms\ of\ K$$
$$= exchange\ rate\ between\ L\ and\ K\ in\ the\ market$$

Changes in Isocost

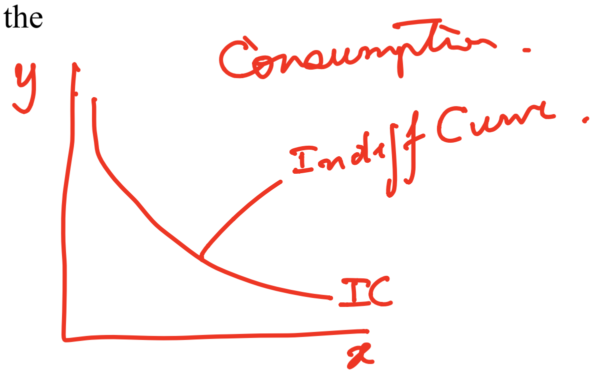
1. C_0 increases to C_1



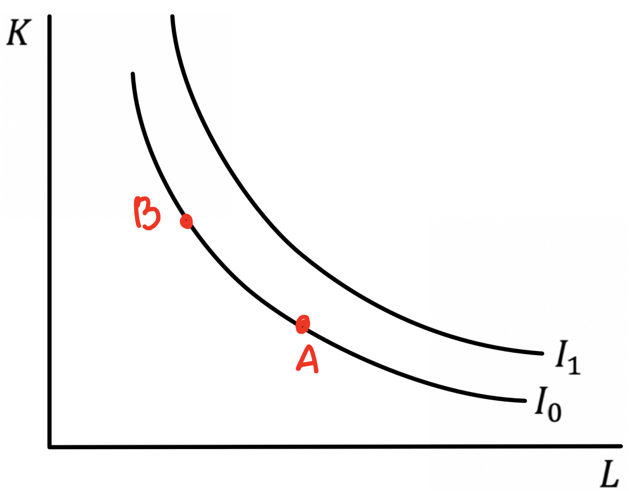
2. w increases to w'



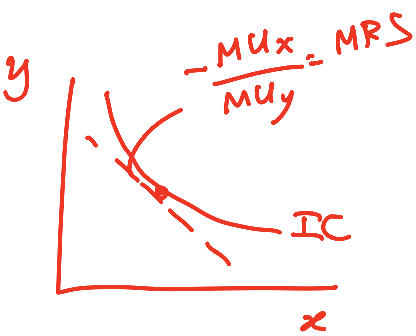
Isoquant Given output level Q_0, $Q_0 = f(L, K)$ is a line called Isoquant where every point on this line is the amounts of L and K that produce the same output Q_0.



• Isoquant is similar to Indifference Curve

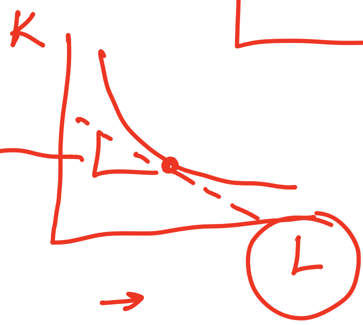


Properties of Isoquant ≈ similar to properties of Indifference Curve.



1. For each point of $A = (L_0, K_0)$, there is exactly one Isoquant passing though it. That is, no two Isoquants can intersect nor be tangent to each other.
2. There are infinite number of Isoquants, each never intersects nor is tangent to another.
3. Higher Isoquant means higher output Q.
4. Each Isoquant always has negative slope—called Marginal Rate of Technical Substitution ($MRTS$). At point $A = (L_0, K_0)$, slope of the Isoquant is

$$MRTS = -\boxed{\frac{MP_L(L_0, K_0)}{MP_K(L_0, K_0)}}. = \cancel{1} -2$$



MRTS is the **instantaneous exchange rate** between L and K **inside the firm** such that output Q is unchanged.

5. **Diminishing** $MRTS$ On a given Isoquant, as the use of L ~~x~~ increases, the value of $|MRTS|$ decreases.

Production Equilibrium As mentioned in Chapter 15 Production: How a firm employs inputs (labor L and capital K) to

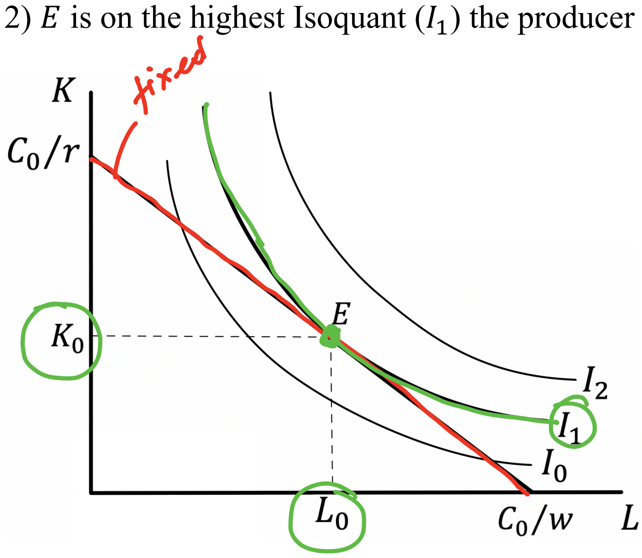
1. Maximize output Q for a given cost level C_0, or
2. Minimize cost C to produce a give level of output Q_0.

Thus, there are two possible cases of productions equilibrium.

1. Maximization of Output Q:

$$\max Q = f(L, K)$$
$$\text{st.}\, wL + rK = C_0$$

The equilibrium is at $E = (L_0, K_0)$ because
1) E is on the Isocost $wL + rK = C_0$ line—E is feasible.
2) E is on the highest Isoquant (I_1) the producer can reach.



At the equilibrium point E, the Isoquant curve I_1 is tangent to the Isocost line $wL + rK = C_0$.

Equilibrium Conditions: Maximization Output The firm has production equilibrium of maximizing Q at $E = (L_0, K_0)$ with the following conditions:

1) $wL_0 + rK_0 = C_0 \Leftrightarrow E$ is feasible,
2) $MRTS_{(at\ E)} = \frac{MP_L(L_0,K_0)}{MP_K(L_0,K_0)} = \frac{w}{r}$, or equivalently $\frac{MP_L(L_0,K_0)}{w} = \frac{MP_K(L_0,K_0)}{r}$.

