Macroeconomics

Lecture 10_2

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The behavior of households

- **The representative household takes the paths of r(t) and w(t) as given.**
- **Its objective function is to maximize**

$$U = \int_{t=0}^{\infty} e^{-\rho t} u\left(C\left(t\right)\right) \frac{L\left(t\right)}{H} dt$$

where $C(t)$ *is the consumption of each member of the household at time* t.

$u(.)$ *is the* $ins\tan\tan eous$ *utility function, and takes the form of a* $cons\tan t - relative - risk - aversion$ *utility*

$$u\left(C\left(t\right)\right) = \frac{C\left(t\right)^{1-\theta}}{1-\theta}, \qquad \theta > 0,\ \rho - n - \left(1-\theta\right) g > 0$$

$L(t)$ *is total population in the economy.*

The behavior of households

- **The household's budget constraint is**

$$\int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \le \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} A(t) w(t) \frac{L(t)}{H} dt \qquad (5)$$

$$where \qquad R(t) = \int_{\tau=0}^{t} r(\tau) d\tau.$$

One unit of capital good invested at time o yields

$e^{R(t)}$ *units of good at* t.

- **Let c(t) be consumption per effective labor, c(t)=[C(t)L(t)/H]/[A(t)L(t)/H].**

The behavior of households

- **Since C(t)=A(t)c(t) ,**

$$u(C(t)) = \frac{C(t)^{1-\theta}}{1-\theta} = \frac{[A(t)c(t)]^{1-\theta}}{1-\theta} = \frac{[A(0)e^{gt}]^{1-\theta} c(t)^{1-\theta}}{1-\theta}$$

$$= A(0)^{1-\theta} e^{(1-\theta)gt} \frac{c(t)^{1-\theta}}{1-\theta}$$

$$\therefore \qquad U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\theta}}{1-\theta} \frac{L(t)}{H} dt = \int_{t=0}^{\infty} e^{-\rho t} \left[A(0)^{1-\theta} e^{(1-\theta)gt} \frac{c(t)^{1-\theta}}{1-\theta} \right] \frac{L(0)e^{nt}}{H} dt$$

$$= A(0)^{1-\theta} \frac{L(0)}{H} \int_{t=0}^{\infty} e^{-\rho t} e^{(1-\theta)gt} e^{nt} \frac{c(t)^{1-\theta}}{1-\theta} dt$$

$$= B \int_{t=0}^{\infty} e^{-\beta t} \frac{c(t)^{1-\theta}}{1-\theta} dt \qquad (13)$$

$$where \qquad B \equiv A(0)^{1-\theta} \frac{L(0)}{H}, \quad and \quad \beta \equiv \rho - n - (1-\theta) g$$

The behavior of households

The Lagrangian function

$$L = B\int_{t=0}^{\infty} e^{-\beta t}\frac{c(t)^{1-\theta}}{1-\theta}dt + \lambda\left[k(0)+\int_{t=0}^{\infty} e^{-R(t)}w(t)e^{(n+g)t}dt - \int_{t=0}^{\infty} e^{-R(t)}c(t)e^{(n+g)t}dt\right] \quad (14)$$

