EE 415 / EE 418 Game theory

Lecture 3 Basic theory: applications of Nash Equilibrium

- More games
- Cournot Nash equilibrium
- Bertrand Nash equilibrium

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Prisoners' Dilemma

		Prisoner B	
		Confess	Don't Confess
Prisoner A	Confess	-5, -5	-1, -10
	Don't Confess	-10, -1	-2, -2

Prisoners' Dilemma

- What is the:
  - Dominant strategy
  - Nash equilibrium
  - Maximin solution
- Dominant strategies are also maximin strategies
- Both confess is both Nash equilibrium and maximin solution.

		Prisoner B	
		Confess	Don't Confess
Prisoner A	Confess	-5, -5	-1, -10
	Don't Confess	-10, -1	-2, -2

Maximin = maximize the minimum gains that can be earned.

A. If playing confess, minimum payoff = -5. If playing Don't confess, minimum payoff = -10

Matching Pennies

		Heads	Tails
Player A	Heads	1, -1	-1, 1
	Tails	-1, 1	1, -1

Matching Pennies

- Pure Strategy
  - Player makes a specific choice or takes a specific action
- Mixed Strategy
  - Player makes a random choice among two or more possible actions, based on a set of chosen probabilities.
  - Implies that player is uncertain about another player choice of a pure strategy.
- In Matching penny game, no pure NE. But the NE theorem tells us that we can mixed NE.

Matching Pennies

- Player A might flip coin playing heads with ½ probability and tails with ½ probability.
- If both players follow this strategy, there is a Nash equilibrium – both player will be doing the best they can given what they opponent is doing.
- Although the outcome is random, the expected payoff is 0 for each player.
- What if you were stick to "head", how about when player 2 sticks to "tail"
- One reason to consider mixed strategies is when there is a game that do not have any Nash equilibriums in pure strategy.
- When allowing for mixed strategies, every game has a Nash equilibrium
- whether mixed strategies are reasonable will depend on the particular game, say, tennis, poker

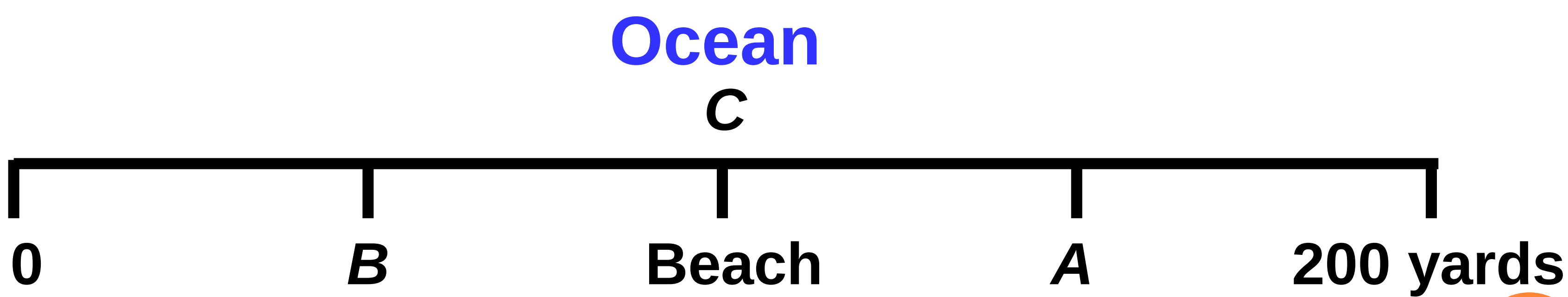
The Battle of the Sexes

		Joan Wrestling	*Joan* Opera
Jim	Wrestling	2,1	0,0
Jim	Opera	0,0	1,2

- Pure Strategy
  - Both watch wrestling
  - Both watch opera
- Mixed Strategy
  - Jim chooses wrestling with prob = 2/3.
  - Joan chooses opera with prob=2/3.
  - Both have expected payoff =2(1/3)+0(2/3)=2/3.

