Chapter 11
Monopoly

Solutions to Problems

1. The marginal revenue for a perfectly competitive firm is equal to the market price. Why is the marginal revenue for a monopolist less than the market price for positive quantities of output?

Marginal revenue is less than price for a monopolist. This is because as it lowers its price two things happen. First, the firm's revenue increases from the additional units it sells (these are the marginal units). Second, the firm's revenue decreases because it loses revenue from selling units at a lower price than it could have had it chosen a lower quantity of output (these are the inframarginal units.) The change in revenue is the sum of the increase from the marginal units and the decrease from the inframarginal units. This change can be summarized as

$$MR = \frac{\Delta TR}{\Delta Q} = P + Q\frac{\Delta P}{\Delta Q}$$

Since demand is downward sloping, the second term will be negative implying marginal revenue will be less than price.

10. Suppose that Intel has a monopoly in the market for microprocessors in Brazil. During the year 2005, it faces a market demand curve given by *P* = 9 - *Q*, where *Q* is millions of microprocessors sold per year. Suppose you know nothing about Intel's costs of production. Assuming that Intel acts as a profit-maximizing monopolist, would it ever sell 7 million microprocessors in Brazil in 2005?

If demand is $P = 9 - Q$, then $MR = 9 - 2Q$. If the firm sets $Q = 7$, then $MR = -5$. At this point, if the firm lowered its output it would increase total revenue, and with the lower level of output total cost would fall. Thus, decreasing output would increase profit. Therefore, a profit-maximizing monopolist facing this demand curve would never choose $Q = 7$.

14. Suppose a monopolist has a constant marginal cost MC = £50 and faces a demand curve *P* = 100 – *Q*/2 (which can be rewritten as *Q* = 200 – 2*P*).

a) **Find the profit-maximizing price and quantity for the monopolist using the monopoly midpoint rule.**
b) **Find the profit-maximizing price and quantity for the monopolist by equating *MR* to *MC*.**

a) The monopoly midpoint rule says that we need to compare the vertical intercept of demand to the intercept of marginal cost. Here, we have a vertical intercept of 100 for demand from the expression of inverse demand, P = 100 – Q/2 and we have a vertical intercept of 50 for marginal cost, as it is constant at 50. Hence, we have price equal to the midpoint between 50 and 100, which equals 75. Substituting this into the expression for inverse demand, we have 75 = 100 – Q/2 or Q = 50.
b) From inverse demand, we can see that the marginal revenue will have twice the slope so that MR = 100 – Q. Equating this to marginal cost we have 100 – Q = 50 or Q = 50. Substituting into the expression for inverse demand we have, then, P = 100 – 50/2 = 75. This answer squares with that of the monopoly midpoint rule in (a).

16 A monopolist serves a market in which the demand is $P = 120 - 2Q$. It has a fixed cost of 300. Its marginal cost is 10 for the first 15 units ($MC = 10$ when $0 \leq Q \leq 15$). If it wants to produce more than 15 units, it must pay overtime wages to its workers, and its marginal cost is then 20. What is the maximum amount of profit the firm can earn?

The marginal revenue curve is $MR = 120 - 4Q$. Initially we are not sure whether the optimal quantity will be less than 15 units (in which case $MC = 10$), or more than 20 units (where $MC = 20$).

There are two regions of output:
Region I: where $MC = 10$ and $0 \leq Q \leq 15$
Region II: where $MC = 20$ and $15 < Q$

Let's assume that the $MC = 10$ and optimal quantity is less than or equal to 15 units. In that case, setting $MR = MC$, we find that $120 - 4Q = 10$, or that $Q = 27.5$. But when $Q = 27.5$, MC is not 10, so the assumption that the optimal quantity is in Region I is not correct.

Now let's assume that the $MC = 20$ and optimal quantity is greater than 15 units. In that case, setting $MR = MC$, we find that $120 - 4Q = 20$, or that $Q = 25$. When $Q = 25$, MC is 20, so that marginal cost we have assumed is correct at the optimal output level we have calculated.
The market price is $P = 120 - 2(25) = 70$.
Revenue = $PQ = 70(25) = 1750$
Variable cost = 10(15) + 20 (25 – 15) = 350
Fixed Cost = 300
Profit = 1750 – 350 – 300 = 1100.

18. Suppose a monopolist has an inverse demand function given by $P = 100Q^{-1/2}$. What is the monopolist's optimal markup of price above marginal cost?

