

Solution to Homework 4**CHAPTER 15**

18. a.

| Maturity (years) | Price | YTM | Forward rate |
|---------------------|----------|-------|-----------------|
| 1 | \$925.93 | 8.00% | |
| 2 | \$853.39 | 8.25% | 8.50% |
| 3 | \$782.92 | 8.50% | 9.00% |
| 4 | \$715.00 | 8.75% | 9.50% |
| 5 | \$650.00 | 9.00% | 10.00% |

b. For each 3-year zero issued today, use the proceeds to buy:

$$\$782.92/\$715.00 = 1.095 \text{ four-year zeros}$$

Your cash flows are thus as follows:

| Time | Cash Flow | |
|------|-----------|---|
| 0 | \$ 0 | |
| 3 | -\$1,000 | The 3-year zero issued at time 0 matures; the issuer pays out \$1,000 face value |
| 4 | +\$1,095 | The 4-year zeros purchased at time 0 mature; receive face value |

This is a synthetic one-year loan originating at time 3. The rate on the synthetic loan is $0.095 = 9.5\%$, precisely the forward rate for year 4.

- c. For each 4-year zero issued today, use the proceeds to buy:

$$\$715.00/\$650.00 = 1.100 \text{ five-year zeros}$$

Your cash flows are thus as follows:

| Time | Cash Flow | |
|------|-----------|---|
| 0 | \$ 0 | |
| 4 | -\$1,000 | The 4-year zero issued at time 0 matures; the issuer pays out \$1,000 face value |
| 5 | +\$1,100 | The 5-year zeros purchased at time 0 mature; receive face value |

This is a synthetic one-year loan originating at time 4. The rate on the synthetic loan is $0.100 = 10.0\%$, precisely the forward rate for year 5.

19. a. For each three-year zero you buy today, issue:

$$\$782.92/\$650.00 = 1.2045 \text{ five-year zeros}$$

The time-0 cash flow equals zero.

- b. Your cash flows are thus as follows:

| Time | Cash Flow | |
|------|-------------|--|
| 0 | \$ 0 | |
| 3 | +\$1,000.00 | The 3-year zero purchased at time 0 matures; receive \$1,000 face value |
| 5 | -\$1,204.50 | The 5-year zeros issued at time 0 mature; issuer pays face value |

This is a synthetic two-year loan originating at time 3.

- c. The effective two-year interest rate on the forward loan is:

$$\$1,204.50/\$1,000 - 1 = 0.2045 = 20.45\%$$

- d. The one-year forward rates for years 4 and 5 are 9.5% and 10%, respectively. Notice that:

$$1.095 \times 1.10 = 1.2045 =$$

$$1 + (\text{two-year forward rate on the 3-year ahead forward loan})$$

The 5-year YTM is 9.0%. The 3-year YTM is 8.5%. Therefore, another way to derive the 2-year forward rate for a loan starting at time 3 is:

$$f_3(2) = \frac{(1+y_5)^5}{(1+y_3)^3} - 1 = \frac{1.09^5}{1.085^3} - 1 = 0.2046 = 20.46\%$$

[Note: slight discrepancies here from rounding errors in YTM calculations]

CHAPTER 20

7. a. From put-call parity:

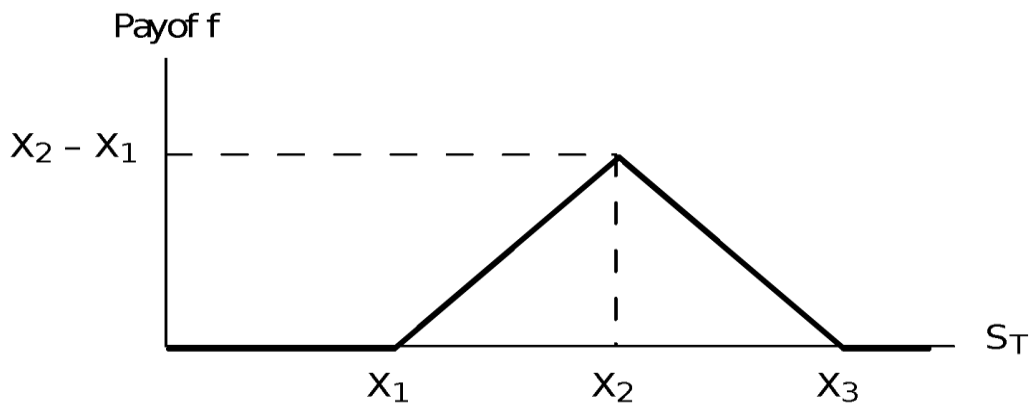
$$P = C - S_0 + \frac{X}{(1+r_f)^T} = 10 - 100 + \frac{100}{1.10^{25}} = \$7.65$$

