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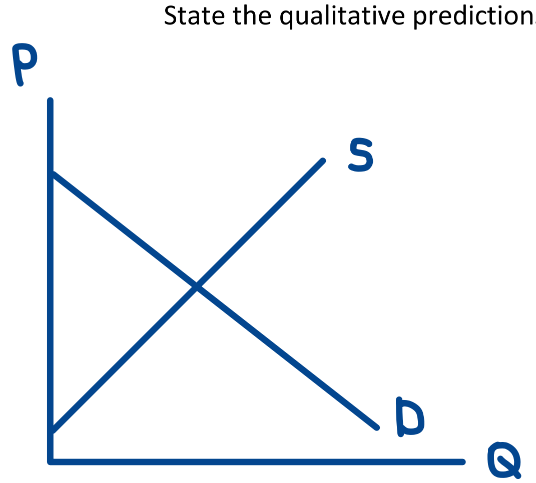
EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If “a” increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1)



$P = 10 - Q^2$

$Q = -14 + P$ $\quad$ $Q + 14 = P$

1.2) $10 - Q^2 = Q + 14$

$-Q^2 - Q - 4 = 0$

$\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-1 \pm \sqrt{1-16}}{-2} = Q^*$

$P = 10 - \left[\dfrac{-1 \pm \sqrt{1-16}}{-2}\right] = P^*$

1.3) I think it both will increase.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$= \frac{d}{dQ}\left(\ln(Q^2+1)\right) + \frac{d}{dQ}\left(\frac{3Q}{Q+1}\right)$$

$$= \left(\frac{1}{Q^2+1} \times 2Q\right) + \frac{\overset{3(Q+1)}{3Q+3} - 3Q}{(Q+1)^2}$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

It is increasing function.

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\frac{d}{d(Q)}\left(-\frac{1}{3}Q^2 - Q^2 + 8Q - 1\right)$$

$$= -\cancel{3} \times \frac{1}{\cancel{3}} Q^2 - 2Q + 8$$

$$= -Q^2 - 2Q + 8 \rightarrow \text{first deivative}$$

$$\frac{d}{d(Q)}\left(-Q^2 - 2Q + 8\right)$$

$$= -2Q - 2 \rightarrow \text{Second derivative}$$

4. Suppose that $\overset{2\times 2}{A} = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $\overset{2\times 3}{B} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B   cannot do

4.2 A*B

$$\begin{bmatrix} (8\times1+9\times4) & (8\times2+9\times5) & (8\times3+9\times6) \\ (10\times1+11\times4) & (10\times2+11\times5) & (10\times3+11\times6) \end{bmatrix} = \overset{2\times 3}{\begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}}$$

4.3 det(A) $8\times11 - 9\times10$

$= 88-90$

$= -2$

4.4 det(B)

cannot do (square matrices)

4.5 det(C)

$$\left|\begin{array}{ccc|cc} 1 & 2 & 3 & 1 & 2 \\ 4 & 5 & 6 & 4 & 5 \\ 7 & 8 & 9 & 7 & 8 \end{array}\right.$$

$= (1\times5\times9) + (2\times6\times7) + (3\times4\times8) - (7\times5\times3) + (8\times6\times1) + (9\times4\times2)$

$= 45 + 84 + 96 - \underline{105 + 48 + 72}$

$= 0$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial u}{\partial x} = a x^{a-1} y^b + \frac{\cancel{x+y}}{x}\left(\frac{(\cancel{x}+y)-\cancel{x}}{\underset{x+y}{\cancel{(x+y)^2}}}\right)$$

$$= a x^{a-1} y^b + \frac{y}{x(x+y)}$$

$$\frac{\partial u}{\partial y} = x^a b y^{b-1} + \frac{\cancel{x+y}}{x}\left(-x\left(\frac{1}{\cancel{x+y}^2}\right)\right)$$

$$= x^a b y^{b-1} - \frac{x}{x(x+y)}$$