The condition (2) means that

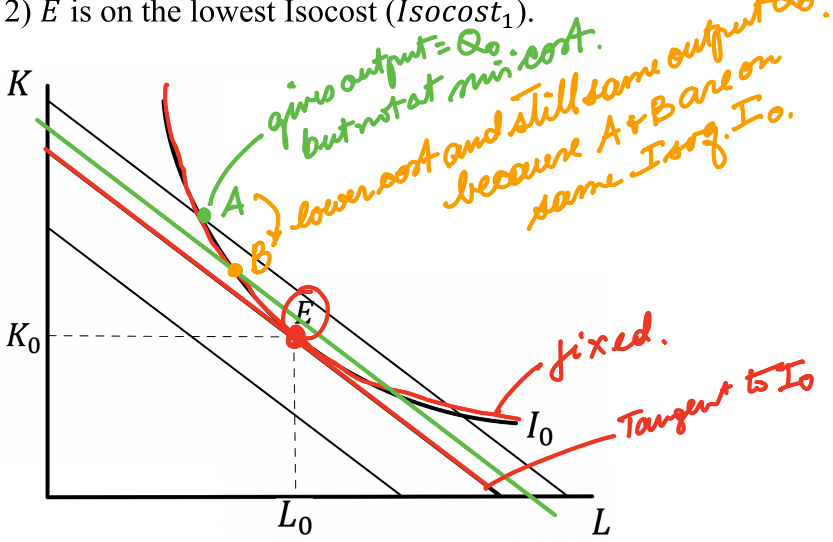
$$\left.\begin{matrix} Exchange\ rate\ between \\ L\ and\ K \\ \color{red}{in\ the\ firm} \end{matrix}\right\} = \left\{\begin{matrix} Exchange\ rate\ between \\ L\ and\ K \\ \color{red}{in\ the\ market} \end{matrix}\right.$$

Or equivalently, the output from the last baht spent on L is equal to that spent on K.

2. Minimization of Cost C:

$$\min C = wl + rK$$
$$\text{st. } f(L, K) = Q_0$$

The equilibrium is at $E = (L_0, K_0)$ because
1) E is on the Isoquant $f(L, K) = Q_0$—E is feasible.
2) E is on the lowest Isocost ($Isocost_1$).



At the equilibrium point E, the Isoquant curve IC_0 is tangent to the Isocost line $wL + rK = C_0$.

Equilibrium Conditions: Minimizing Cost The firm has production equilibrium of maximizing Q at $E = (L_0, K_0)$ with the following conditions:

$f(L_0, K_0) = Q_0$

1) ~~$wL_0 + rK_0 = C_0$~~ $\Leftrightarrow E$ is feasible,

2) $MRTS_{(at\ E)} = \frac{MP_L(L_0,K_0)}{MP_K(L_0,K_0)} = \frac{w}{r}$, or equivalently

$\frac{MP_L(L_0,K_0)}{w} = \frac{MP_K(L_0,K_0)}{r}$.

The condition (2) means that

$$\left.\begin{matrix} Exchange\ rate\ between \\ L\ and\ K \\ \underline{in\ the\ firm} \end{matrix}\right\} = \left\{\begin{matrix} Exchange\ rate\ between \\ L\ and\ K \\ \underline{in\ the\ market} \end{matrix}\right.$$

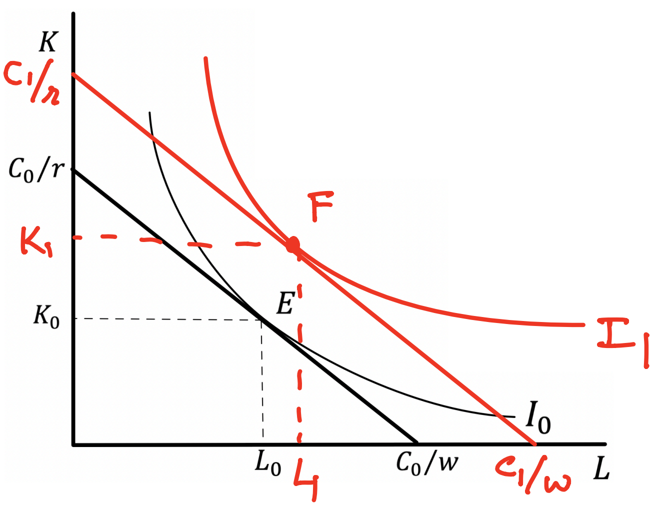
Or equivalently, the output from the last baht spent on L is equal to that spent on K.

Comparison of Consumption and Production Problems

Consumption	Production
Budget Line: $p_x x + p_y y = B$ Slope $= -\frac{p_x}{p_y}$	Isocost: $wL + rK = C_0$ Slope $= -\frac{w}{r}$
Indifference Curve: Slope $= -\frac{MU_x}{MU_y}$	~~Indifference Curve:~~ Isoquant. Slope $= -\frac{MP_L}{MP_K}$
Equilibrium Conditions at $E = (x_0, y_0)$ 1) $p_x x_0 + p_y y_0 = B$ 2) $MRS_{(at\ E)} = -\frac{MU_x(x_0,y_0)}{MU_y(x_0,y_0)} = -\frac{p_x}{p_y}$ is negative. max utility $U(x,y)$ st $p_x x + p_y y = B$	Equilibrium Conditions at $E = (L_0, K_0)$ Maximization of Output: 1) $wL_0 + rK_0 = C_0$ - L_0, K_0 is on the isocost. 2) $MRTS_{(at\ E)} = -\frac{MP_L(L_0,K_0)}{MP_K(L_0,K_0)} = -\frac{w}{r}$ Minimization of Cost: 1) $f(L_0, K_0) = Q_0$ - L_0, K_0 is on Isoq of Q_0. 2) $MRTS_{(at\ E)} = -\frac{MP_L(L_0,K_0)}{MP_K(L_0,K_0)} = -\frac{w}{r}$ slope isoq = slope isocost.
Income Consumption Curve	Expansion path

Change in Equilibrium: Maximization of Output

1) Cost C_0 increases to C_1

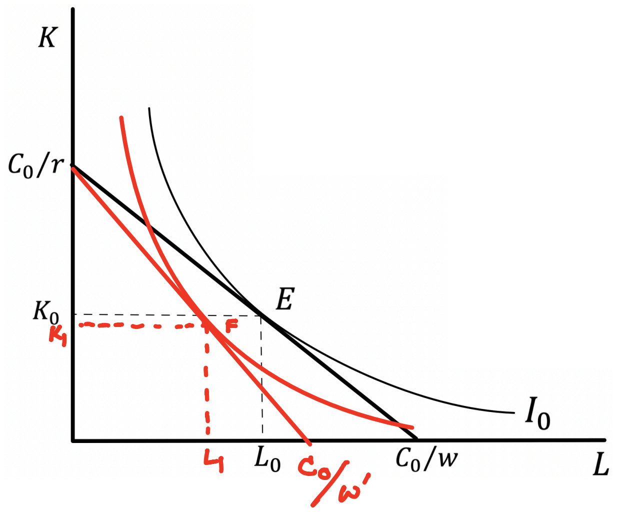


- Same as higher income in consumption case
- No discussion of Necessary/Luxury goods
- Equilibrium conditions: at F