MORE GAMES: BEACH LOCATION

- Scenario
  - Two competitors, Y and C, selling soft drinks
  - Beach 200 yards long
  - Sunbathers are spread evenly along the beach
  - Price Y = Price C
  - Customer will buy from the closest vendor



Beach Location

- Where will the competitors locate (i.e. where is the Nash equilibrium)?
- Will want to all locate in center of beach.
  - Similar to groups of gas stations, car dealerships, political votes, etc.

Summary on NE

- Best response given other players' strategies.
- All finite simultaneous game has NE. Mixed or pure or both.
- Mixed NE. : indifferent between playing pure or specific strategy. Expected payoff of playing each strategy must be equal.
- NE does not guarantee the best outcome

Duopoly model with output competition: Cournot game

- When we have few firms in the market, why don't they collude and act like a single monopolist, which earn more for both?
- If they collude indeed, will they keep their promise?
- Will the collusive equilibrium stable?

Duopoly model with output competition: Cournot game

- Let a simple market consisting of J firms.
- Each producing output q^j.
- Suppose each firm's profit is adversely affected by an increase in the output of other firms
- $\Pi^j = \Pi^j(q^1, \ldots, q^J)$ and $\partial\Pi^j / \partial q^k < 0$ for $j \neq k$.
- Now, suppose firms cooperate to maximize joint profits $\sum_{j=1}^{J} \Pi^j$.

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Duopoly model with output competition: Cournot game

Let $\bar{\mathbf{q}}$ maximizes joint profits.

$$\frac{\partial \Pi^k(\bar{\mathbf{q}})}{\partial q^k} + \sum_{j \neq k} \frac{\partial \Pi^k(\bar{\mathbf{q}})}{\partial q^k} = 0,\ k = 1, \ldots, J.$$

Since the second term is negative, so the first term must be positive.

$$\frac{\partial \Pi^k(\bar{\mathbf{q}})}{\partial q^k} > 0,\ k = 1, \ldots, J.$$

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Duopoly model with output competition: Cournot game

- At $\bar{\mathbf{q}}$ each firm can increase its own profit by increasing outputs away from assigned output given that other firms keeps their promises.
- If each has incentive to cheat, $\bar{\mathbf{q}}$ will not be kept in the equilibrium.
- The promising equilibrium for noncooperative actions (self-interested firms) is when no one has incentives to deviate from it.

Duopoly model with output competition: Cournot game

- Nash Equilibrium: each firm's output must maximize its own profit given the other firm's output choices/
- Let **q*** be Nash equilibrium outputs so

$$\frac{\partial \Pi^k(q^1*,\ldots,q^J*)}{\partial q^k} = 0,\ k = 1,\ldots,J.$$

Duopoly model with output competition: Cournot game

- When few firms operate in the market
- Their actions becomes more strategic.
- Since they perceive their interdependence, they have incentive to take into account of their rival's action before deciding their own action.
- Strategic actions : choosing output, price

Cournot game

- Assume homogenous product market
- There are J identical firms
- More entry is blocked.
- Each has identical costs: $C(q^j) = cq^j$
- Constant marginal cost, no fixed cost.
- Let $p=a-bQ$ is the inverse market demand
- Where $Q = q^1 + \ldots + q^J$, $a>0$, $b>0$, $a>c$.

Cournot game

- Profit for firm j is

$$\Pi^j(q^1,\ldots,q^J) = (a - b\sum_{k=1}^{J} q^k)q^j - cq^j.$$

- If $\bar{\boldsymbol{q}}$ is Cournot-Nash equilibrium, it must maximize each firm's profit

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Cournot game

- FOC wrt q^j ....

$$a - 2b\overline{q}^j - b\sum_{k \neq j} \overline{q}^k - c = 0.$$

Or

$$b\overline{q}^j = a - c - b\sum_{k=1}^{J} \overline{q}^k.$$

COURNOT GAME

- Since all firms are identical, they must produce the same amount of output in equilibrium.
- Let $\overline{q}$ be the common equilibrium output.
- We have $b\overline{q} = a - c - J\overline{q} \;\; or$

$$\overline{q} = (a-c)/b(J+1).$$

- The equilibrium price each firm is facing is

$$\overline{p} = a - J(a-c)/(J+1) < a,$$

- Each firm profit is

$$\overline{\Pi}^j = (a-c)^2 / b(J+1)^2.$$

- We can see how price deviates from marginal cost,

$$\overline{p} - c = (a-c)/(J+1) > 0,$$

- When J=1, we have the pure monopoly case.
- When $J \rightarrow \infty$, we have *lim* $(\overline{p} - c) = 0$.
- Price approaches marginal cost as no. of competitors becomes large.

