



EE 320 Introductory Mathematical Economics

Semester 1/2015

Homework 1

Due 8 September 2015

There are five questions in total. Each of them is worth equally.

1. Mr. Push produces shoes for sale, and his total cost function is given by $C(Q) = 105 + 23Q$, where Q is the output level.

- a. (2 points) Suppose that Mr. Push sells his shoes in a perfectly competitive shoe market at the price \$30 per pair. How many pairs of shoes should Mr. Push sell in order to break-even?

Ans. $\pi_{BE} = 30Q - (105 + 23Q) = 0 \rightarrow Q_{BE} = 15.$

- b. (3 points) Suppose now that Mr. Push is the only producer in this market (because his shoes have special designs), and the demand function he faces is $Q_d = 153 - P$. If he is still able to sell his shoes at the break-even amount in the competitive market (i.e. the quantity found in part a.), what is his new profit?

Ans. $Q = 153 - P \rightarrow P = 153 - Q$

$$\pi = (153 - Q)Q - (105 + 23Q)$$

$$\pi = 153Q - Q^2 - 105 - 23Q = -Q^2 + 130Q - 105$$

At $Q = 15$, $\pi = 1620$

2. Consider the following system of equations:

$$\begin{aligned}Q_{d1} &= 33 - 4P_1 - 4P_2 + P_3 & Q_{s1} &= 2 + P_1 \\Q_{d2} &= 18 - 2P_1 - 3P_2 + 4P_3 & Q_{s2} &= 3 + 2P_2 \\Q_{d3} &= 15 + P_1 + 5P_2 - 4P_3 & Q_{s3} &= 7 + 5P_3\end{aligned}$$

a. (2 points) What are the relationships among the three goods?

Ans. Good 1 and good 2 are complements to one another, but they are substitutes for good 3.

b. (3 points) Find the equilibrium price and quantity for the three goods.

$$\begin{aligned}\text{Ans. } (Q_1^*, P_1^*) &= (5, 3) \\(Q_2^*, P_2^*) &= (13, 5) \\(Q_3^*, P_3^*) &= (27, 4)\end{aligned}$$

3. Consider the following IS-LM model:

Commodity market:

$$\begin{aligned}Y &= C + I + G_0 \\C &= C_0 + bY, \quad (C_0 > 0, 0 < b < 1) \\I &= I_0 - ar, \quad (I_0 > 0, a > 0)\end{aligned}$$

Money market:

$$\begin{aligned}M_S &= M_0 \\M_D &= mY - hr, \quad (m > 0, h > 0)\end{aligned}$$

a. (2 points) Write out the explicit IS-LM system of equations, and determine the equilibrium national income and equilibrium interest rate.

$$\begin{aligned}\text{Ans. IS: } Y &= \frac{C_0 + I_0 + G_0 - ar}{(1-b)} \\ \text{LM: } Y &= \frac{M_0 + hr}{m}\end{aligned}$$

$$Y^* = \frac{h(C_0 + I_0 + G_0) + aM_0}{am + h(1-b)} ; \quad r^* = \frac{m(C_0 + I_0 + G_0) - (1-b)M_0}{am + h(1-b)}$$

b. (1 points) Find the impact of an exogenous increase in money supply (M_0) on the equilibrium interest rate found in part (a). Assume everything else remains constant.

Ans.
$$\frac{\Delta r^*}{\Delta M_0} = \frac{-(1-b)}{am + h(1-b)}$$

c. (1 points) Suppose that $C_0 = 250$, $I_0 = 150$, $G_0 = 50$, $b = 0.8$, $a = 1000$, $h = 1500$, $M_0 = 200$, and $m = 0.4$. Find the equilibrium national income and interest rate.

Ans.
$$Y^* = 1250; \quad r^* = 0.2$$

d. (1 points) Based on the information in part (c), if the government expenditure (G_0) increases to 120, all else constant, what is the *change* in the equilibrium national income?

Ans.
$$\frac{\Delta Y^*}{\Delta G_0} = \frac{h}{am + h(1-b)} = \frac{1500}{700} = \frac{15}{7} \approx 2.14$$

If $\Delta G_0 = 70$, then $\Delta Y^* = 150$.

4. Consider a café that serves both pizza and cookies. Suppose the demand curve and supply curve for pizza in a café are given by

Demand: $Q = 300 - 20P - 20P_c$

Supply: $Q = 10P - 10$

where P is the price of a pizza, and P_c is price of a cookie.

Answer the following questions:

a. (3 points) Suppose the price of a cookie is $P_c = 5$. Find the equilibrium price and quantity of pizza.