1) $wL_1 + rK_1 = C_1$

2) $MRTS_P = -\dfrac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \dfrac{w}{r}$

2. Wage w increases to w'



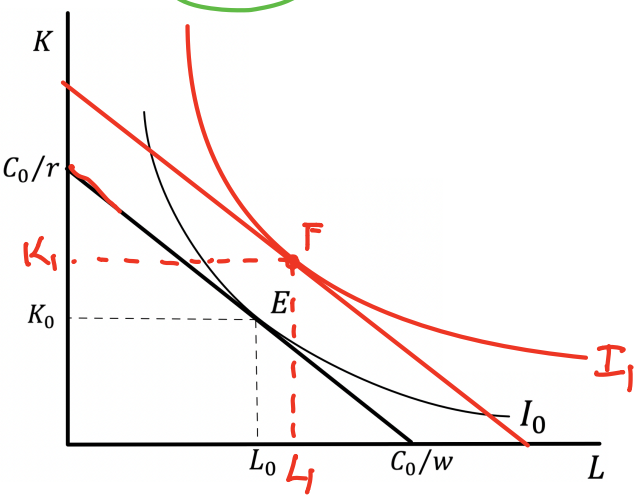
- Same as when p_x increases in consumption case
- No discussion of Substitution and Income Effects
- Equilibrium conditions: at F

1) $w'L_1 + rK_1 = \cancel{C_1}\ C_0$

2) $MRTS_P = -\dfrac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \dfrac{w'}{r}$

Change in Equilibrium: Minimization of Cost

1) Output Q_0 increases to Q_1



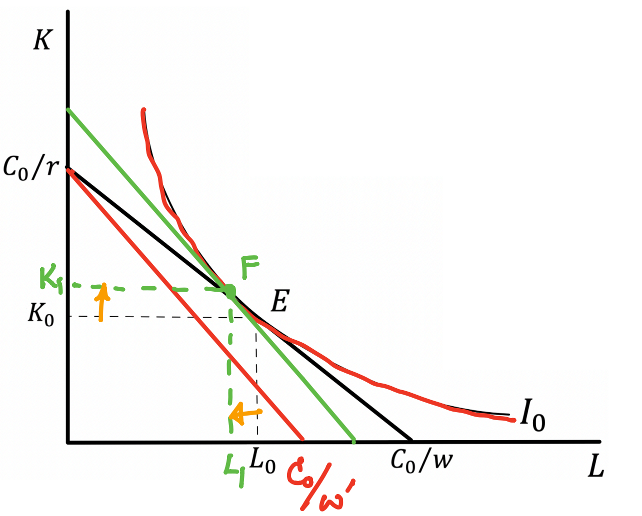
- Equilibrium conditions: F

1) $f(L_1, K_1) = Q_1$

2) $MRTS_F = -\dfrac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \dfrac{w}{r}.$

2. Wage w increases to w' – want the same output Q_0 → Isoq I_0 does not change.

How about r changes?



- Equilibrium conditions: at F → when w increases to w' the firm always employs less of L and more of K.

1) $f(L_1, K_1) = \cancel{Q_1}\ Q_0$

2) $MRTS_F = -\dfrac{MP_L(L_1, K_1)}{MP_K(L_1, K_1)} = \dfrac{w'}{r}.$

why? – Because of property 5 of Isoq. – Diminishing MRTS.

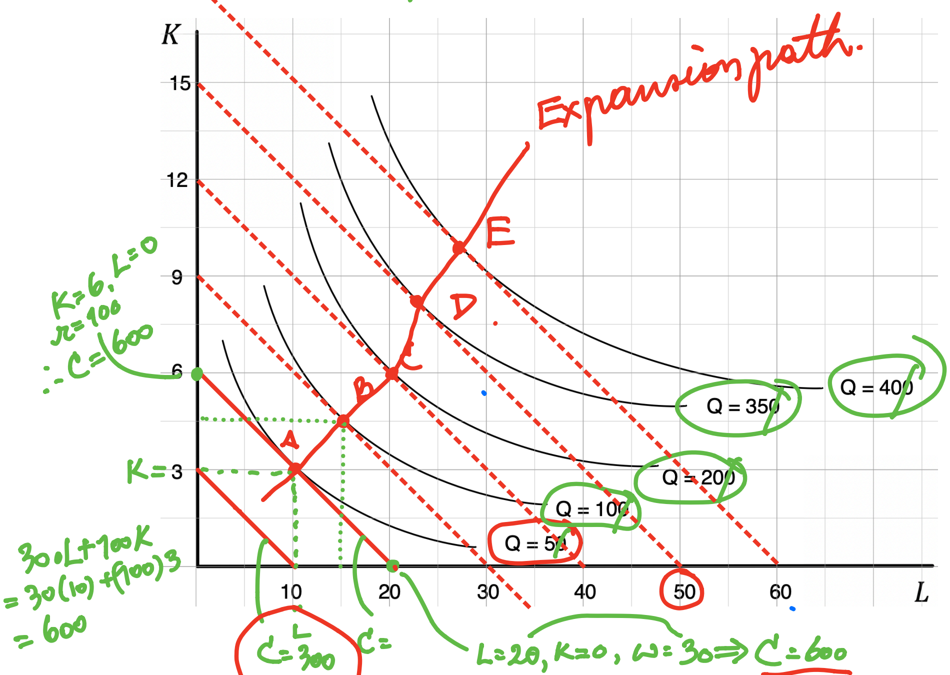
Production Expansion Path: a line whose every point is a (least cost) production equilibrium for a given output level at given fixed input prices of w and r.

Suppose $w = 30, r = 100$, the Isocost is given by — Slope of Isocost $= -\frac{w}{r} = -\frac{30}{100}$

$$\boxed{30L + 100K = C}$$

Let $C = 300$ $30L + 100K = 300$
$L = 0,\ K = 3$
$K = 0,\ L = 10.$

- We have the following equilibria at various levels of output $Q = 50, 100, 200, 350, 450, etc.$ — cost minimization



(LRTC.) Long Run Total Cost

	Q	L	K	Cost
A	50	10	3	600
B	100	15	4.5	900
C	200	20	6	1200
D	350	24	8	1500
E	400	26	11	1800

least cost.

given Q what is the least cost

given C what is the max Q.

- An alternative definition of Expansion Path: a line a line whose every point is a (maximum output) production equilibrium for a given cost level at given fixed input prices of w and r.

Expansion Path and Long-Run Total Cost

Long-Run Total Cost (LRTC) the least cost of producing a given output Q by the use of inputs (labor L and capital K) in the Long-Run.

