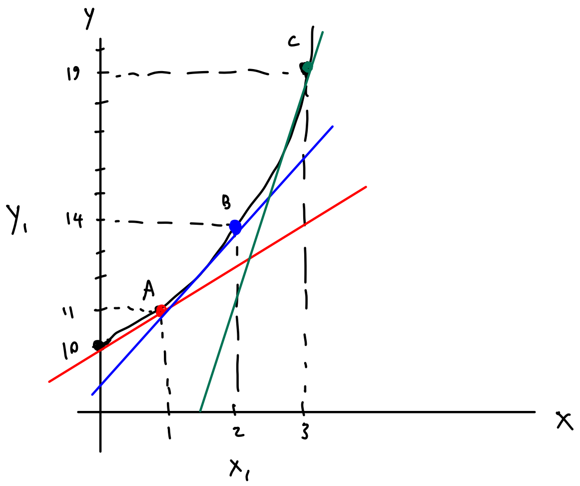
Approximation of change of y as a result of change of x

at B, slope = 4



- When $x_1 = 2, y_1 = 14$. If $\Delta x = 0.1$, we can approximate

$$\begin{aligned}\Delta y &\approx f'(x_1).\Delta x\\ &= f'(2).0.1\\ &= 2(2)\cdot 0.1 = 0.4\end{aligned}$$

- What is the real Δy?

$x_1 = 2$
$\Delta x = 0.1$
$x_2 = 2.1$

$$y_2 = f(2.1) = 10 + (2.1)^2 = 14.41$$
$$\Delta y = y_2 - y_1 = 14.41 - 14 = 0.41$$

- We underestimate the real change of y.
- What if $\Delta x = -0.2$? Approximate the change of y.

$\Delta y \approx f'(x_1)\, \Delta x$

60 km/hr

$\Delta x = 1$ min $= \frac{1}{60}$ hr

$\Delta y \approx 60$ km/hr $\cdot \frac{1}{60}$ hr

$= 1$ km

when x & y have linear relationship

$y = 10 + 2x \Rightarrow \frac{dy}{dx} = 2$ — constant

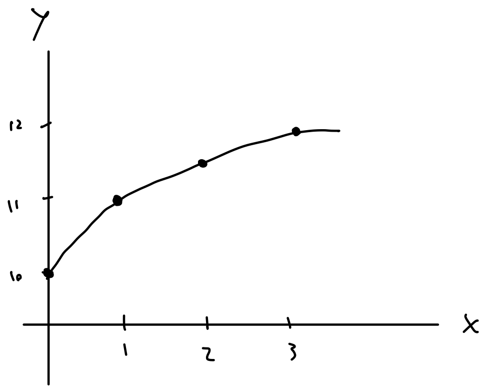
$y = 10 - 3x \Rightarrow \frac{dy}{dx} = -3$ — constant

HW Given $y = 10 + \sqrt{x}$,

a) Find the derivative $f'(x)$. a. $f'(x) = \frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

b) Fill in the table

Point	X	Y	$f'(x)$
	0	10	
A	1	11	$\frac{1}{2} = 0.5$
B	2	11.414	0.3535
C	3	11.732	0.2887



c) Does the slope increase as x increases? c. NO

d) Approximate the change in Y when $\Delta x = 0.2$ at $x_1 = 3$. Is the approximation under- or over-estimate?

d. $\Delta y \approx f'(x_1)\cdot \Delta x$
$= f'(3)\cdot 0.2$
$= 0.2887 \times 0.2 = 0.05774$

$\therefore$ over estimate

real Δy

$y_2 = f(3.2) = 10 + \sqrt{3.2} = 11.7889$

$\Delta y = y_2 - y_1 = 11.7889 - 11.732 = 0.0565$

Note: If the function $f(x)$ is linear, the approximation is exact.

$y = 10 + 2x$ — slope $= \frac{dy}{dx} = f'(x) = 2$

at $x_1 = 2$ $\Delta x = 0.1$

$\Delta y = f'(x)\cdot \Delta x$

$= 2 \cdot 0.1 = 0.2$