B.E. International Program

Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics (Section 046402)

Semester 1/2013

Homework 6

Due 28 November 2013

There are four questions in total. Each of them is worth 5 points.

1. Suppose that Mr. Pai's utility depends on two commodities: x_1 and x_2. His utility function is given by

$$U = x_1 x_2 + 2x_1 + x_2.$$

Suppose that Pai's income is \$76, and the per-unit prices for the commodities x_1 and x_2 are \$2 and \$3, respectively.

(a) (*3 points*) Determine the values x_1^* and x_2^* that maximize Pai's utility, given that Pai spent all of his income on these two commodities.

(b) (2 *points*) Show the second-order sufficient condition for the constrained utility maximization.

2. Suppose that a monopolistic firm sells a single product in three separate markets (say, three different countries), where the firm can set different prices and reselling the products in different markets is not possible. The inverse demand functions in the three markets are:

$$P_1 = a_1 - b_1 Q_1$$
$$P_2 = a_2 - b_2 Q_2$$
$$P_3 = a_3 - b_3 Q_3$$

where a_i and b_i, $i = 1, 2, 3$, are positive constants. The total cost function is given by: $TC = C_0 + cQ$, where $Q = Q_1 + Q_2 + Q_3$. C_0 is the total fixed cost, and c is the variable cost per unit.

(a) (*3 points*) Suppose that there is a shortage of a key raw material so that the total production must be restricted to $\bar{Q}$ units. Find the profit-maximizing output levels of Q_1, Q_2, and Q_3, and determine the maximum profit.

(b) *(2 points*) Write down the bordered Hessian matrix, and state the second-order sufficient conditions for the profit maximization problem.

3. Given the production function

$$Q = f(K, L) = K^{0.5}L^{0.5}$$

Suppose that the per unit input prices for K and L are \$20 and \$5, respectively.

(a) (*3 points*) If the producer needs to maintain the output level at $\bar{Q} = 20$ units, what are the values K^* and L^* that minimizes the total cost, and what is the corresponding minimum cost? Verify that the second-order sufficient condition is met.

(b) (*2 points*) Suppose now that the output level is not fixed, but the producer has a budget constraint at \$400. Assume that this producer spends his entire budget on the production. Verify that the values K^* and L^* that maximizes the total output are the same as the values found in part (a).

4. Integration

(a) (*2.5 points*) Suppose that the marginal cost function is given by $C'(Q) = ae^{bQ}$, where $a > 0$ and $0 < b < 1$. Given that the total fixed cost is C_0, find the total cost function, $C(Q)$.

(b) (*2.5 points*) The demand function is given by $P = 100 - 0.5Q$. Calculate the consumer surplus when P = 70.