- with fixed input prices (wage w and interest r)
- in the most efficient way (no unnecessary wastes) — minimized cost.
- with the available best technology

From the Expansion Path above, we have the following relationship between the output and the least cost in the Long Run,

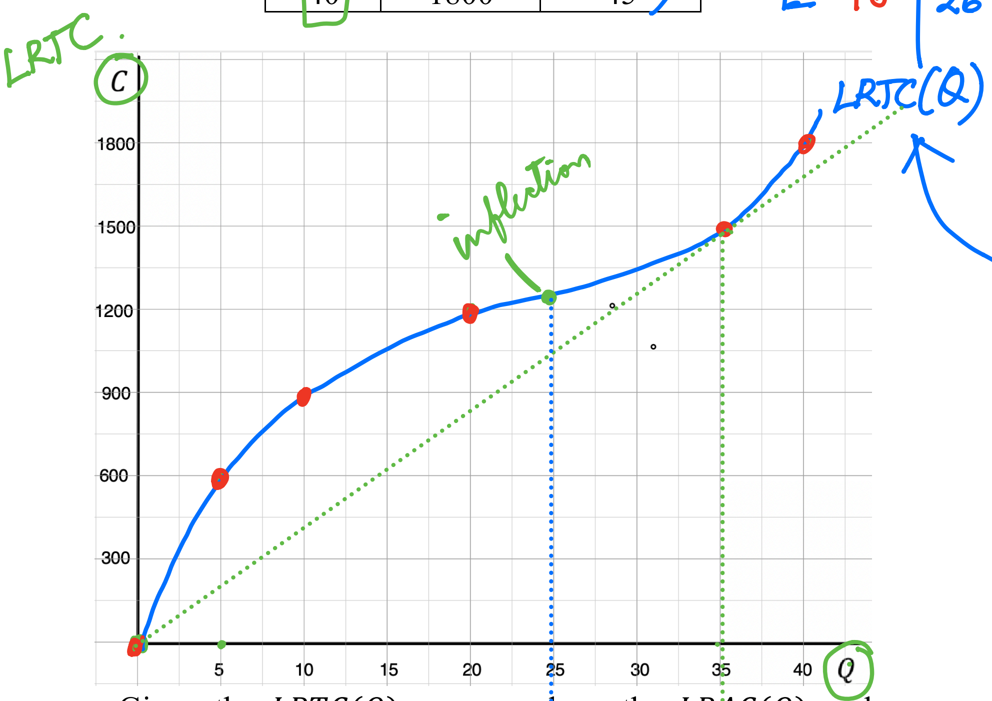
$LRAC(Q) = \frac{LRTC(Q)}{Q}$

Q	$LRTC(Q)$	$LRAC(Q)$
0	0	—
5	600	120 ✓
10	900	90 ✓
20	1200	60 ✓
35	1500	42.9 ✓
40	1800	45 ✓

	Q	L	K	Cost.
A	5	10	3	600
B	10	15	4.5	900
C	20	20	6	1200
D	35	24	8	1500
E	40	26	11	1800

least cost

LRTC.

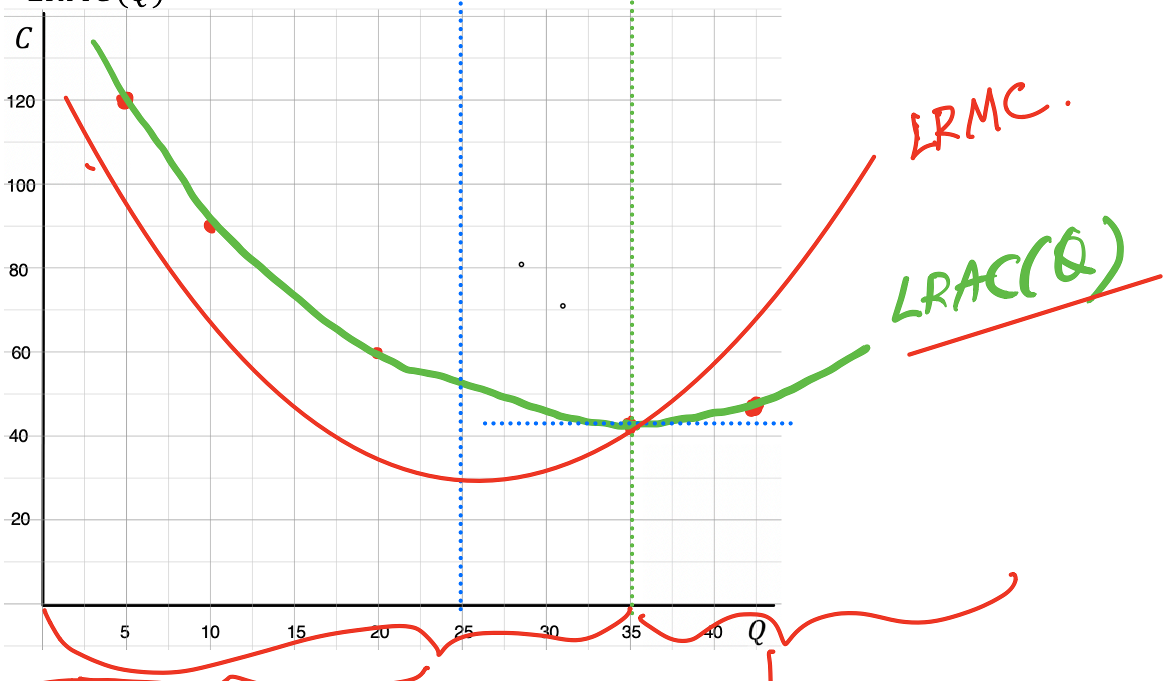


$Q=0$, $LRTC=0$.
There is no fixed cost.

similar to $TVC(Q)$ in short run because no fixed cost in LR.

$TC(Q) \to SRTC(Q)$
$MC(Q) \to SRMC(Q)$
$AC(Q) \to SRAC(Q)$
$AVC(Q) \to SRAVC(Q)$

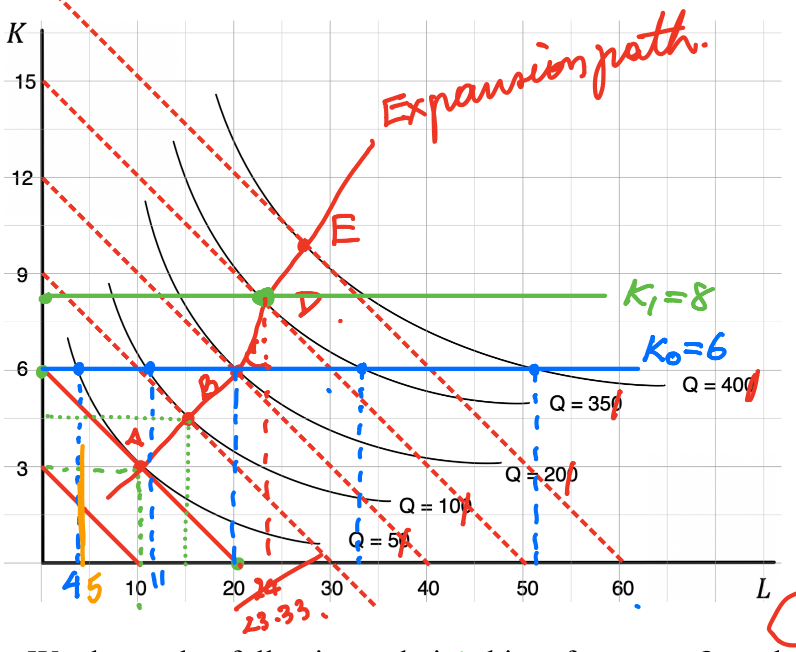
- Given the $LRTC(Q)$, we can draw the $LRAC(Q)$ and $LRMC(Q)$



