



FOUNDATIONS OF FINANCE: EXPECTED UTILITY THEORY?



EE 434 Behavioral Finance, SEM1/2022

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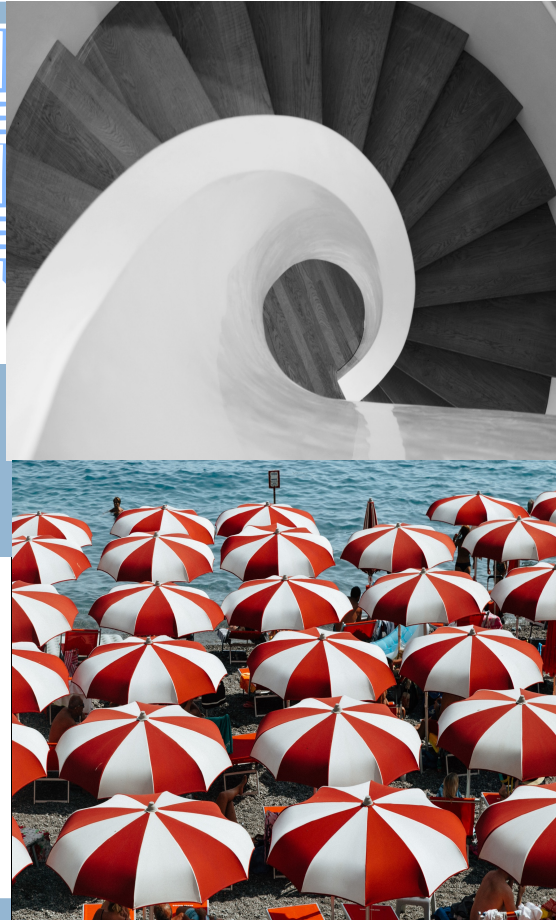
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01

ALLAIS PARADOX



Allais Paradox



<https://www.menti.com/live7tt3i4m>



Question 1: Which do you prefer between A and A*?

| Prospect A | | $A \succ A^*$ | Prospect A* |
|-------------|------|---------------|-------------|
| \$1,000,000 | 100% | \$ 0 | 1% |
| | | \$ 1,000,000 | 89% |
| | | \$ 5,000,000 | 10% |

Question 2: Which do you prefer between B and B*?

| Prospect B | | $B^* \succ B$ | Prospect B* |
|-------------|-----|---------------|-------------|
| \$0 | 89% | \$ 0 | 90% |
| \$1,000,000 | 11% | \$5,000,000 | 10% |

Allais Paradox

A large number of people choose A over A* and B* over B.

$$A > A^*$$

$$E(U(A)) > E(U(A^*))$$

$$U(\$1M) > 0.89U(\$1M) + 0.1U(\$5M)$$

$$0.11U(\$1M) > 0.1U(\$5M)$$

$$B^* > B$$

$$E(U(B^*)) > E(U(B))$$

$$0.1U(\$5M) > 0.11U(\$1M)$$

Contradiction

Allais Paradox



<https://www.menti.com/3jqphyrwj8>



Question 1: Which do you prefer between A and A*?

| <i>\$1M with 100%</i> Prospect A | | Prospect A* | |
|-------------------------------------|----------------|-------------------------|----------------|
| \$1,000,000 | 89% | \$ 1,000,000 | 89% |
| \$1,000,000 | 11% | \$ 0 | 1% |
| | | \$ 5,000,000 | 10% |

Question 2: Which do you prefer between B and B*?

| Prospect B | | <i>\$0 with 90%</i> Prospect B* | |
|----------------|----------------|------------------------------------|----------------|
| \$0 | 89% | \$ 0 | 89% |
| \$1,000,000 | 11% | \$ 0 | 1% |
| | | \$5,000,000 | 10% |

if $A \succ A^* \ \& \ B \succ B^*$ or $A^* \succ A \ \& \ B^* \succ B$, then EUT holds.

Allais paradox

After removing commonalities, the choices between prospects A and A* or B and B* are exactly the same.

Thus, people should choose A and B or A* and B*.

An axiom known as "context independence" is contradicted by the Allais paradox.

if $x \succ z$ $\sum y$
 $x \succ z$ $\sum y \succ z$

Alias paradox

Suppose a person is indifferent between two prospects, A and B. If we consider another prospect, C, independence implies that this person should also be indifferent between one gamble that combines A with C and another that combines B with C with fixed probability.