- b. Purchase a straddle, i.e., both a put and a call on the stock. The total cost of the straddle is: $\$10 + \$7.65 = \$17.65$
9. a. i. A long straddle produces gains if prices move up or down, and limited losses if prices do not move. A short straddle produces significant losses if prices move significantly up or down. A bullish spread produces limited gains if prices move up.
- b. i. Long put positions gain when stock prices fall and produce very limited losses if prices instead rise. Short calls also gain when stock prices fall but create losses if prices instead rise. The other two positions will not protect the portfolio should prices fall.
10. Note that the price of the put equals the revenue from writing the call, net initial cash outlays = \$38.00

| Position | $S_T S_T < 35$ | $35 \leq S_T S_T \leq 40$ | $40 < S_T$ |
|-------------------|----------------|---------------------------|------------|
| Buy Stock | S_T | S_T | S_T |
| Write call (\$40) | 0 | 0 | $40 - S_T$ |
| Buy put (\$35) | $35 - S_T$ | 0 | 0 |
| Total | \$35 | S_T | \$40 |

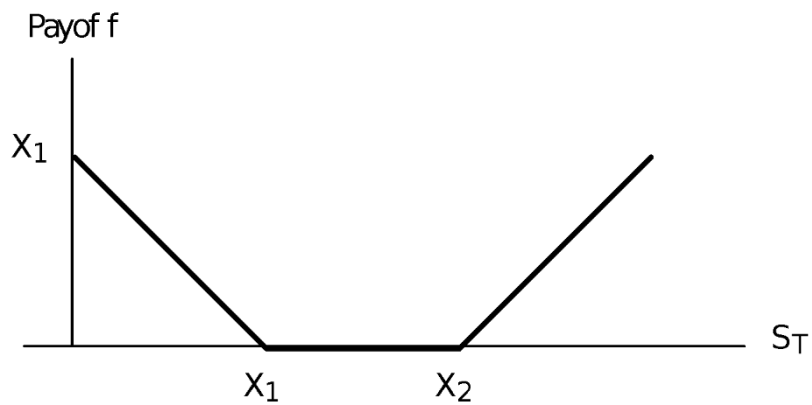
13. a.

| Position | $S_T < X_1$ | $X_1 \leq S_T \leq X_2$ | $X_2 < S_T \leq X_3$ | $X_3 < S_T$ |
|-------------------------|-------------|-------------------------|----------------------|---------------------------------|
| Long call (X_1) | 0 | $S_T - X_1$ | $S_T - X_1$ | $S_T - X_1$ |
| Short 2 calls (X_2) | 0 | 0 | $-2(S_T - X_2)$ | $-2(S_T - X_2)$ |
| Long call (X_3) | 0 | 0 | 0 | $S_T - X_3$ |
| Total | 0 | $S_T - X_1$ | $2X_2 - X_1 - S_T$ | $(X_2 - X_1) - (X_3 - X_2) = 0$ |



b.

| Position | $S_T < X_1$ | $X_1 \leq S_T \leq X_2$ | $X_2 < S_T$ |
|--------------------|-------------|-------------------------|-------------|
| Buy call (X_2) | 0 | 0 | $S_T - X_2$ |
| Buy put (X_1) | $X_1 - S_T$ | 0 | 0 |
| Total | $X_1 - S_T$ | 0 | $S_T - X_2$ |

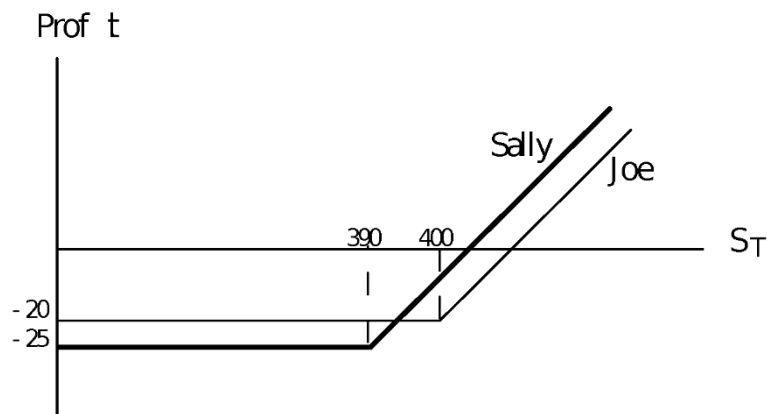


26. a. Joe's strategy

| Position | Cost | Payoff | |
|-------------------------|------|----------------|-------------|
| | | $S_T \leq 400$ | $S_T > 400$ |
| Stock index | 400 | S_T | S_T |
| Put option, $X = \$400$ | 20 | $400 - S_T$ | 0 |
| Total | 420 | 400 | S_T |
| Profit = payoff - \$420 | | -20 | $S_T - 420$ |