- Economies of scale--$LRAC$ is decreasing
- Diseconomies of scale--$LRAC$ is increasing

Relationship between $LRTC$ and $SRTC$

- Given the fixed factor is capital $\boxed{K \text{ at } K_0 = 6,}$



	Q	L	K	Cost
A	5	10	3	600
B	10	15	4.5	900
C	20	20	6	1200
D	35	~~24~~ 23.33	8	1500
E	40	26	11	1800

least cost

- We have the following relationship of output Q and $SRTC(Q)$

$w = 30$
$r = 100$

330

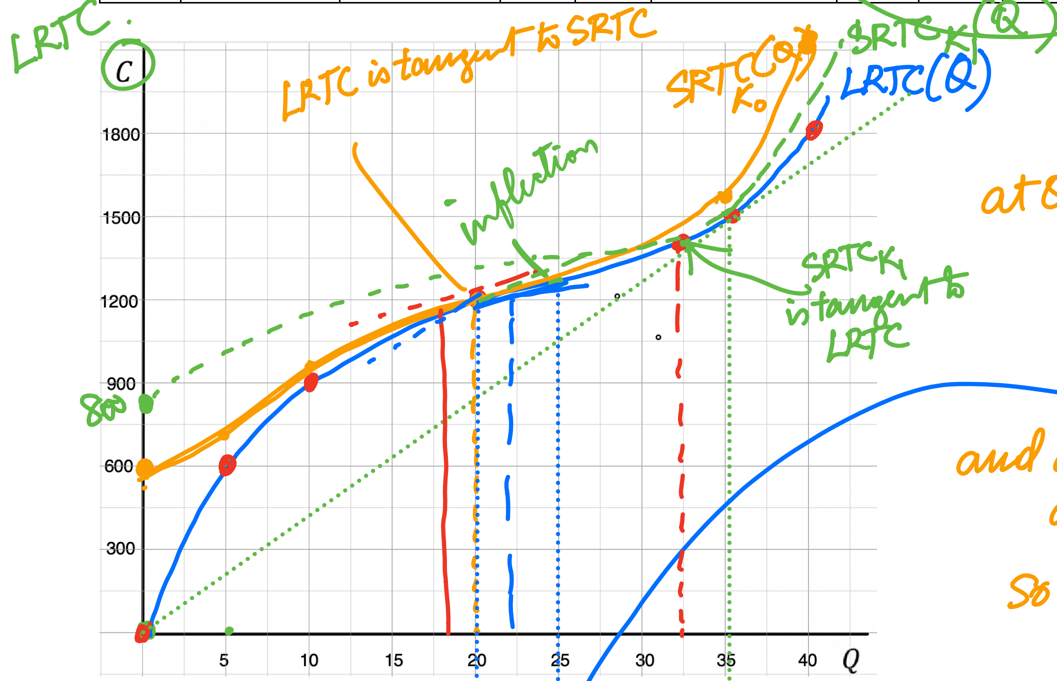
Q	$LRTC(Q)$	$LRAC(Q)$	SR with $K_0 = 6$			SR with $K_1 = 8$		
			K 100	L 30	$SRTC_{K_0}(Q)$	K	L	$SRTC_{K_1}(Q)$
0	0		6	0	600	8		
5	600	120	6	4	720	8		
10	900	90	6	11	930	8		
20	1200	60	6	20	1,200	8		
35	1500	42.9	6	33	1590	8	23.33	1500
40	1800	45	6	51	2,130	8		

1530

$SRTC(Q) \geq LRTC(Q)$ for any Q because in LR you can choose any level of L+K while in SR you are stuck with K_0.

$SRTC(Q_0) = LRTC(Q_0)$ at Q_0 where in LR you produce Q_0 by using $K = K_0 = 6$.

Q	$LRTC(Q)$	$LRAC(Q)$	SR with $K_0 = 6$			SR with $K_1 = 8$		
			K	L	$SRTC_{K_0}(Q)$	K	L	$SRTC_{K_1}(Q)$
0	0		6	0	600			
5	600	120	6	4	720			
10	900	90	6	11	930			
20	1200	60	6	20	1,200			
35	1500	42.9	6	39	1590			
40	1800	45	6	57	2,130			



at Q = 20.

