

EE434 Sem1/2021

Suggested Answers for Review Question8

1. Differentiate the following terms/concepts:

(1.a) Risk and uncertainty

Risk is measurable using probability, but uncertainty is not. Uncertainty is when probabilities can't be assigned or the possible outcomes are unclear.

(1.b) Risk aversion, risk seeking, and risk neutrality

Risk aversion describes someone who prefers the expected value of a lottery to the lottery itself. Risk seeking describes someone who prefers a lottery to the expected value of a lottery. And risk neutrality describes someone whose utility of the expected value of a lottery is equal to the expected utility of the lottery.

2. When eating out, Rory prefers spaghetti over a hamburger. Last night, she had a choice of spaghetti or macaroni and cheese and decided on the spaghetti again. The night before, Rory had a choice of spaghetti, pizza, or a hamburger, and this time she had pizza. Then, today, she chose macaroni and cheese over a hamburger. Does her selection today indicate that Rory's choices are consistent with economic rationality? Why or why not?

Rory's preferences are consistent with rationality. They are complete and transitive. We see that her preference ordering is:

Pizza > spaghetti > macaroni and cheese > hamburger

3. Consider a person with the following utility function over wealth: $u(w) = e^w$, where e is the exponential function (approximately equal to 2.7183) and w = wealth in hundreds of thousands of dollars. Suppose that this person has a 40% chance of wealth of \$50,000 and a 60% chance of wealth of \$1,000,000 as summarized by (\$50,000, 0.40; \$1,000,000, 0.60).

(3.a) What is the expected value of wealth?

$$E(w) = .4 * .5 + .6 * 10 = 6.2$$

(3.b) What is the expected utility from (\$50,000, 0.40; \$1,000,000, 0.60) ?

$$U(P) = .4e^{0.50} + .6e^{10} = 13,216.54$$

(3.c) Construct a graph of this utility function.

(3.d) Is this person risk averse, risk neutral, or a risk loving?

The utility function is convex, hence the person is risk loving.

4. An individual has the following utility function: $u(w) = w^{0.5}$ where w = wealth.

(4.a) Using expected utility, order the following prospects in terms of preference, from the most to the least preferred:

$$P1 = (1,000,0.80; 600,0.20),$$

$$P2 = (1,200,0.70; 600,0.30),$$

$$P3 = (2,000,0.5; 300,0.5)$$

Ranking: P2, P3, P1 with expected utilities 31.5972, 31.0209, and 30.1972 for prospects 2, 3, and 1, respectively

(4.b) What is the certainty equivalent for prospect P2?

$$998.3830$$

(4.c) Without doing any calculations, would the certainty equivalent for prospect P1 be larger or smaller? Why?

The certainty equivalent for P1 would be smaller because P2 is ranked higher than P1.

5. Consider two prospects:

Problem 1: Choose between

Prospect A: \$2,500 with probability .33,
\$2,400 with probability .66,
Zero with probability .01.
And Prospect B: \$2,400 with certainty.

Problem 2: Choose between

Prospect C: \$2,500 with probability .33,
Zero with probability .67.
And Prospect D: \$2,400 with probability .34,
Zero with probability .66.

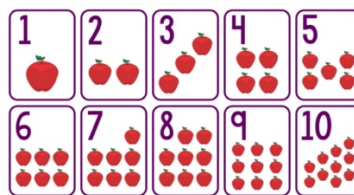
It has been shown by Daniel Kahneman and Amos Tversky (1979, "Prospect theory: An analysis of decision under risk," *Econometrica* 47(2), 263-291) that more people choose B when presented with problem 1 and when presented with problem 2, most people choose C. These choices violate expected utility theory. Why?

This is an example of the Allais paradox. The first choice suggests that $u(2,400) > .33u(2,500) + .66u(2,400)$ or $.34u(2,400) > .33u(2,500)$ while the second choice suggests just the opposite inequality.

6. Risk attitude is a fundamental concept in finance, with risk aversion commonly assumed. A 2002 paper by Charles Holt and Susan Laury provides a simple choice experiment that allows us to measure the extent of risk aversion.¹

In this experiment you will complete the table below indicating your choice of option A or option B. You have ten decisions to make and should record your choice in the final column of the table.

The experimenter has a set of cards numbered one to ten. The back of all cards are the same and are what you can only see. The card will be **randomly drawn**. The number on the card will determine the payoff as stated in each option in the table.