- Ex. Duopoly (J=2)
- Assume $p=a-bQ$, $c(q^j)=cq^j$.
- Firm 1's profit = $\{a-b(q_1+q_2)\}q_1-cq_1$
- FOC: $a-2bq_1-bq_2 = c$
- Or $q_1 = (a-c-bq_2)/ 2b$
- Notice that from the FOC, we get $q_1(q_2)$ a reaction function for firm 1.
- Similarly for firm 2, thus we have two equations for two unknowns q_1 and q_2. We can solve this.

- Or we can use the fact that $q^* = q_1^* = q_2^*$
- So, $a - 2bq^* - bq^* = c$ or
- $q^* = (a-c)/3b$.
- $p^* = (a+2b)/3$ and
- profit $= (a-c)^2 / 9b$.

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HOMEWORK 1

- 1. Duopoly with different marginal costs, c_1, and c_2. Find Nash equilibrium.
- 2. Collusive outcome when there are two identical firms competing in a Cournot fashion. Verify that it is not stable.

Bertrand game

- Think of firms competing in price rather than quantity.
- So, Bertrand = price competition
- Assume identical cost, no fixed cost.
- Simultaneous play or declare the prices and supply all demanded at their price.
- Consumers buy from the cheapest firm.
- The firm with the lowest price will take all the market, assume they share market equally if they declare the same price.

Bertrand game

- Each firm profit will be

$$\Pi^1(p^1, p^2) = \begin{cases} (p^1 - c)(a - bp^1), & c < p^1 < p^2 \\ 0.5(p^1 - c)(a - bp^1), & c < p^1 = p^2 \\ 0, & otherwise. \end{cases}$$

Bertrand game

- As long as price exceeds marginal cost, they will make positive profits.
- At worst, they can set p=mc, receiving zero profits.
- What is the Nash Equilibrium?
- How about setting price = marginal cost and both earn zero profit.
- Here, we cannot use calculus, use logic instead.

Bertrand game

- First, we argue that firm has an incentive to undercut its rival.
- If $p_1 > c$, firm 2 will cut price such that $p_2 > c$ and still $p_2 < p_1$. When see this, firm 1 will want to undercut p_2.
- So, this can't be equilibrium since firm 1 wants to change its price.
- How about if $p_1 = p_2 > c$, firms will have incentive to undercut. This can't be either.

Bertrand game

- How about when $p_1 = p_2 = c$.
- No one wants to undercut beyond c, since this will give negative profits.
- So when p=c, they have no incentive to cut down or increase their price. Also, this is the best response or the profit-maximizing choice given the rival price =c.
- Notice that we get different results from quantity or Cournot competition in which both firms make profits and $p > MC$, while in price competition, both firms make zero profits.

BERTRAND GAME

- Strategic variable chosen by firms could affect outcome.
- Some comments on this price competition:
- When good is homogenous, should firms compete by setting price? why not using quantity competition
- Why identical or 50:50
- Model Variation:
- Model with capacity constraints: competition will not drive down price to marginal cost with price =c.
- Model with heterogeneous products.

Bertrand game

- Model with heterogeneous products.
  - Firm 1's demand is $Q_1 = 12 - 2P_1 + P_2$
  - Firm 2's demand is $Q_2 = 12 - 2P_2 + P_1$
  - P_1 and P_2 are prices firms 1 and 2 charge respectively
  - Q_1 and Q_2 are the resulting quantities they sell
  - FC = \$20
  - VC = 0