$\frac{LRTC(20)}{20} = \frac{SRTC_{K_0}(20)}{20}$

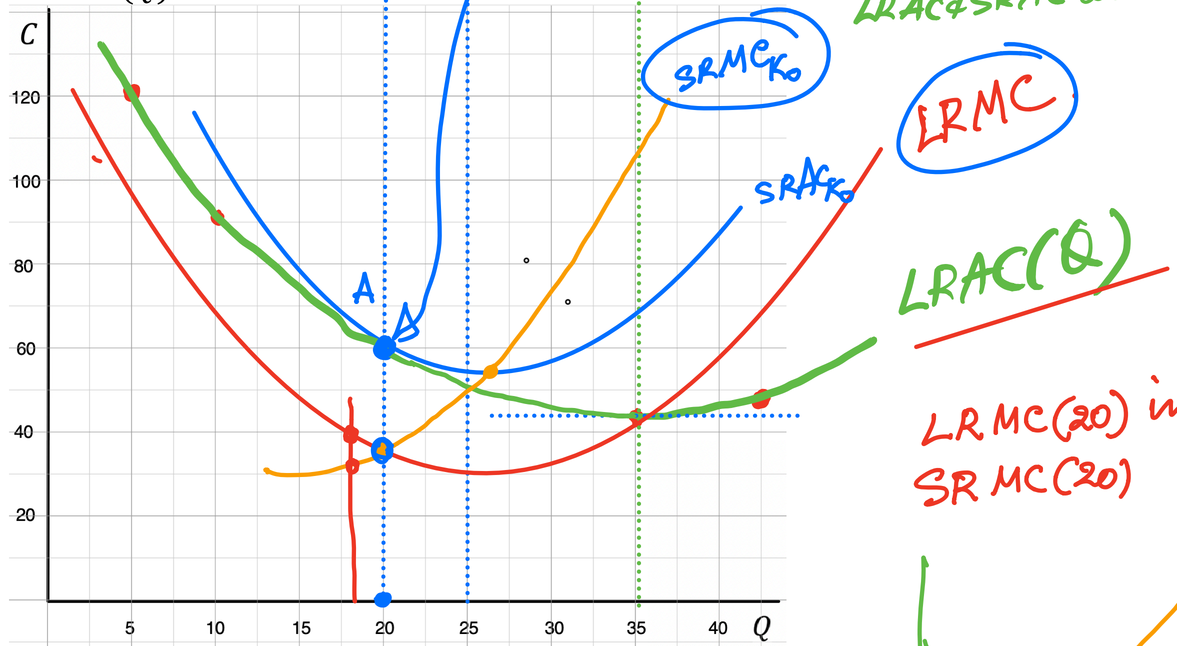
$LRAC(20) = SRAC_{K_0}(20)$

and at Q = 20, LRTC & $SRTC_{K_0}$ are tangent.

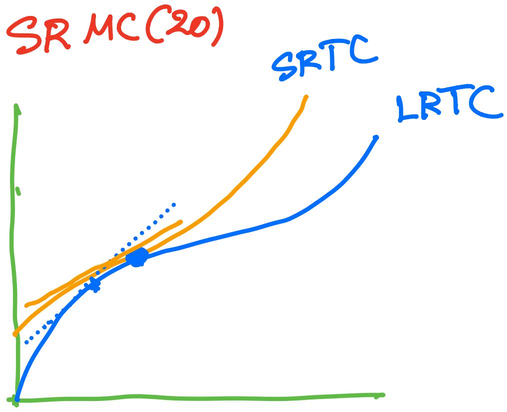
So $LRMC(20) = SRMC_{K_0}(20)$

• Given the $SRTC(Q)$, we can draw the $SRAC(Q)$ and $SRMC(Q)$

If LRTC + SRTC are tangent LRAC & SRAC are also tangent.



LRMC(20) intersects SR MC(20)



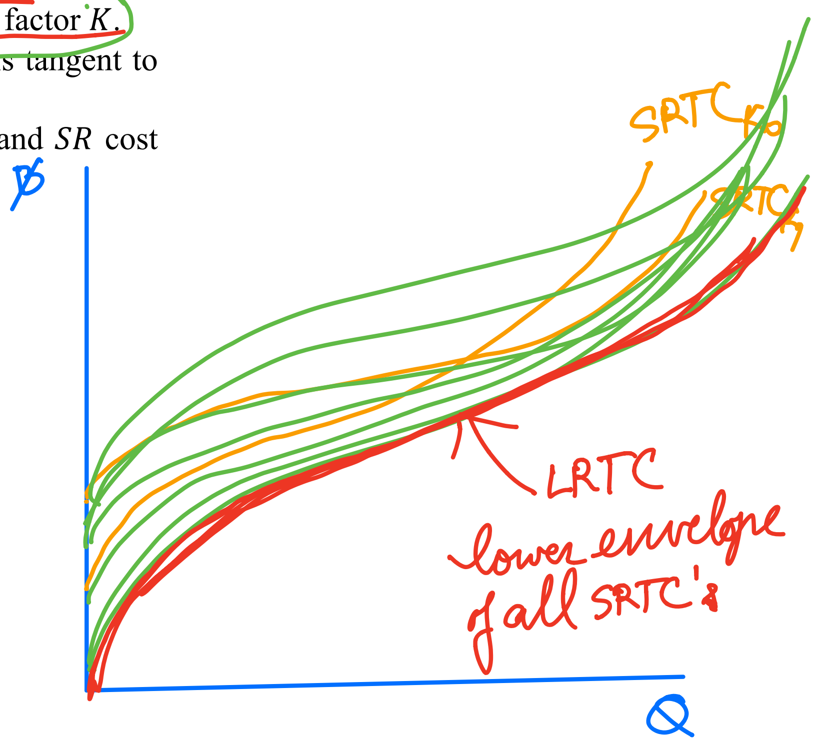
- When $K_0 = 6$, $SRTC_{K_0}(Q) \geq LRTC(Q)$ at all output Q, and $Q = 20$ where $SRTC_{K_0}(20) = LRTC(20)$.
- Thus, $SRTC_{K_0}$ and $LRTC$ are tangent at $Q = 20$

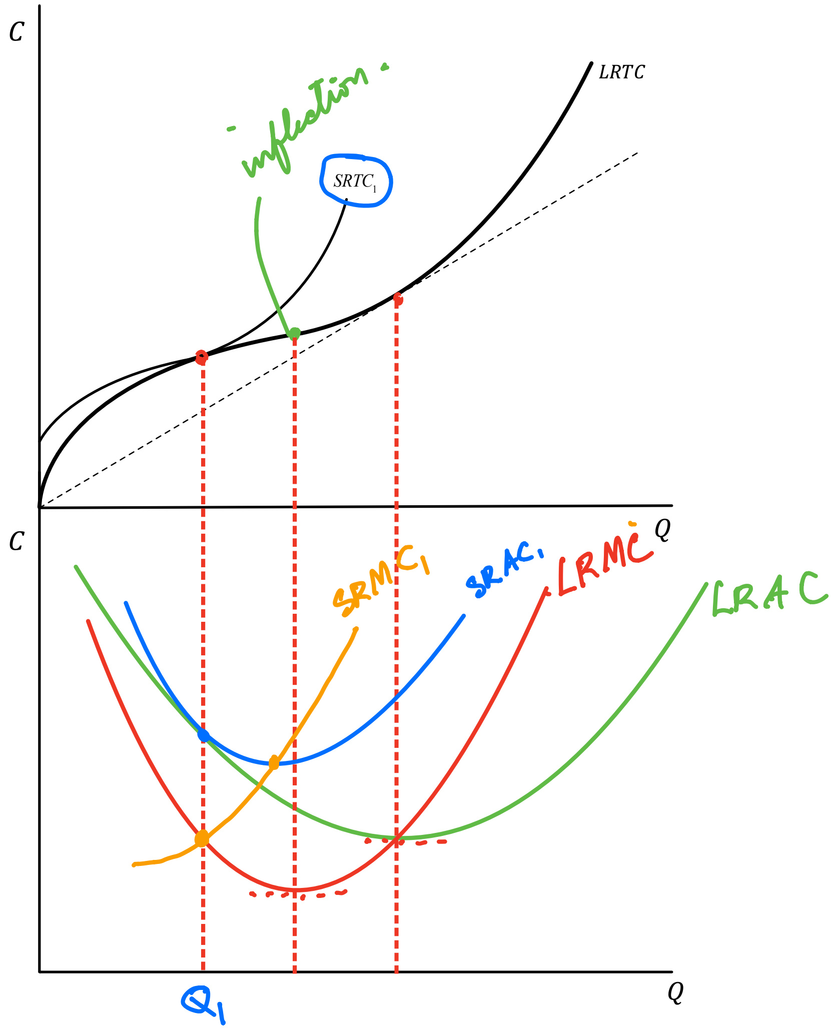
$$\Longrightarrow \begin{array}{ll} 1) & SRMC_{K_0}(20) = LRMC(20) \\ 2) & SRAC_{K_0}(Q) \geq LRAC(Q) \\ 3) & SRAC_{K_0}(20) = LRAC(20) \end{array}$$