Given $P_c = 5$, demand equation would be $Q = 300 - 20P - 20 \cdot 5 = 200 - 20P$

Setting demand equal to supply, we obtain that

$$10P - 10 = 200 - 20P \Rightarrow P^* = 7$$

$$Q^* = 10(7) - 10 = 60 \text{ units}$$

- b. (4 points) Due to a change in the price of cookies, the new equilibrium price of pizza changes to $P^* = 5$. What must be the price of a cookie now?

Solve for the solution of equilibrium price/quantity in term of P_c .

$$300 - 20P - 20P_c = 10P - 10 \Rightarrow P^* = \frac{310 - 20P_c}{30} \Rightarrow Q^* = 10 * \left(\frac{310 - 20P_c}{30} \right) - 10.$$

Set $P^* = 5$ and solve for P_c . The answer is that $P_c = 8$.

- c. (3 points) Are pizza and cookies substitutes or complements in this example? Use your economic intuition to justify your answer.

They are complementary product. Under the new equilibrium, quantity is equal to 40 units. As quantity decreases when price of “c” increases, this implies that both products are complementing to each other.

5. Alex, Zander, and Clark are the only three consumers in the market for oatmeal. They have the following individual demand curves for oatmeal. The market supply curve for oatmeal is also provided to you.

$$\begin{array}{ll} \text{Alex:} & P = 2 - Q_d \\ \text{Zander:} & Q_d = 40 - 2P \\ \text{Clark:} & Q_d = 20 - 4P \\ \text{Supply:} & Q_s = \left(\frac{3}{2}\right)P - 12 \end{array}$$

Answer the following questions:

- a. (4 points) Derive equation for market demand. Make sure you correctly specify all the domain sets of variables that make your equation economically meaningful.

First, rewrite demand into Q-equals form.

Alex would be in the market only when price is less than 2.

Zander would be in the market only when price is less than 20.

Clark would be in the market only when price is less than 5.

All the above three number are threshold that when price gradually drops below those number, new person will be attracted into the market.

Summing up Q-equal form of the demand equation for each associated region of prices, we shall obtain that

$$\begin{aligned}Q &= 40 - 2P; & 5 < p \leq 20 \\ &= 40 - 2P + 20 - 4p = 60 - 6p; & 2 < p \leq 5 \\ &= 40 - 2P + 20 - 4p + 2 - p = 62 - 7p; & p \leq 2\end{aligned}$$

- b. (3 points) What is the equilibrium price *and* quantity of oatmeal?

Market supply will intersect with market demand for the part that $5 < p \leq 20$. Setting the equation equal to supply, we yield that $P = \frac{104}{7}$ and $Q = \frac{72}{7}$. Having set supply equation equal to the other two equations would result in p-solution that does not lie in the interval that justify the use of those two equations at the first place.

- c. (3 points) Under equilibrium, are there consumers rationed or excluded from the market?

Yes, all, **except Zander**, are rationed. Producer will only sell to **Zander**.

6. The demand and supply curves in the market for whiskey are given by the following equations:

$$\text{Demand: } Q_d = 120 - 3P$$

$$\text{Supply: } P = 0.5Q_s$$

where Q is the quantity of whiskey in bottles and P is the price per bottle of whiskey.

Answer the following questions:

- a. (1 point) Find the pre-tax equilibrium price and quantity, i.e. (P*,Q*).

Setting demand equal to supply, we obtain that $P^*=24$ and Q^*48 .

- b. (1 point) Calculate the price elasticities of demand and supply at the equilibrium.

elasticity of demand = $-3*(24/48) = -3/2$ and elasticity of supply = $2*(24/48) = 1$

- c. (1 point) Based on the values of calculated elasticities, can you tell us anything about the change in the total revenue if the market price can be increased by 1%?

Demand is elastic. So, it's above the midpoint. If price increases by 1%, quantity would decrease by more than 1%, which implies that total revenue would drop accordingly.

Now suppose that the government in this economy has levied an excise tax of \$5 on the *suppliers* of whiskey.

- d. (2 points) Find the post-tax equilibrium price and quantity.

Tax at the producer would imply that $p_s = p_d - t = p_d - 5$.

$$p_s = p_d - 5$$

$$0.5Q = (\frac{1}{3})*(120-Q) - 5$$

$$Q = 42.$$

$$P_s = 0.5*(42) = 21$$

$$P_d = P_s + 5 = 26.$$

- e. (2 point) How much total revenue can government collect?

$$\text{Tax revenue} = t*Q = 5*42 = 210.$$

- f. (3 point) In terms of economic incidence, what percentage of the tax is actually borne by suppliers?

Initial price was \$24. After imposing \$5 unit tax, $P_s = 21$ and $P_d = 26$. Producers get \$3 less than before. Consumers pay \$2 more than before. These number of tax burdens that each would have to bear to a total of \$5 unit tax. Consumers' burden is then 40% and producers' burden is 60%. This makes sense because consumers' demand is relatively more elastic than producers' supply.