Bertrand game

- Each firm will max profits

$$\Pi_1 = P_1 Q_1 - FC - VC$$

$$= P_1(12 - 2P_1 + P_2) - 20$$

$$\frac{\partial \Pi_1}{\partial P_1} = 12 - 4P_1 + P_2 = 0$$

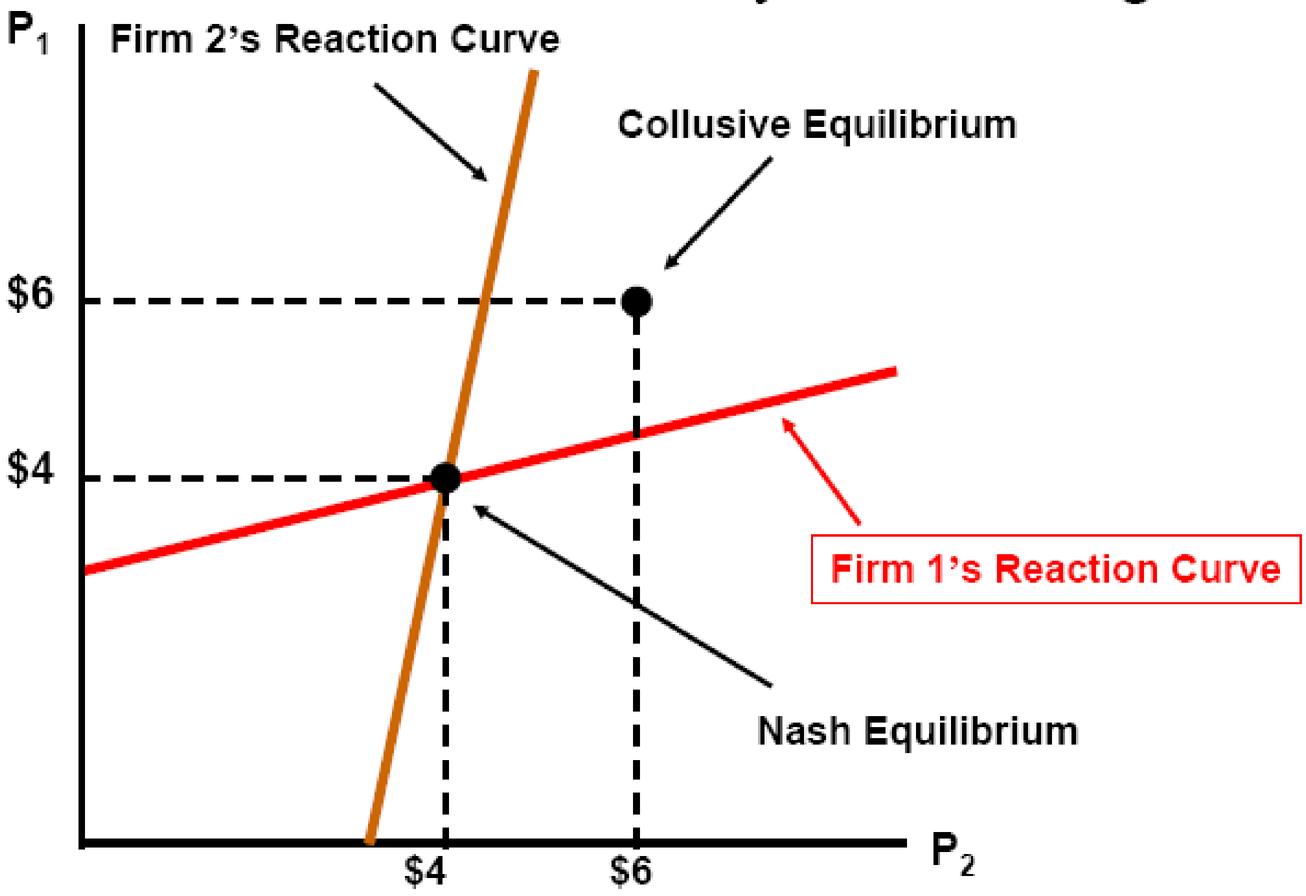
$$\Rightarrow P_1 = (12 + P_2)/4$$

$$= 3 + 0.25P_2$$

- This is Firms 1's reaction function.

Nash Equilibrium in Prices

Why not selecting $6?



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LET'S PLAY: CLASSROOM GAMES

- 2. The Ultimatum Game
- Two players
- Player A offers a split of a 100 Baht bill
- If B agrees, then the game is over.
- If B refuses, it is then B's turn to offer a split, but now the bill reduced to 80 Baht
- If A agrees, then the game is over. Both get paid the agreed split.
- If A refuses, the game is over and neither get anything.