The first-order condition for an individual $c(t)$ *is*

$$Be^{-\beta t}c(t)^{-\theta} = \lambda e^{-R(t)}e^{(n+g)t} \quad (15)$$

Take ln *of both sides*

$$\ln B - \beta t - \theta \ln c(t) = \ln \lambda - R(t) + (n+g)t \quad (16)$$

The derivative of both sides of $Eq(16)$, *w.r.t.* t, *must be the same*

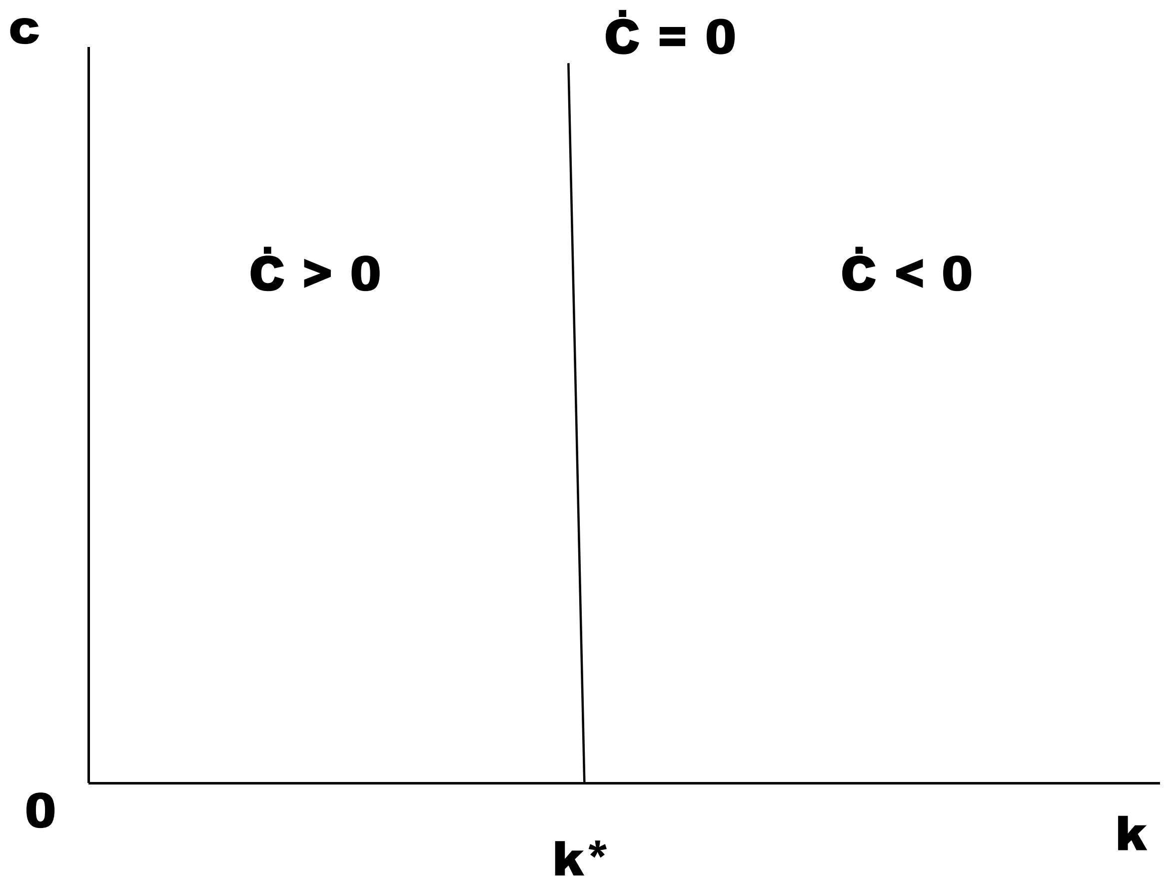
$$-\beta - \theta\frac{\dot{c}(t)}{c(t)} = -r(t) + (n+g)$$

$$\therefore \frac{\dot{c}(t)}{c(t)} = \frac{r(t)-n-g-\beta}{\theta} = \frac{r(t)-\rho-\theta g}{\theta} = \frac{f'(k(t))-\rho-\theta g}{\theta} \quad (17)$$