Decision	Option A	Option B	Your choice (A or B)
1	\$2.00 if card is 1 \$1.60 if card is 2-10	\$3.80 if card is 1 \$0.10 if card is 2-10	
2	\$2.00 if card is 1-2 \$1.60 if card is 3-10	\$3.80 if card is 1-2 \$0.10 if card is 3-10	
3	\$2.00 if card is 1-3 \$1.60 if card is 4-10	\$3.80 if card is 1-3 \$0.10 if card is 4-10	
4	\$2.00 if card is 1-4 \$1.60 if card is 5-10	\$3.80 if card is 1-4 \$0.10 if card is 5-10	
5	\$2.00 if card is 1-5 \$1.60 if card is 5-10	\$3.80 if card is 1-5 \$0.10 if card is 6-10	
6	\$2.00 if card is 1-6 \$1.60 if card is 7-10	\$3.80 if card is 1-6 \$0.10 if card is 7-10	
7	\$2.00 if card is 1-7 \$1.60 if card is 8-10	\$3.80 if card is 1-7 \$0.10 if card is 8-10	
8	\$2.00 if card is 1-8 \$1.60 if card is 9-10	\$3.80 if card is 1-8 \$0.10 if card is 9-10	
9	\$2.00 if card is 1-9 \$1.60 if card is 9-10	\$3.80 if card is 1-9 \$0.10 if card is 10	
10	\$2.00 if card is 1-10	\$3.80 if card is 1-10	

(6.a) In each decision, which option is more risky? **Option B is riskier than A.**

(6.b) What are your choices in each option? Holt and Laury categorize those as taking 0 to 1 safe choices as “highly risk loving” and, at the other extreme, those taking 9 to 10 safe choices as “stay in bed.” How many safe choices did you make?

¹ See Holt, C. A. and S. K. Laury, 2002, “Risk aversion and incentive effects,” *American Economic Review* 92(5), 1644-1655.

7. Differentiate the following two concepts: Risk aversion and Loss aversion. Is there any aspect that the two concepts can be related? If so, how?

A person who is risk averse prefers the expected value of a prospect to the prospect itself, whereas for a person who is loss averse, losses loom larger than gains.

People who are loss-averse can exhibit risk-averse behavior, rejecting small-stake gamble like (\$10, 0.5; -\$6, 0.5) .

8. According to prospect theory, which is preferred?

(8.a) Choose between:

$$A = (\$50, 0.8; 0, 0.2) \text{ and } B = (\$100, 0.4; 0, 0.6)$$

Prospect A is preferred due to risk aversion for gains. While both have the same expected change in wealth, A has less risk.

(8.b) Choose between:

$$C = (-\$50, 0.8; 0, 0.2) \text{ and } D = (-\$100, 0.4; 0, 0.6)$$

Prospect D is preferred due to risk seeking in the loss domain

(8.c) Are these choices consistent with expected utility theory? Why or why not?

No, these choices are not consistent with EUT because in EUT risk preference doesn't change because of the change in frame. And, risk preference can either be risk averse, risk seeking, or risk neutral for a range of wealth. It cannot be both at the same time.

9. Consider a person with the following value function under prospect theory:

$$v(x) = \begin{cases} x^{0.5} & , x \geq 0 \\ -2(-x)^{0.5} & , x < 0 \end{cases}$$

(9.a) Is this individual loss-averse? Explain.

This person is loss averse. Losses are felt twice as much as gains of equal magnitude.

(9.b) Assume that this individual weights values by objective probabilities, instead of using a prospect theory weighting function. Which of the following prospects would be preferred?

$$\begin{aligned} P1 &= (1000, 0.8; -800, 0.2) \\ P2 &= (1200, 0.7; -600, 0.3) \\ P3 &= (2000, 0.5; -1000, 0.5) \end{aligned}$$

$$\begin{aligned} V(P1) &= .8(31.62) + .2(-2)(28.27) = 13.982 \\ V(P2) &= .7(34.64) + .3(-2)(24.49) = 9.55 \\ V(P3) &= .5(44.72) + .5(-2)(31.62) = 9.265 \end{aligned}$$

Therefore prospect P1 is preferred.

10. Now consider a person with the following value function under prospect theory:

$$v(x) = \begin{cases} x^{0.8} & , x \geq 0 \\ -3(-x)^{0.8} & , x < 0 \end{cases}$$

This individual has the following weighting function:

$$\pi(p) = \frac{p^{0.65}}{(p^{0.65} + (1-p)^{0.65})^{\frac{1}{0.65}}}$$

(10.a) Which of the following prospects would he choose?

$$A = (-5000, 0.001; 0, 0.999) \text{ and } B = (-5, 1)$$

(10.b) Repeat the calculation but using probabilities instead of weights.

What does this illustrate?

This question deals with probability weighting function and is not covered in the midterm exam.