02

RABIN'S CALIBRATION



Rabin's calibration (Rabin, Econometrica 2000)

People tend to dislike risky prospects even when they involve an expected gain.

For example, Will you turn down a 50-50 gamble of losing \$100 vs. gaining \$105?

Based on Expected Value theory,

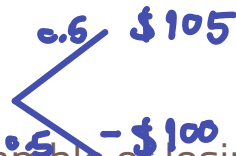
EV accept the bet

$$0.5(\$105) + 0.5(-\$100) > 0 \\ = \$2.5$$

\therefore accept the bet

Small-stake mixed gamble
have gain & loss

note: fair bet: Expected value of the bet = 0

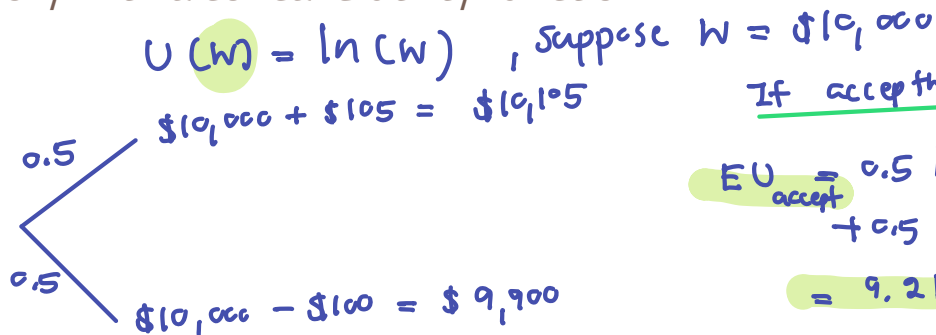


"Better-than-fair bet"

Rabin's calibration (Rabin, Econometrica 2000)

Standard explanation:

EU theory with a concave utility function.



Based on Expected Utility Theory,

\therefore accept the bet \sim reject the bet
 \Rightarrow risk neutrality

If accept the bet,

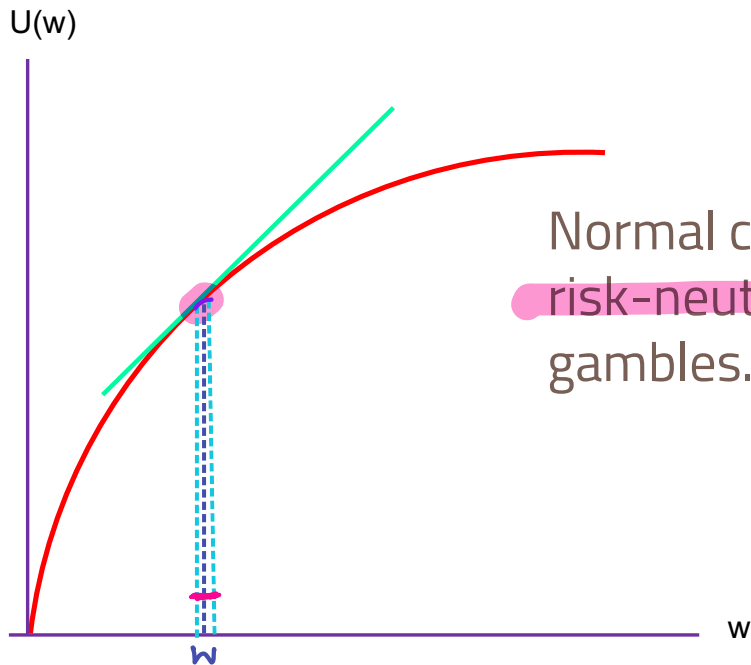
$$\begin{aligned} EU_{\text{accept}} &= 0.5 \ln(\$10,105) \\ &\quad + 0.5 \ln(\$9,900) \\ &= 9.21 \end{aligned}$$

If reject the bet,

$$\begin{aligned} EU_{\text{reject}} &= 1 \times \ln(\$10,000) \\ &= 9.21 \end{aligned}$$

$$EU_{\text{accept}} = EU_{\text{reject}}$$

Rabin's calibration (Rabin, Econometrica 2000)

