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Quiz 6: Oligopoly and Game Theory

Answer all the questions in 30 minutes. (***There are 2 pages)

1. There are only two firms in the market, deciding on the quantity produced. Let the inverse demand function of the market be:

$$P = 100 - Q,$$

where $Q = q_1 + q_2$ represents the total amount of output in the market, q_i be the quantity produced by firm *i*, and P represents the market price. Both firms have constant marginal cost $MC_1 = MC_2 = 10$.

a) Suppose the firms are competing in the market. Let q_i^C and π_i^C be the optimal quantity produced and the profit value of firm *i*, respectively. Calculate q_i^C and π_i^C.

For firm 1, the profit function is $\pi_1 = (100 - q_1 - q_2)q_1 - 10q_1$. *Then, we can figure out best response function by:*

$$\frac{\partial \pi_1}{\partial q_1} = 100 - 2q_1 - q_2 - 10 = 0$$

$$q_1^* = 45 - q_2/2.$$

This is the same for firm 2, and the optimal quantity produced is the cross between best response functions of firm 1 and firm 2. That is:

$$\begin{aligned} q_1^C &= 45 - \tfrac{1}{2}(45 - \tfrac{1}{2}q_1^C) \\ &= (4/3)(45/2) \\ &= 30. \end{aligned}$$

This is the same for firm 2. Hence, we get $q_1^C = q_2^C = 30$. *Substitute these into the profit function, we get* $\pi_1 = \pi_2 = (100 - 60)30 - 10(30) = 900.$

b) Now, suppose both firms decide to collude and agree to produce equally on the total output supplied to the market. Let q_i^M and π_i^M be the optimal quantity produced and the profit value of firm *i*, respectively, for this case. Calculate for q_i^M and π_i^M.

After the collusion, the behavior would be like a monopolist. Hence, the monopolist maximizes profit:

$$\pi = (100 - Q)Q - 10Q$$

$$\frac{\partial \pi}{\partial Q} = 100 - 2Q - 10 = 0$$

$$Q^M = 45.$$

Since the firms agree to produce equally, each produces $q_1^M = q_2^M = 22.5$ *units and gets profit of* $\pi_1^M = \pi_2^M = (100-45)(22.5)-10(22.5) = 1,012.5.$

		Firm 2	
		q_2^C	q_2^M
Firm 1	q_1^C	π_1^C , π_2^C	*1125, 843.75*
	q_1^M	*843.75, 1125*	π_1^M , π_2^M

c) Calculate the relevant profit values to fill up the above payoff matrix. After that figure out the Nash Equilibrium of the above game.

For firm 1, the profit function is $\pi_1 = (100-q_1-q_2)q_1 - 10q_1$.

- *If* $q_1 = q_1^M = 22.5$ *and* $q_2 = q_2^C = 30$, *the profit is*

$$\pi_1 = (100-22.5-30)(22.5)-10(22.5) = 843.75.$$

- *If* $q_1 = q_1^C = 30$ *and* $q_2 = q_2^M = 22.5$, *the profit is*

$$\pi_1 = (100-22.5-30)(30)-10(30) = 1,125.$$

These values are the same for firm 2. Hence, we get the payoff matrix as above.

There is only one Nash Equilibrium, which is (q_1^C, q_2^C *). For the other pair of actions, at least one firm wants to deviate out of that point. For example, at (* q_1^M, q_2^M *), both firms want to deviate to* q_i^C *because the profit of each firm is higher (1125 >* $\pi_i^M = 1,012.5$ *).*