- When $K_1 = 8$, $SRTC_{K_1}(Q) \geq LRTC(Q)$ at all output Q, and $Q = 35$ where $SRTC_{K_1}(35) = LRTC(35)$.
- Thus, $SRTC_{K_1}$ and $LRTC$ are tangent at $Q = \cancel{20}$ 35

$$\Longrightarrow \begin{array}{ll} 1) & SRMC_{K_1}(35) = LRMC(35) \\ 2) & SRAC_{K_1}(Q) \geq LRAC(Q) \\ 3) & SRAC_{K_1}(35) = LRAC(35) \end{array}$$

- We can think of $LRTC$ as the lower envelope of all the possible $SRTC$, each at a given level of fixed factor K.
- At any pint on $LRTC$, there is a $SRTC$ that is tangent to that point on $lRTC$.
- We can demonstrate the relationship of LR and SR cost curves in the following three cases.



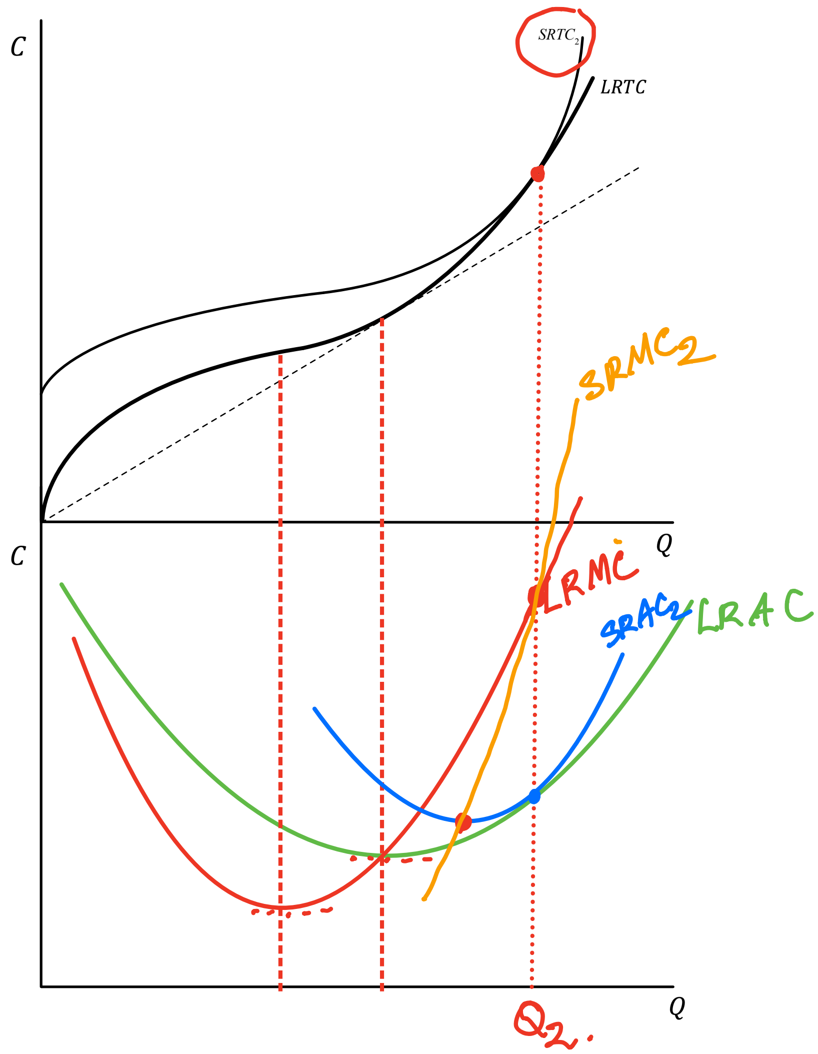


tangent.

at Q_1, $SRAC_1(Q_1) = LRAC(Q_1)$

$SRMC_1(Q_1) = LRMC(Q_1)$

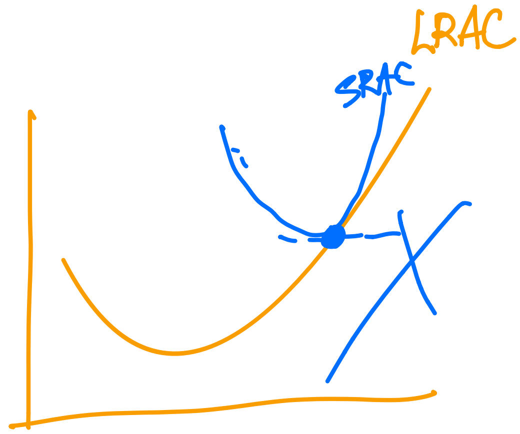
intersection.