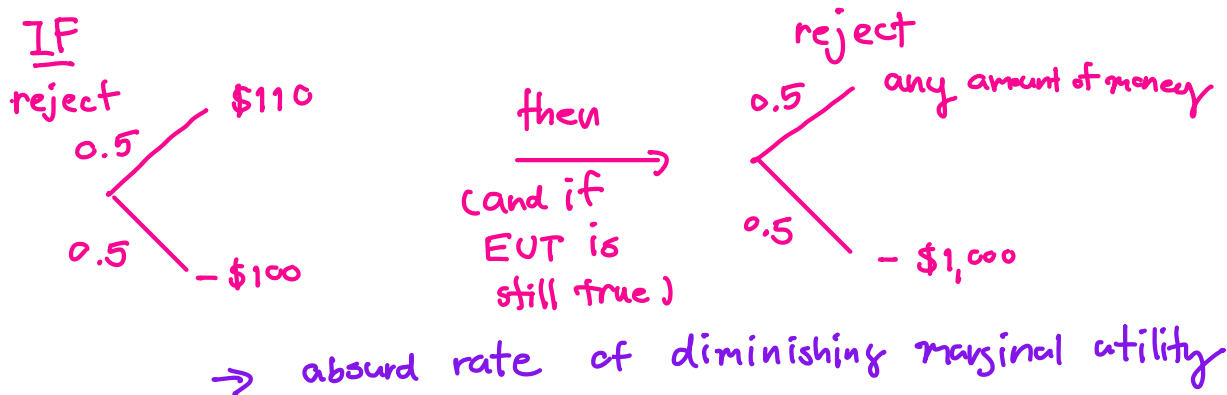


Normal concave utility function is risk-neutral for very small stake gambles.

Rabin's calibration (Rabin, Econometrica 2000)

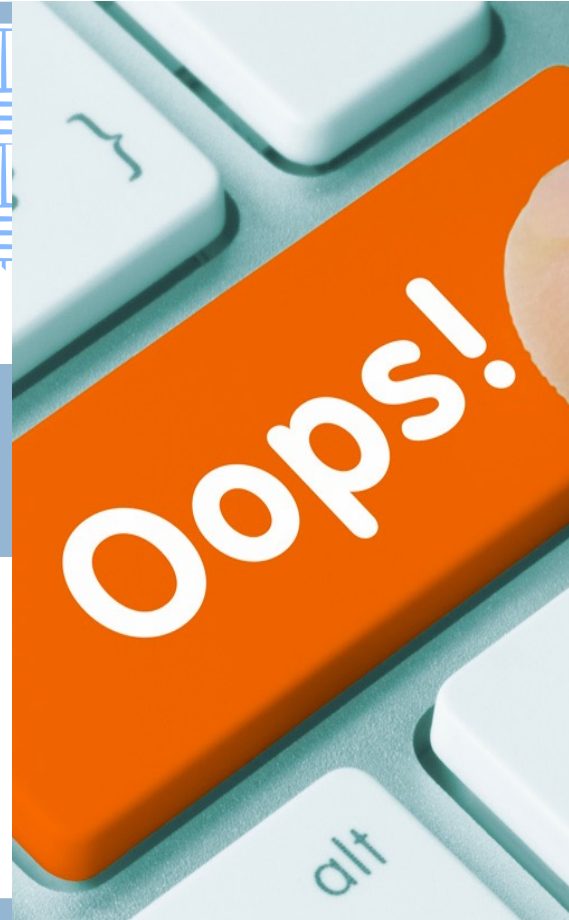
Rabin's Point:

This explanation doesn't work, because according to EU theory, anything but virtual risk neutrality over modest stakes implies manifestly unrealistic risk aversion over large stakes.

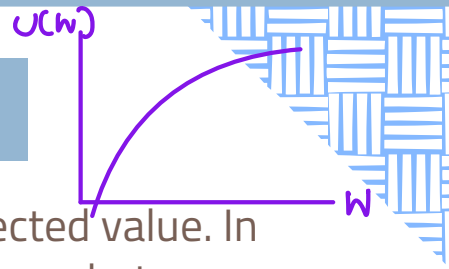


03

Bernoulli's Error



Bernoulli's Error



One way to evaluate a prospect is to calculate expected value. In 1738, Daniel Bernoulli noted that there was a difference between the actual value of money and the psychological value of money. Bernoulli noticed that people dislike risk, and want to avoid the worst outcomes at great cost, that is, they are **risk-averse**.

This can be explained by **diminishing marginal utility of wealth**. Hence, a **concave utility function** is implicitly assumed in EUT.

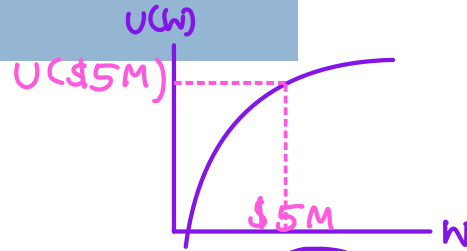
Expected Utility Theory has been widely accepted and used for many years, but it has its own issues.

