Options on Stock Indices and Currencies

Chapter 13

Index Options

- The most popular underlying indices in the U.S. are
  - The Dow Jones Index times 0.01 (DJX)
  - The Nasdaq 100 Index (NDX)
  - The Russell 2000 Index (RUT)
  - The S&P 100 Index (OEX)
  - The S&P 500 Index (SPX)
- Contracts are on 100 times index; they are settled in cash; OEX is American and the rest are European.

LEAPS

- Leaps are options on stock indices that last up to 3 years
- They have December expiration dates
- They are on 10 times the index
- Leaps also trade on some individual stocks

European Options on Stocks Paying Dividend Yields

We get the same probability distribution for the stock price at time T in each of the following cases:

1. The stock starts at price S_0 and provides a dividend yield = q
2. The stock starts at price S_0e^{-qT} and provides no income

European Options on Stocks Paying Dividend Yield continued

We can value European options by reducing the stock price to $S_0 e^{-qT}$ and then behaving as though there is no dividend

Extension of Chapter 12 Results (Equations 13.4 and 13.5)

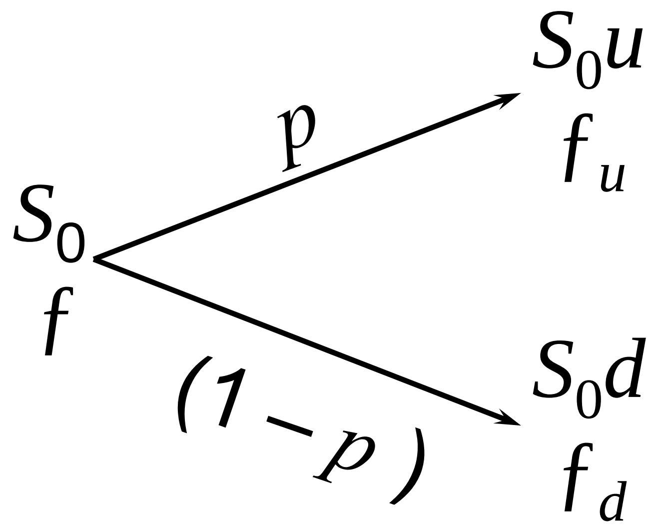
$$c = S_0 e^{-qT} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-qT} N(-d_1)$$

where $$d_1 = \frac{\ln(S_0/K) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0/K) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}}$$

The Binomial Model



$$F = e^{-rT}[pf_u + (1-p)f_d]$$

The Binomial Model

- In a risk-neutral world the stock price grows at $r-q$ rather than at r when there is a dividend yield at rate q
- The probability, p, of an up movement must therefore satisfy

$$pS_0u+(1-p)S_0d=S_0e^{(r-q)T}$$

so that

$$p=\frac{e^{(r-q)T}-d}{u-d}$$

Currency Options

- Currency options trade on the Philadelphia Exchange (PHLX)
- There also exists an active over-the-counter (OTC) market
- Currency options are used by corporations to buy insurance when they have an FX exposure

The Foreign Interest Rate

- We denote the foreign interest rate by r_f
- When a U.S. company buys one unit of the foreign currency it has an investment of S_0 dollars
- The return from investing at the foreign rate is $r_f S_0$ dollars
- This shows that the foreign currency provides a "dividend yield" at rate r_f

Valuing European Currency Options

- A foreign currency is an asset that provides a continuous "dividend yield" equal to r_f
- We can use the formula for an option on a stock paying a continuous dividend yield :

  Set S_0 = current exchange rate

  Set $q = r_f$

Formulas for European Currency Options (Equations 13.9 and 13.10, page 297)

$$c = S_0 e^{-r_f T} N(d_1) - K e^{-rT} N(d_2)$$

$$p = K e^{-rT} N(-d_2) - S_0 e^{-r_f T} N(-d_1)$$

where $$d_1 = \frac{\ln(S_0 / K) + (r - r_f + \sigma^2 / 2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S_0 / K) + (r - r_f - \sigma^2 / 2)T}{\sigma\sqrt{T}}$$