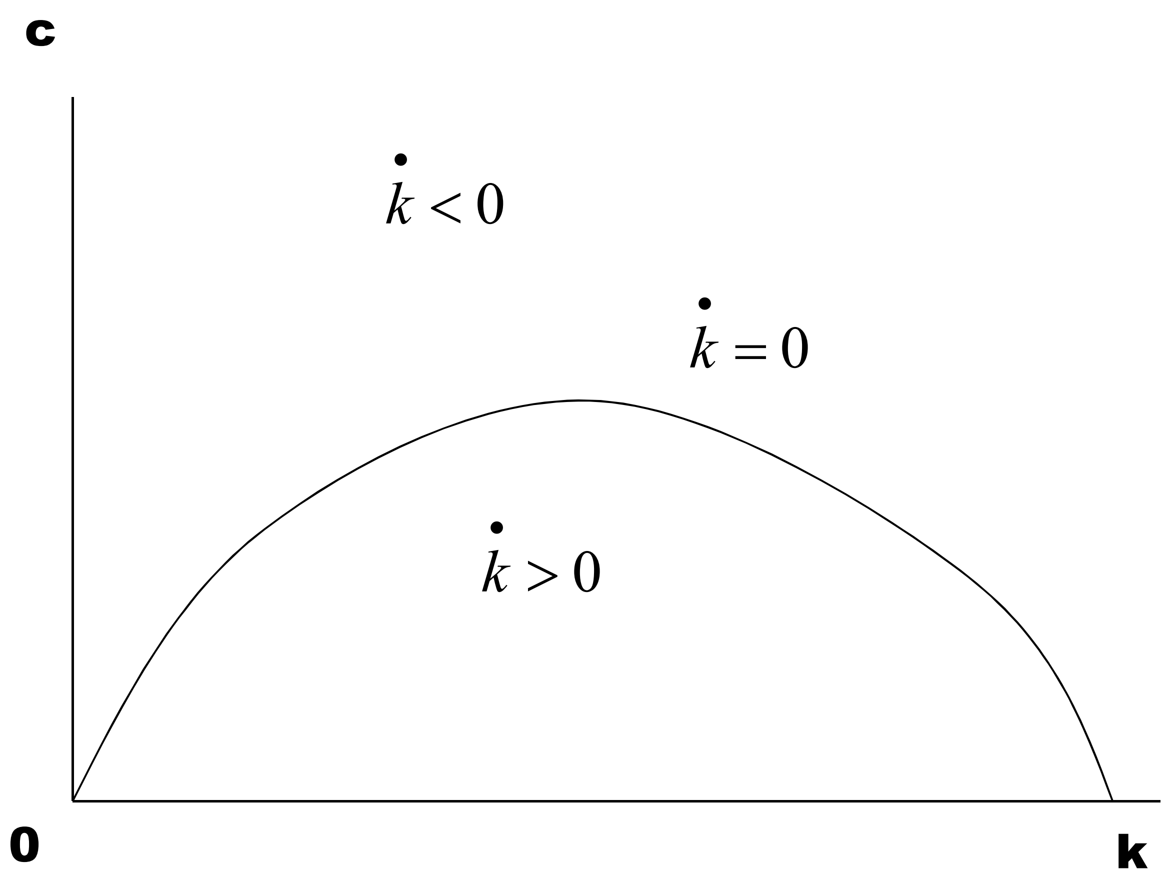
Since actual investment is output minus consumption, thus from $Eq(4)$,