Bernoulli's Error

Consider this example:

Today **Wanda** and **Loki** each have a wealth of \$5 million.

Yesterday, **Wanda** had \$1 million and **Loki** had \$9 million.



equally happy based on EUT

For today,

Are they equally happy?

Bernoulli's Error

The expected utility model only takes into account final states of wealth, not changes in wealth.

Wanda and Loki both have final wealth of \$5 mil (same final state). But they had different initial wealth (different changes).

04



Thought experiments



Thought experiment A

<https://www.menti.com/y4radnvp3i>



Problem 1: Choose between...

- ① Sure thing: win \$900
- ② Gamble: 90% win \$1000, 10% win \$0

Problem 2: Choose between...

Sure thing: lose \$900

Gamble: 90% lose \$1000, 10% lose \$0

Problem 1: Choose between...

① Sure thing: win \$900

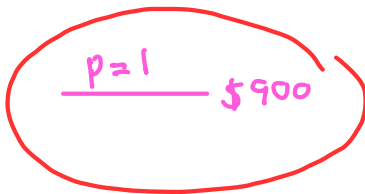
② Gamble: 90% win \$1000, 10% win \$0

Problem 2: Choose between...

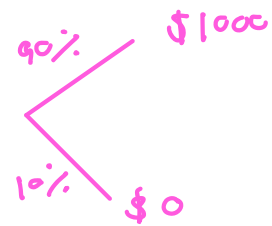
Sure thing: lose \$900

Gamble: 90% lose \$1000, 10% lose \$0

Problem 1



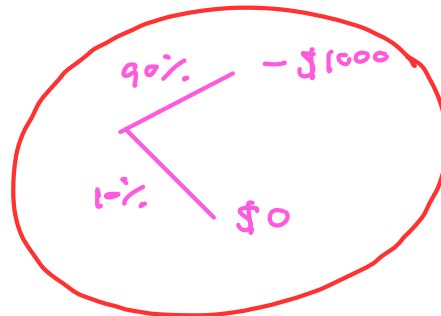
vs.



Problem 2



vs.



Thought experiment A

In Problem 1, most people...
Choose sure thing
Explained by risk aversion

in gain domain

In Problem 2, most people...
Choose the gamble *in loss domain*
Explained if people are risk-loving



Thought experiment B

<https://www.menti.com/6cgv7m1exa>



Problem 3: In addition to whatever you own, you have been given \$1000.
Choose between...

Sure thing:

win \$500

Gamble:

50% win \$1000, 50% win \$0

presented to be
in the gain domain
 $0.5 \left\{ \begin{array}{l} \$1000 + \$1000 = \$2000 \\ \$1000 + \$0 = \$1000 \end{array} \right.$

Problem 4: In addition to whatever you own, you have been given \$2000.
Choose between...

Sure thing:

lose \$500

Gamble:

50% lose \$1000, 50% lose \$0

presented to be
in the loss domain
 $0.5 \left\{ \begin{array}{l} \$2000 - \$1000 = \$1000 \\ \$2000 - \$0 = \$2000 \end{array} \right.$

Thought experiment B

In this thought experiment, the final payoffs in problem 3 and 4 are exactly the same.

Sure thing: \$1500

Gamble: 50% win \$1000 and 50% win \$2000

Findings suggest that people are risk-averse over potential gains, and risk-loving over potential losses.

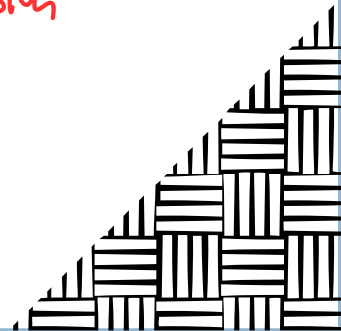
HOW CAN YOU BE BOTH RISK AVERSE/NEUTRAL IN GAINS
AND RISK SEEKING IN LOSSES?



Thought experiments C

People feel the pain of a loss more strongly than the desire of an equally-sized gain.

loss aversion



The slide features a light blue background with decorative elements. On the left, there is a large pink and blue geometric shape. Below it, a series of blue and white horizontal bars form a staircase-like pattern. In the bottom right corner, there is a blue and white striped pattern. In the top right corner, there are two overlapping orange circles and some blue lines.

DANKE!

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