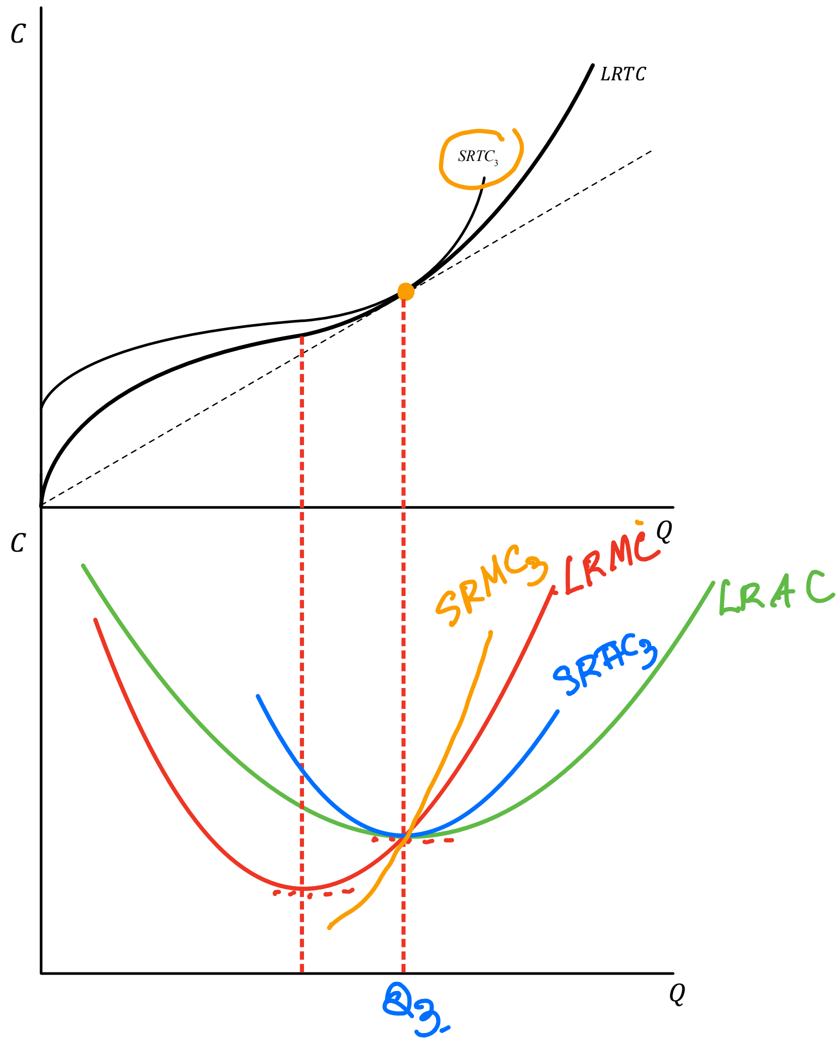


At Q_2. $SRTC_2(Q_2) = LRTC(Q_2)$

$\therefore$ $SRAC_2(Q_2) = LRAC(Q_2)$

$SRMC_2(Q_2) = LRMC(Q_2)$

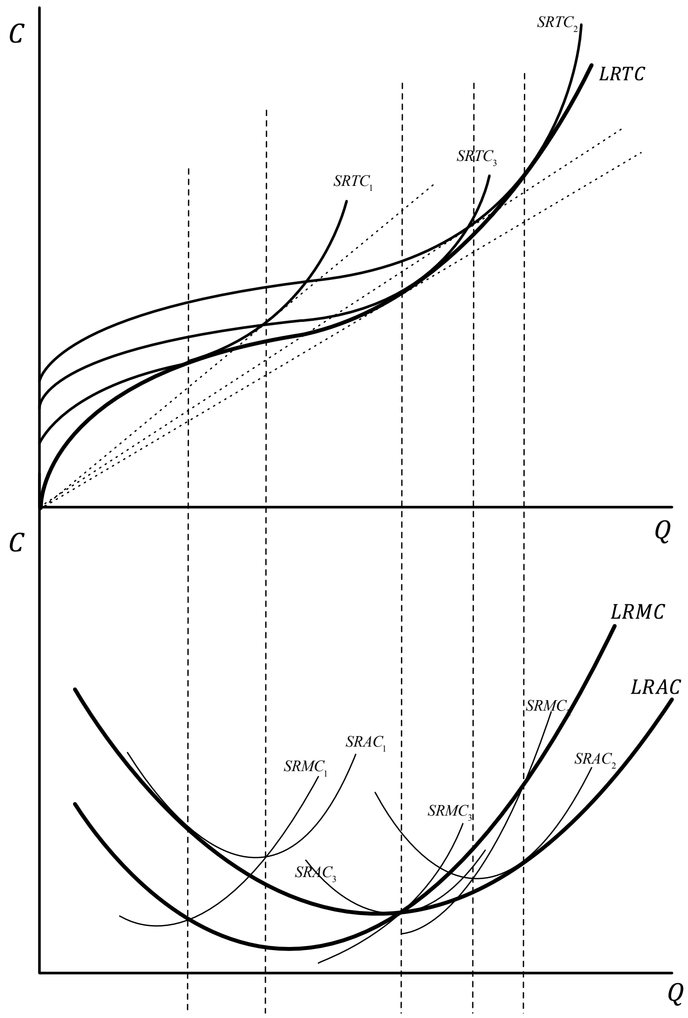


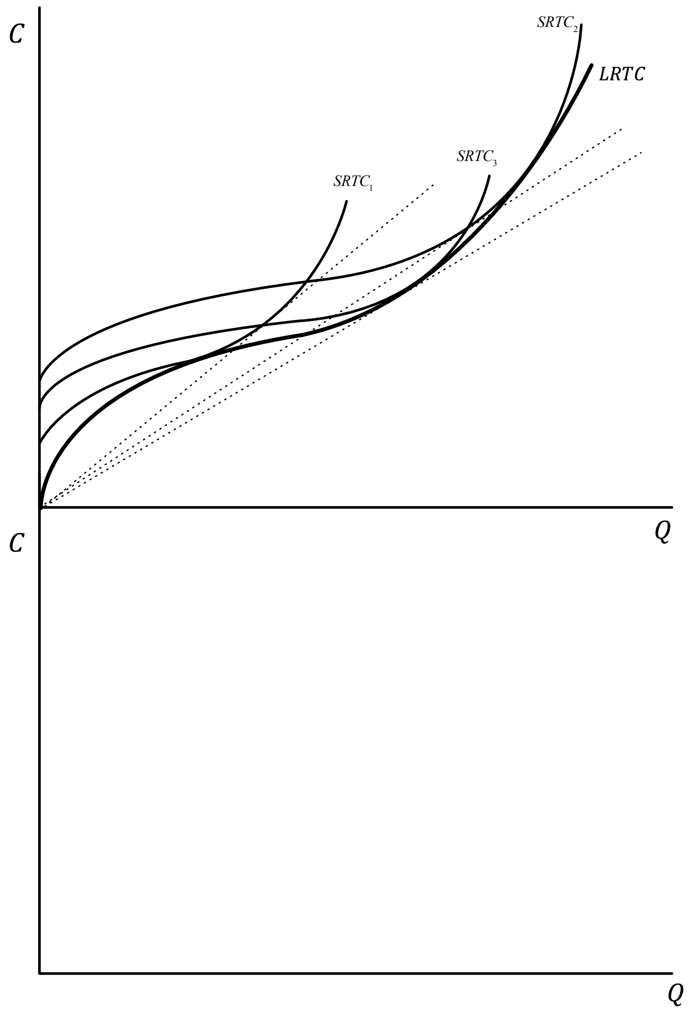


at Q_3, $SRTC_3(Q_3) = LRTC(Q_3)$

$\therefore$ $SRAC_3(Q_3) = LRAC(Q_3)$

$SRMC_3(Q_3) = LRMC(Q_3)$


C
SRTC2
LRTC
SRTC3
SRTC1
Q
C
LRMC
LRAC
SRMC
SRAC1
SRMC1
SRAC2
SRMC3
SRAC3
Q


C
$SRTC_2$
$LRTC$
$SRTC_3$
$SRTC_1$
Q
C
Q