$$\dot{k}(t) = f(k(t)) - c(t) - (n+g)k(t) \quad (18)$$

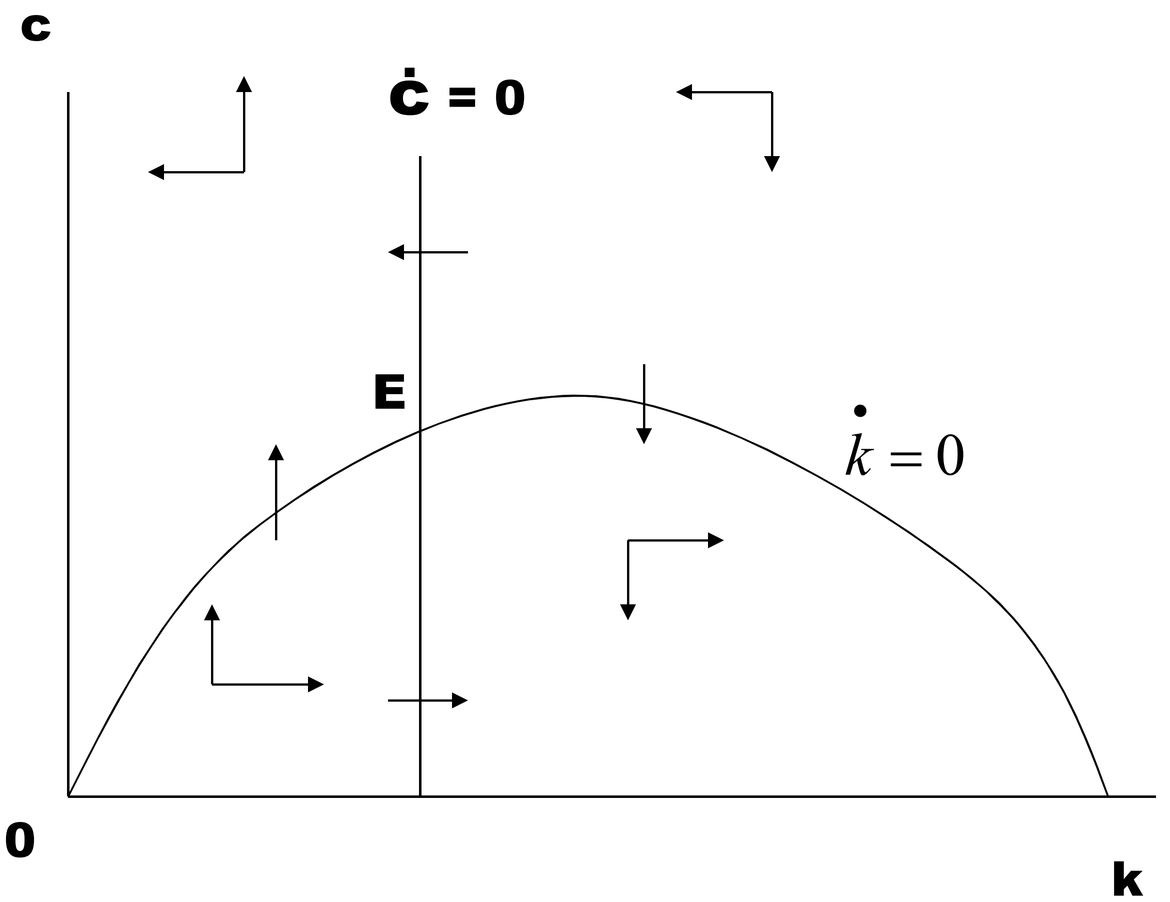
The Dynamic of c



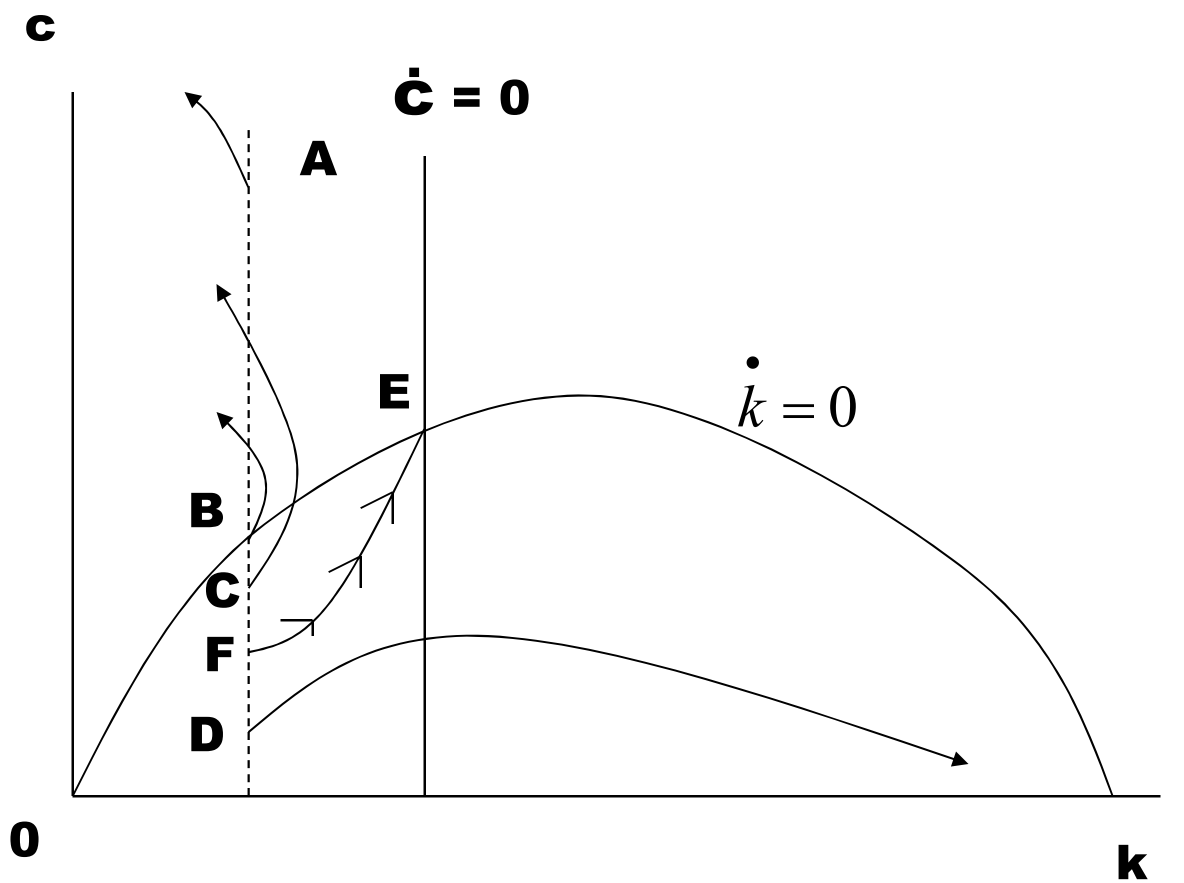
The Dynamic of k



The Phase Diagram



The Initial Value of c



