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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^{5}(a+bx_i)$ $= \sum_{i=1}^{5} a + \sum_{i=1}^{5} bx_i$

$= 5a + b\sum_{i=1}^{5} x_i$

$= 5a + b(x_1+x_2+x_3+x_4+x_5)$

b. $\sum_{y=0}^{5} f(x+y)$ $= f(x+0)+f(x+1)+f(x+2)+f(x+3)+f(x+4)+f(x+5)$

c. $\sum_{i=1}^{10} i^2$ $= 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$

$= 385$ #

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y)$ $= \sum_{x=1}^{2}\left(\sum_{y=2}^{3}(2x+y)\right) = \sum_{x=1}^{2}\left((2x+2)+(2x+3)\right)$

$= \sum_{x=1}^{2}(4x+5) = (4(1)+5)+(4(2)+5) = 22$ #

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$\sum_x f(x) = 0.5b + b + 2.25b + 2b + 1.5b + 0.5b + 0.25b$

$= 8b$

$1 = 8b$ $b = \frac{1}{8}$ #

b. Find the answer for $P(X \le 2)$

$P(X \le 2) = P(-2) + P(-1) + P(0) + P(1) + P(2)$

$= 0.5b + b + 2.25b + 2b + 1.5b$

$= 7.25b = 7.25(\frac{1}{8}) = 0.90625$ #

c. Find the answer for $P(-2 \le X \le 3)$

$P(-2 \le X \le 3) = P(-2) + P(-1) + P(0) + P(1) + P(2) + P(3)$

$= 0.5b + b + 2.25b + 2b + 1.5b + 0.5b$

$= 7.75b = 7.75(\frac{1}{8}) = 0.96875$ #

d. Find the answer for $P(X \ge 1)$

$P(X \ge 1) = P(1) + P(2) + P(3) + P(4)$

$= 2b + 1.5b + 0.5b + 0.25b$

$= 4.25b = 4.25(\frac{1}{8}) = 0.53125$ #

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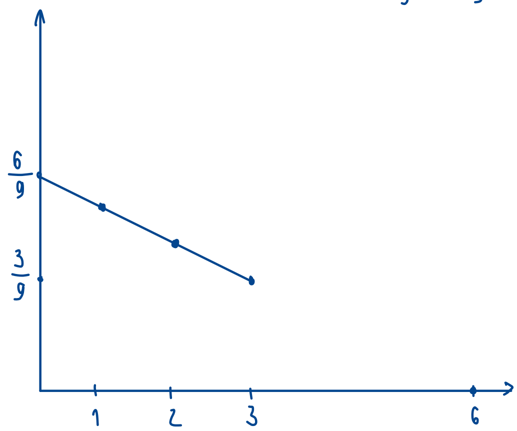
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$

$f(x) = \frac{1}{9}x + \frac{6}{9}$



$f(0) = -\frac{1}{9}(0) + \frac{6}{9} = \frac{6}{9}$

$f(1) = -\frac{1}{9}(1) + \frac{6}{9} = \frac{5}{9}$

$f(2) = -\frac{1}{9}(2) + \frac{6}{9} = \frac{4}{9}$

$f(3) = -\frac{1}{9}(3) + \frac{6}{9} = \frac{3}{9}$

b. Find the answer for $P(1 \le X \le 3)$

$\int_1^3 -\frac{1}{9}x + \frac{6}{9}\,dx = \left(-\frac{1}{9}x^2\right)\frac{1}{2} + \frac{6}{9}x \Big|_1^3$

$= \left(-\frac{1}{\cancel{9}}(\cancel{3})^2\right)\frac{1}{2} + \frac{6}{\cancel{9}_3}(\cancel{3}) - \left(-\frac{1}{9}(1)^2\right)\frac{1}{2} + \frac{6}{9}(1)$

$= \left(-\frac{1}{2} + 2\right) - \left(-\frac{1}{18} + \frac{6}{9}\right) = \frac{3}{2} - \frac{11}{18} = \frac{16}{18}$ #

c. Find the answer for $P(X \ge 2)$

$\int_2^3 -\frac{1}{9}x + \frac{6}{9}\,dx = \left(-\frac{1}{9}x^2\right)\frac{1}{2} + \frac{6}{9}x \Big|_2^3$

$= \left(-\frac{1}{\cancel{9}_3}(\cancel{3})^2\right)\frac{1}{2} + \frac{6}{\cancel{9}_3}(\cancel{3}) - \left(-\frac{1}{9}(2)^2\right)\frac{1}{\cancel{2}} + \frac{6}{9}(2)$