Sally's strategy

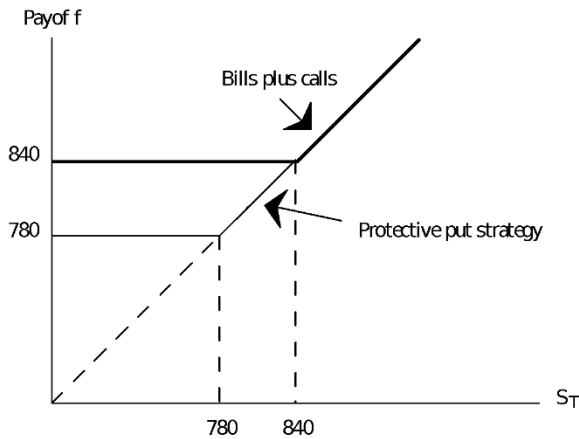
| Position | Cost | Payoff | |
|-------------------------|------|----------------|-------------|
| | | $S_T \leq 390$ | $S_T > 390$ |
| Stock index | 400 | S_T | S_T |
| Put option, $X = \$390$ | 15 | $390 - S_T$ | 0 |
| Total | 415 | 390 | S_T |
| Profit = payoff - \$415 | | -25 | $S_T - 415$ |



- b. Sally does better when the stock price is high, but worse when the stock price is low. The break-even point occurs at $S_T = \$395$, when both positions provide losses of \$20.
- c. Sally's strategy has greater systematic risk. Profits are more sensitive to the value of the stock index.

29. a.

| Position | $S_T \leq 780$ | $S_T > 780$ |
|-----------|----------------|-------------|
| Buy stock | S_T | S_T |
| Buy put | $780 - S_T$ | 0 |
| Total | 780 | S_T |



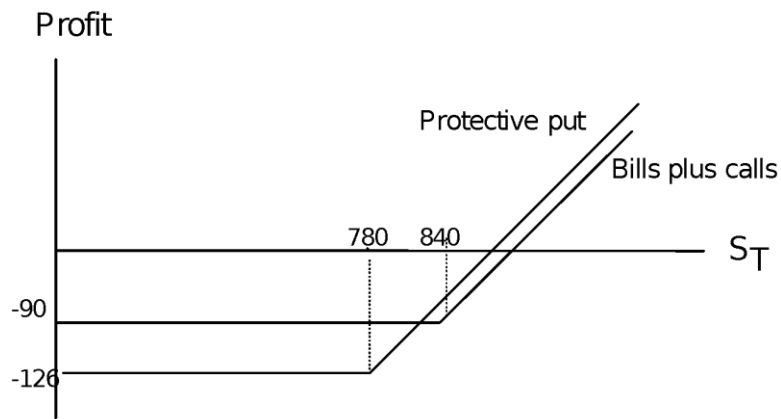
| Position | $S_T \leq 840$ | $S_T > 840$ |
|-------------|----------------|-------------|
| Buy call | 0 | $S_T - 840$ |
| Buy T-bills | 840 | 840 |
| Total | 840 | S_T |

b. The bills plus call strategy has a greater payoff for some values of S_T and never a lower payoff. Since its payoffs are always at least as attractive and sometimes greater, it must be more costly to purchase.

c. The initial cost of the stock plus put position is: $\$900 + \$6 = \$906$

The initial cost of the bills plus call position is: $\$810 + \$120 = \$930$

| | $S_T = 700$ | $S_T = 840$ | $S_T = 900$ | $S_T = 960$ |
|--------|-------------|-------------|-------------|-------------|
| Stock | 700 | 840 | 900 | 960 |
| + Put | 80 | 0 | 0 | 0 |
| Payoff | 780 | 840 | 900 | 960 |
| Profit | -126 | -66 | -6 | 54 |
| Bill | 840 | 840 | 840 | 840 |
| + Call | 0 | 0 | 60 | 120 |
| Payoff | 840 | 840 | 900 | 960 |
| Profit | -90 | -90 | -30 | +30 |



- d. The stock and put strategy is riskier. This strategy performs worse when the market is down and better when the market is up. Therefore, its beta is higher.
- e. Parity is not violated because these options have different exercise prices. Parity applies only to puts and calls with the same exercise price and expiration date.