Remember that the demand elasticity in a constant elasticity demand function is the exponent on P when the demand function is written in the regular form, i.e. $Q = f(P)$. We can manipulate the inverse demand function to get the regular demand function,

$Q = 10{,}000P^{-2}$. This implies that the demand elasticity is –2. Therefore, using the IEPR, $\frac{P - MC}{P} = \frac{1}{2}$. So the optimal percentage mark-up of price over marginal cost is ½, or 50 percent.

19. The marginal cost of preparing a large latte in a specialty coffee house is $1. The firm's market research reveals that the elasticity of demand for its large lattes is constant, with a value of about -1.3. If the firm wants to maximize profit from the sale of large lattes, about what price should the firm charge?

Since the elasticity of demand is constant, we can use the inverse elasticity rule for a monopolist.
$[P - MC]/P = -1/e_{Q,P}$. The inverse elasticity rule then becomes $[P - 1]/P = -1/1.3$. Thus we would expect to see the firm charge about $4.33 for a large latté.

20. Imagine that Gillette has a monopoly in the market for razor blades in Mexico. The market demand curve for blades in Mexico is $P = 968 - 20Q$, where P is the price of blades in cents and Q is annual demand for blades expressed in millions. Gillette has two plants in which it can produce blades for the Mexican market: one in Los Angeles and one in Mexico City. In its L.A. plant, Gillette can produce any quantity of blades it wants at a marginal cost of 8 cents per blade. Letting Q_1 and MC_1 denote the output and marginal cost at the L.A. plant, we have $MC_1(Q_1) = 8$. The Mexican plant has a marginal cost function given by $MC_2(Q_2) = 1 + 0.5Q_2$.
a) Find Gillette's profit-maximizing price and quantity of output for the Mexican market overall. How will Gillette allocate production between its Mexican plant and its U.S. plant?

a) Profit-maximizing firms generally allocate output among plants so as to keep marginal costs equal. But notice that $MC_2 < MC_1$ whenever $1 + 0.5Q_2 < 8$, or $Q_2 < 14$. So for small levels of output, specifically $Q < 14$, Gillette will only use the first plant. For $Q > 14$, the cost-minimizing approach will set $Q_2 = 14$ and $Q_1 = Q - 14$.
Suppose the monopolist's profit-maximizing quantity is $Q > 14$. Then the relevant MC = 8, and with $MR = 968 - 40Q$ we have

$$968 - 40Q = 8$$
$$Q = 24$$

Since we have found that $Q > 14$, we know this approach is valid. (You should verify that had we supposed the optimal output was $Q < 14$ and set $MR = MC_2 = 1 + 0.5Q$, we would have found $Q > 14$. So this approach would be invalid.) The allocation between plants will be $Q_2 = 14$ and $Q_1 = 10$. With a total quantity $Q = 24$, the firm will charge a price of $P = 968 - 20(24) = 488$. Therefore the price will be $4.88 per blade.

21. Market demand is $P = 64 - (Q/7)$. A multiplant monopolist operates three plants, with marginal cost functions:

$$MC_1(Q_1) = 4Q_1$$
$$MC_2(Q_2) = 2 + 2Q_2$$
$$MC_3(Q_3) = 6 + Q_3$$

a) Find the monopolist's profit-maximizing price and output at each plant.
b) How would your answer to part (a) change if $MC_2(Q_2) = 4$?

a) Equating the marginal costs at MC_T, we have $Q = Q_1 + Q_2 + Q_3 = 0.25MC_T + 0.5MC_T - 1 + MC_T - 6$, which can be rearranged as $MC_T = (4/7)Q + 4$. Setting $MR = MC$ yields

$$64 - (2/7)*Q = (4/7)*Q + 4$$

or $Q = 70$ and $P = 54$. At this output level, $MC_T = 44$, implying that $Q_1 = 11$, $Q_2 = 21$, and $Q_3 = 38$.

b) In this case, using plant 3 is inefficient because its marginal cost is *always* higher than that of plant 2. Hence, the firm will use only plants 1 and 2. Moreover, the firm will not use plant 1 once its marginal cost rises to $MC_2 = 4$, so we can immediately see that it will only produce $4Q_1 = 4$ or $Q_1 = 1$ unit at plant 1. Its total production can be found by setting $MR = MC_2$, yielding

$$64 - (2/7)*Q = 4$$

or $Q = 210$ and $P = 34$. So it produces $Q_1 = 1$ unit in plant 1 and $Q_2 = 209$ units in plant 2, while producing no units in plant 3 (i.e. $Q_3 = 0$).