$= \left(-\frac{1}{2} + 2\right) - \left(-\frac{2}{9} + \frac{12}{9}\right) = \frac{3}{2} - \frac{10}{9} = \frac{27-20}{18} = \frac{7}{18}$ #

d. Find the expected value of X

$E(X) = \int_0^3 x\,fx\,dx$

$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$

$= \left(\left(-\frac{1}{9}x^3\right)\frac{1}{3} + \left(\frac{6}{9}x^2\right)\frac{1}{2}\right)\Big|_0^3$

$= \left(-\frac{1}{\cancel{27}}(\cancel{3})^3 + \frac{6}{\cancel{18}_2}(3)^2\right) - \left(-\frac{1}{27}(0)^3 + \frac{6}{18}(0)^2\right)$

$= (-1 + 3) - 0$

$= 2$ #

Student Name Vipharath Charoensukanun ID 6104640559

4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

a. Construct the joint probability distribution function (PDF) table of X and Y

$y \backslash x$	1	2	3	4	5	6	$P(X=x)$
0	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{6}{12}$
1	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{1}{2}\cdot\frac{1}{6}=\frac{1}{12}$	$\frac{6}{12}$
$P(Y=y)$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	$\frac{2}{12}$	

b. Find the marginal probability distribution function (PDF) of X

$f(X=1) = \sum f(X=1, y=y) = f(X=1, y=0) + f(X=1, y=1) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

$f(X=2) = \sum f(X=2, y=y) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

$f(X=3) = \sum f(X=3, y=y) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

$f(X=4) = \sum f(X=4, y=y) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

$f(X=5) = \sum f(X=5, y=y) = \frac{1}{6}$

$f(X=6) = \sum f(X=6, y=y) = \frac{1}{6}$

c. Find the marginal probability distribution function (PDF) of Y

$f(y=0) = \sum f(X=x, y=0) = 6\left(\frac{1}{12}\right) = \frac{1}{2}$

$f(y=1) = \sum f(X=x, y=1) = 6\left(\frac{1}{12}\right) = \frac{1}{2}$

d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$P(X=x \mid y=1) = p(X=x_i \mid y=1) = \frac{f(X=x_i, y=1)}{f(y=1)}$

$p(X=1 \mid y=1) = \frac{f(X=1, y=1)}{f(y=1)} = \frac{1}{12}(2) = \frac{1}{6}$

$P(X=2 \mid y=1) = \frac{1}{6}$

$p(X=3 \mid y=1) = \frac{1}{6}$

$p(X=4 \mid y=1) = \frac{1}{6}$

$P(X=5 \mid y=1) = \frac{1}{6}$

$P(X=6 \mid y=1) = \frac{1}{6}$

e. Find the expected value of X given Y is equal to 1

$E(X \mid y=1) = \sum x f(x \mid y=1) = 1\left(\frac{f(x=1, y=1)}{f(y=1)}\right) + 2\left(\frac{f(x=2, y=1)}{f(y=1)}\right) + 3\left(\frac{f(x=3, y=1)}{f(y=1)}\right) + 4\left(\frac{f(x=4, y=1)}{f(y=1)}\right)$

$+ 5\left(\frac{f(x=5, y=1)}{f(y=1)}\right) + 6\left(\frac{f(x=6, y=1)}{f(y=1)}\right)$

$= \frac{1}{6} + \frac{2}{6} + \frac{3}{6} + \frac{4}{6} + \frac{5}{6} + \frac{6}{6} = \frac{21}{6} = 3.5$ #

f. Find the variance of X given Y is equal to 1

$Var(X \mid y=1) = \sum [X - E(X \mid y=1)]^2 \cdot f(X \mid y=1)$

$= (1-3.5)^2\left(\frac{1}{6}\right) + (2-3.5)^2\left(\frac{1}{6}\right) + (3-3.5)^2\left(\frac{1}{6}\right) + (4-3.5)^2\left(\frac{1}{6}\right) + (5-3.5)^2\left(\frac{1}{6}\right) + (6-3.5)^2\left(\frac{1}{6}\right)$

$= 6.25\left(\frac{1}{6}\right) + 2.25\left(\frac{1}{6}\right) + 0.25\left(\frac{1}{6}\right) + 0.25\left(\frac{1}{6}\right) + 2.25\left(\frac{1}{6}\right) + 6.25\left(\frac{1}{6}\right)$