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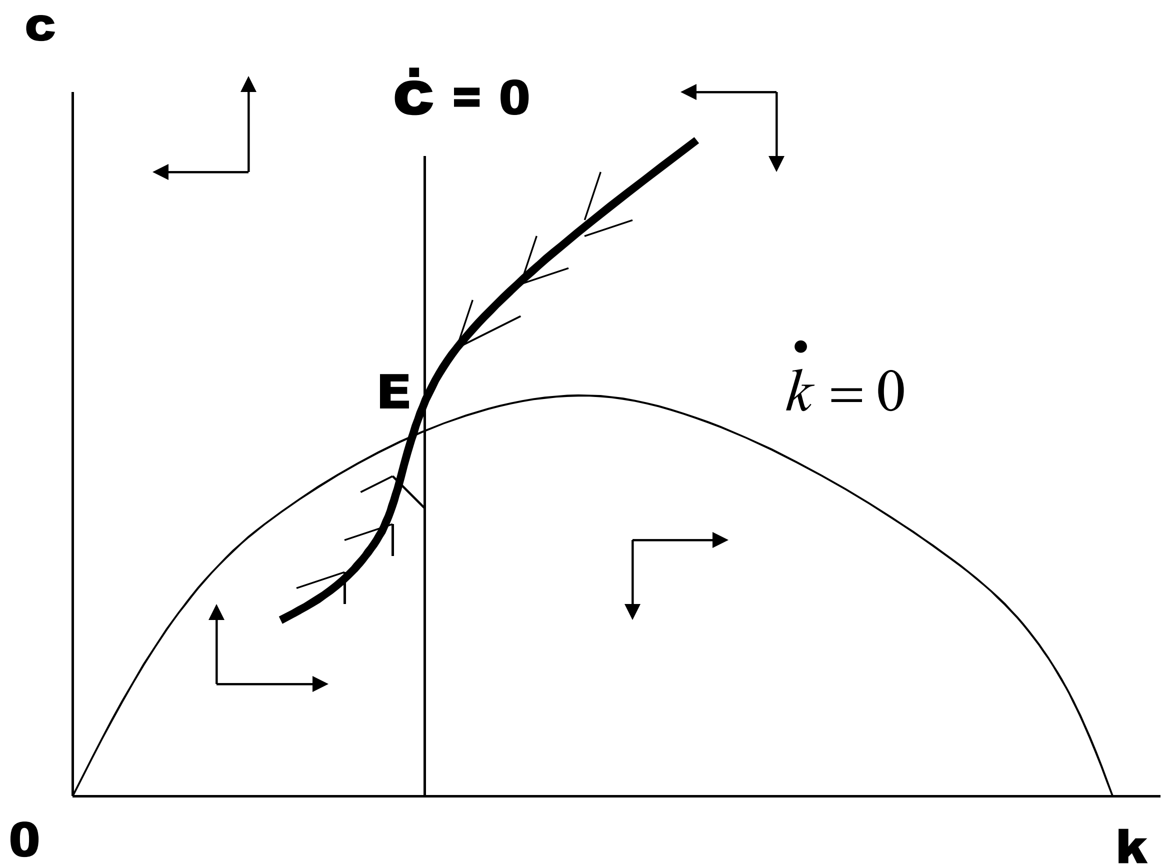
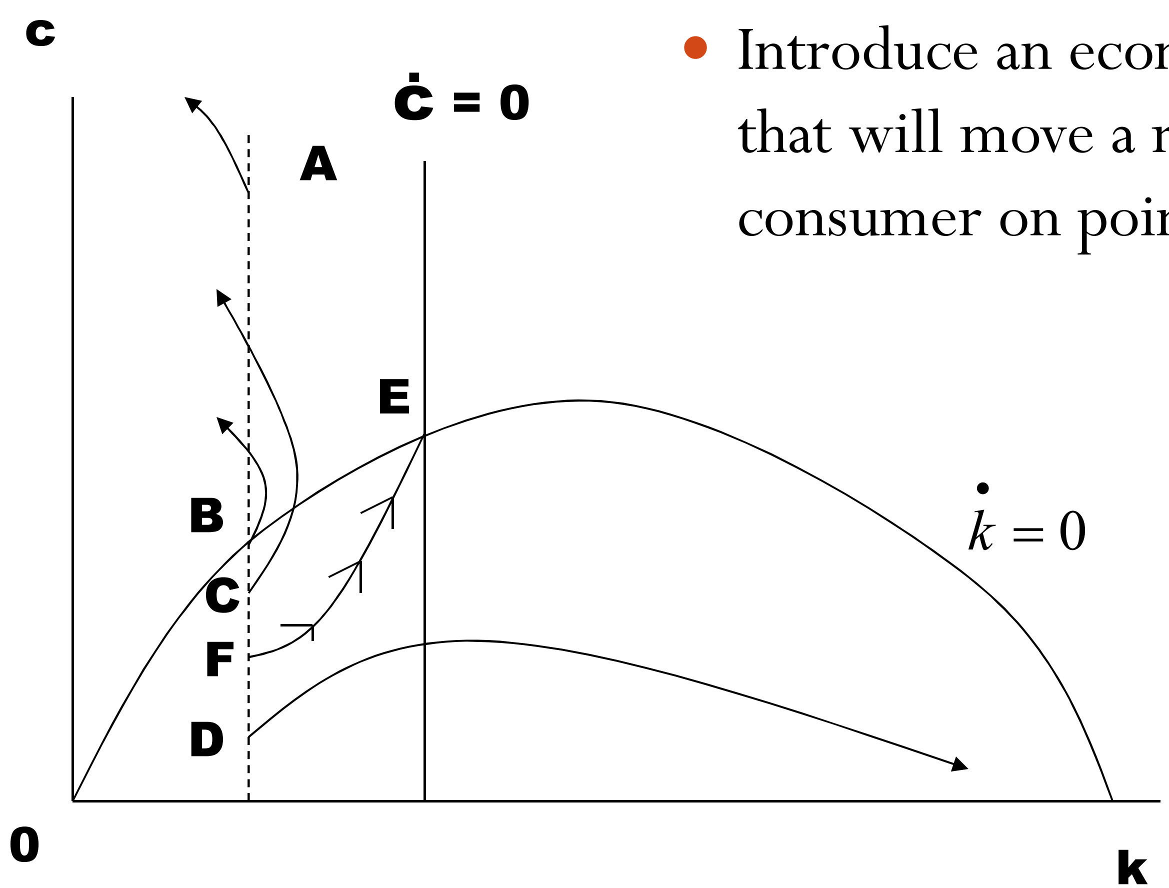
The Saddle Path



Welfare

- The First Welfare Theorem:
- If markets are competitive and there are no externalities, the decentralized equilibrium is Pareto-efficient. (It is impossible to make anyone better off without making someone else worse off.)
- Hence, the equilibrium in this model must be Pareto-efficient.

Homework



- Introduce an economic policy that will move a representative consumer on point C to F.