$= 17.5\left(\frac{1}{6}\right) = 2.91667$ #

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\operatorname{var}(\bar{X})$

$$\begin{aligned}
E(\bar{X}) &= E\left(\frac{X_1+X_2+X_3}{3}\right) \\
&= \frac{1}{3}\left(E(X_1)+E(X_2)+E(X_3)\right) \\
&= \frac{1}{3}(\mu_x+\mu_x+\mu_x) \\
&= \frac{1}{3}\,3\mu_x \\
&= \mu_x \quad \#
\end{aligned}$$

$$\begin{aligned}
Var(\bar{X}) &= Var\left(\frac{1}{3}\sum_{i=1}^{3} X_i\right) \\
&= \left(\frac{1}{3}\right)^2 Var(X_1+X_2+X_3) \\
&= \frac{1}{9}\Big[Var(X_1)+Var(X_2)+Var(X_3)+2Cov(X_1,X_2)+ \\
&\qquad 2Cov(X_1,X_3)+2Cov(X_2,X_3)\Big] \\
&= \frac{1}{9}\left[\sigma^2+\sigma^2+\sigma^2+\cancel{2}\left(\frac{1}{\cancel{4}_2}\sigma^2\right)+\cancel{2}\left(\frac{1}{\cancel{4}_2}\sigma^2\right)+\cancel{2}\left(\frac{1}{\cancel{4}_2}\sigma^2\right)\right] \\
&= \frac{1}{9}\left[3\sigma^2+3\left(\frac{1}{2}\sigma^2\right)\right] \\
&= \frac{1}{\cancel{9}\,3}(\cancel{3})\left[\sigma^2+\frac{1}{2}\sigma^2\right] \\
&= \frac{1}{\cancel{3}}\left(\frac{\cancel{3}}{2}\sigma^2\right) = \frac{\sigma^2}{2} \quad \#
\end{aligned}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\operatorname{var}(\bar{X})$ in term of μ and σ

$$\begin{aligned}
E(\bar{X}) &= E\left(\frac{X_1+X_2+X_3+X_4}{4}\right) \\
&= \frac{1}{4}\left(E(X_1)+E(X_2)+E(X_3)+E(X_4)\right) \\
&= \frac{1}{4}(\mu_x+\mu_x+\mu_x+\mu_x) \\
&= \frac{1}{4}(4\mu_x) \\
&= \mu_x
\end{aligned}$$

$$\begin{aligned}
Var(\bar{X}) &= Var\left(\frac{1}{4}\sum_{i=1}^{4} X_i\right) \\
&= \left(\frac{1}{4}\right)^2 Var(X_1+X_2+X_3+X_4) \\
&= \frac{1}{4^2}\left(Var(X_1)+Var(X_2)+Var(X_3)+Var(X_4)\right) \\
&= \frac{1}{4^2}(\sigma_x^2+\sigma_x^2+\sigma_x^2+\sigma_x^2) \\
&= \frac{1}{4^2}4\sigma_x^2 \\
&= \frac{\sigma_x^2}{4}
\end{aligned}$$

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b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$E(\tilde{x}) = E\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4\right)$$

$$= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4)$$

$$= \frac{1}{8}\mu_x + \cancel{\frac{1}{4}}^{\frac{2}{8}}\mu_x + \frac{1}{8}\mu_x + \cancel{\frac{1}{2}}^{\frac{4}{8}}\mu_x$$

$$= \frac{8}{8}\mu_x$$

$$E(\tilde{x}) = \mu_x$$

$\tilde{x}$ is an biased estimator of μ_x.

$$Var(\tilde{x}) = Var\left(\frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right)$$

$$= \left(\frac{1}{8}\right)^2 Var(X_1) + \left(\frac{1}{4}\right)^2 Var(X_2) + \left(\frac{1}{8}\right)^2 Var(X_3) + \left(\frac{1}{2}\right)^2 Var(X_4)$$

$$= \frac{1}{64}\sigma_x^2 + \cancel{\frac{1}{16}}^{\frac{4}{64}}\sigma_x^2 + \frac{1}{64}\sigma_x^2 + \cancel{\frac{1}{4}}^{\frac{16}{64}}\sigma_x^2$$

$$= \frac{22}{64}\sigma_x^2$$

$$= \frac{11}{32}\sigma_x^2$$

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

$$Var(\bar{x}) = \frac{\sigma^2}{4} \qquad Var(\tilde{x}) = \frac{11}{32}\sigma^2$$

$$Var(\tilde{x}) > Var(\bar{x})$$

so, $\bar{x}$ is more effient estimator of μ_x than $\tilde{x}$ because it has smaller variance.